Microelectromechanical Systems (MEMs) Physical Fundamentals - Part 1 - Mechanical Fundamentals

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OUTLINE

Force
Pressure
Stress, Strain and Hooke's Law
Poisson's Ratio
Simple Cantilever
Springs
Thermal Strain
Bi-metalic Actuator
Diaphragm

FORCE

F = ma (newton)

where **m** is mass in Kg

a is acceleration in m/sec^2

gravitational acceleration is 9.8 m/sec^2

newton (N) = 1 Kg 1 meter/s^2
dyne = 1 gm 1 cm/s^2

How do you convert dynes to newtons?

$$\frac{N}{dyne} = \frac{1000gm\ 100cm / s^2}{1gm\ \ 1\ cm / s^2}$$

so 1 newton = 10^5 dynes

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PRESSURE

Pressure is used to describe force per unit area.

$$P = F/A$$

Pressure due to the weight of liquid

$P = h\ d\ g$ where h is the height
d is the density of the liquid
g is acceleration due to gravity
980 cm/sec2

PRESSURE UNITS

Table of Pressure Conversions

1 atm = 14.696 lbs/in^2 = 760.00 mmHg

1 atm = 101.32 kPa = 1.013×10^6 $dynes/cm^2$

1Pascal = 1.4504×10^{-4} lbs/in^2 =1 N/m^2

1 mmHg = 1 Torr (at 0°C)

1SPL (Sound Pressure Levels) = 0.0002 $dynes/cm^2$

Average speech = 70 dB_{SPL} = 0.645 $dynes/cm^2$

Pain = 130 dB_{SPL} = 645 $dyne/cm^2$

Whisper = 18 dB_{SPL} = 1.62×10^{-3} $dyne/cm^2$

Example:

$$5\,mmHg \times \frac{14.696\ lbs/in^2}{760.00\ mmHg} = 0.0967\ lbs/in^2 = 0.00658\ atm$$

$$= 666.60\ Pascals = 5\ Torr$$

GAUGE PRESSURE – ABSOLUTE PRESSURE

Gauge pressure is a differential pressure measured relative to an ambient pressure. The output of a gauge pressure sensor is not sensitive to a changing barometric pressure

Absolute pressure is differential pressure measured relative to an absolute zero pressure (perfect vacuum). The output of the absolute sensor will change as a result of barometric pressure change and thus can be used as a barometer.

HOOKE'S LAW

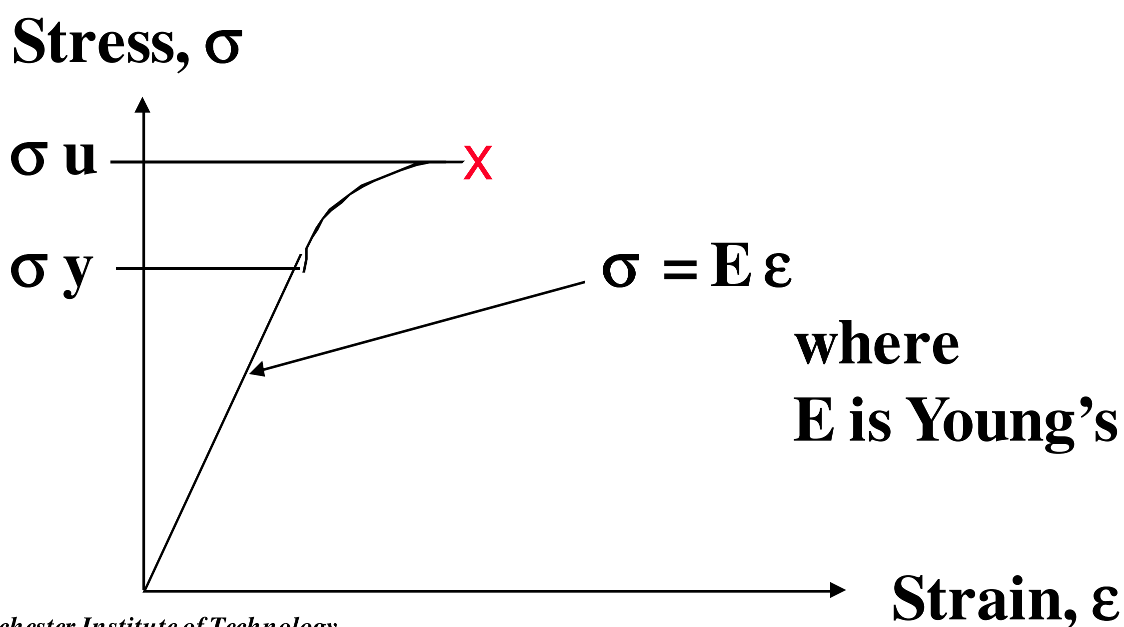
Stress is the applied force per crossectional area, $\sigma = F/A$

Strain is the elongation per unit length, $\varepsilon = \delta L/L$

σy = yield strength

σu = ultimate or maximum strength

σb = rupture strength (for brittle material $\sigma u = \sigma b$)



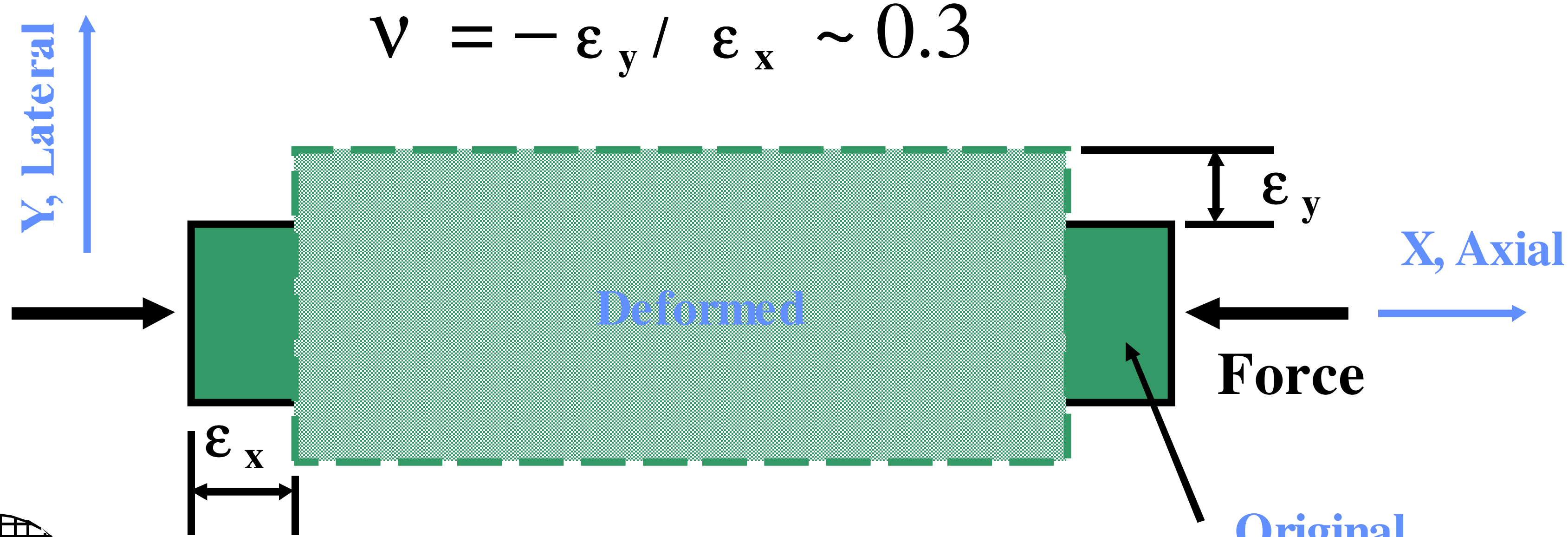
where

E is Young's modulus

POISSON'S RATIO

Poisson's ratio (ν) gives the relationship between lateral strain and axial strain. In all engineering materials the elongation produced by an axial tensile force in the direction of the force is accompanied by a contraction in any transverse direction.

$$\nu = -\varepsilon_y / \varepsilon_x \sim 0.3$$



PROPERTIES OF SOME MATERIALS

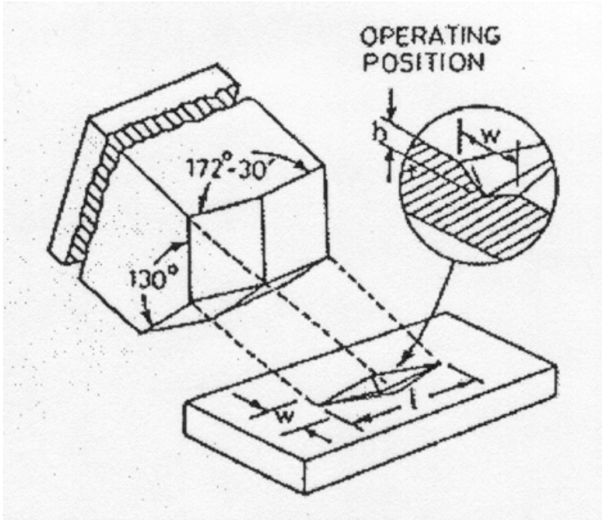
	σy Yield Strength (10^{10} dyne/cm2)	σu Ultimate Strength (10^{10} dyne/cm2)	Knoop Hardness (Kg/mm2)	E Youngs Modulus (10^{12}dyne/cm2)	d Density (gr/cm3)	Thermal Conductivity (w/cm°C)	Thermal Expansion (10^{-6}/°C)	ν Poisso's Ratio
Si3N4	14	28	3486	3.85	3.1	0.19	0.8	0.3
SiO2	8.4	16	820	0.73	2.5	0.014	0.55	0.3
Si	12	14	850	1.9	2.3	1.57	2.33	0.32
Al	0.17		130	0.68	2.7	2.36	25	0.334

10 dyne/cm^2 = 1 newton/m^2

HARDNESS

Hardness is a well known physical parameter, but many different methods have been derived for the measurement and classification of materials on a hardness scale. The Knoop scale is the most commonly used, others being: Moh, Vickers, Rockwell and Brinell.

The experimental procedure for the derivation of a value on the Knoop scale is to use a pyramidal diamond point which is pressed into the material in question with a known force. The indentation made by the point is then measured and the Knoop number calculated from this measurement. A soft material would have a Knoop number of 4, whereas a hard material like sapphire has a Knoop number of 2000 and the Knoop number for diamond is 7000.



STRESS - COMPRESSIVE, TENSILE

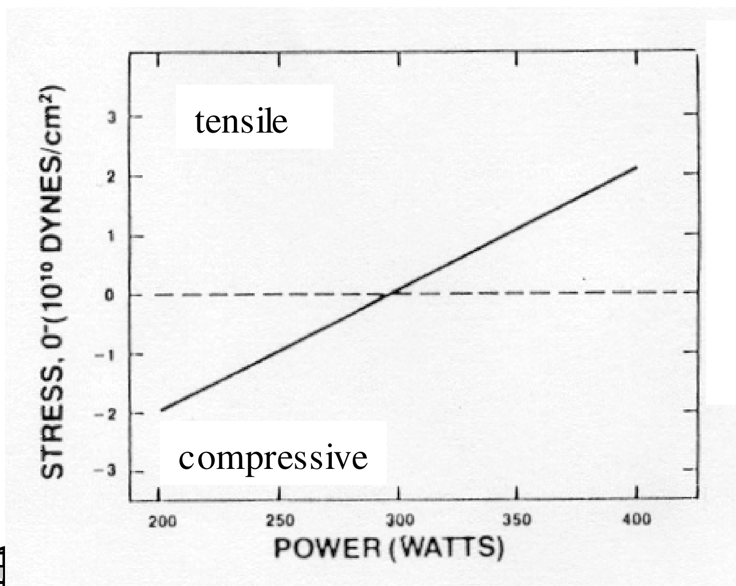
Stress is the force per unit area applied to a mechanical member (Pressure)

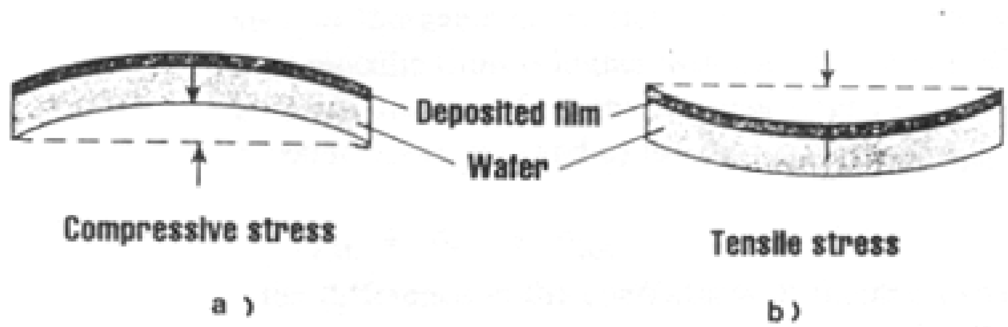
A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression)

These forces can be axial, transverse, oblique, shearing, torsional, thermal, etc

STRESS IN SPUTTERED FILMS

Compressively stressed films would like to expand parallel to the substrate surface, and in the extreme, films in compressive stress will buckle up on the substrate. Films in tensile stress, on the other hand, would like to contract parallel to the substrate, and may crack if their elastic limits are exceeded. In general stresses in films range from 1E8 to 5E10 dynes/cm2.





For AVT sputtered oxide films Dr. Grande found Compressive 18MPa stress, 1-29-2000

10 dyne/cm^2 = 1 newton/m^2 = 1 Pascal

MEASUREMENT OF STRESS IN Si3N4

Kenneth L. Way, Jr. did his senior project on stress in silicon nitride films as a function of the ratio of ammonia to dichlorosilane. Samples were coated with various flows and stress was measured at ADE corporation. The silicon nitride was etched off of the backside of the wafer so that the stress curvature was due to the layer on the front side only. Dr. Lane said the nitride runs at 1:10 (ammonia:dichlorosilane) ratios are rough on the pumping system.

Dr. Grande sent samples to Kodak for stress measurement. He found stress of 900 MPa Tensile for the standard Nitride recipe for 1500 A thickness, 1-29-2000

10 dyne/cm^2 = 1 newton/m^2 = 1 Pascal

LOW STRESS SILICON RICH Si3N4

ADE Measured stress for various Ammonia:Dichlorosilane Flow Ratios

Flow	Stress x E 9 dynes/cm2
10:1	+ 14.63
5:1	+ 14.81
2.5:1	+ 12.47
1:1	+ 10.13
1:2.5	+ 7.79*
1:5	+ 3
1:10	0

*standard recipe

Stress; $\sigma = (E/(6(1-v)))*(D^2/(rt))$

where E is Youngs modulus,

v is Poissons ratio,

D and t are substrate and film thickness

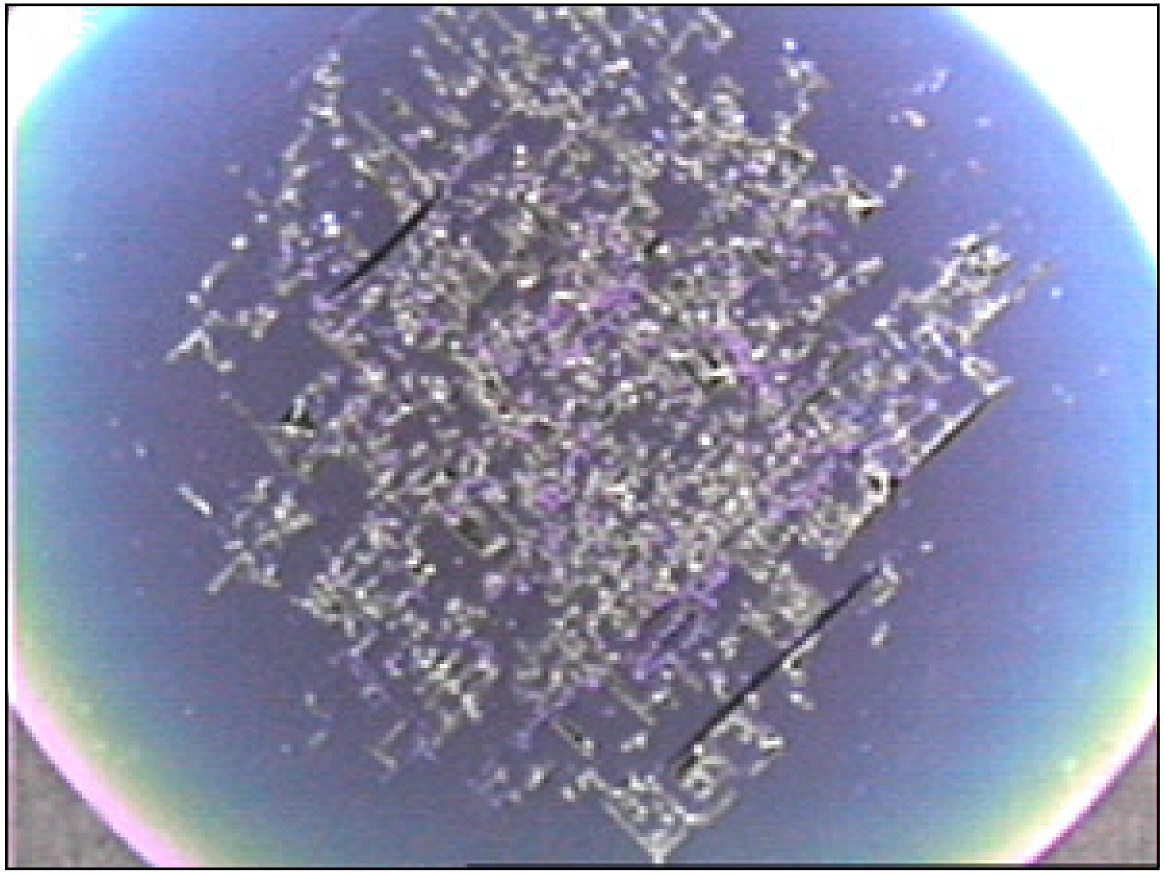
r is radius of curvature (+ for tensile)

T.H Wu, "Stress in PSG and Nitride Films as Related to Film Properties and Annealing", Solid State Technology, p 65-71,May '92

10 dyne/cm^2 = 1 newton/m^2 = 1 Pascal

STRESS IN SILICON NITRIDE FILMS

Stress in an 8000 A Nitride Film



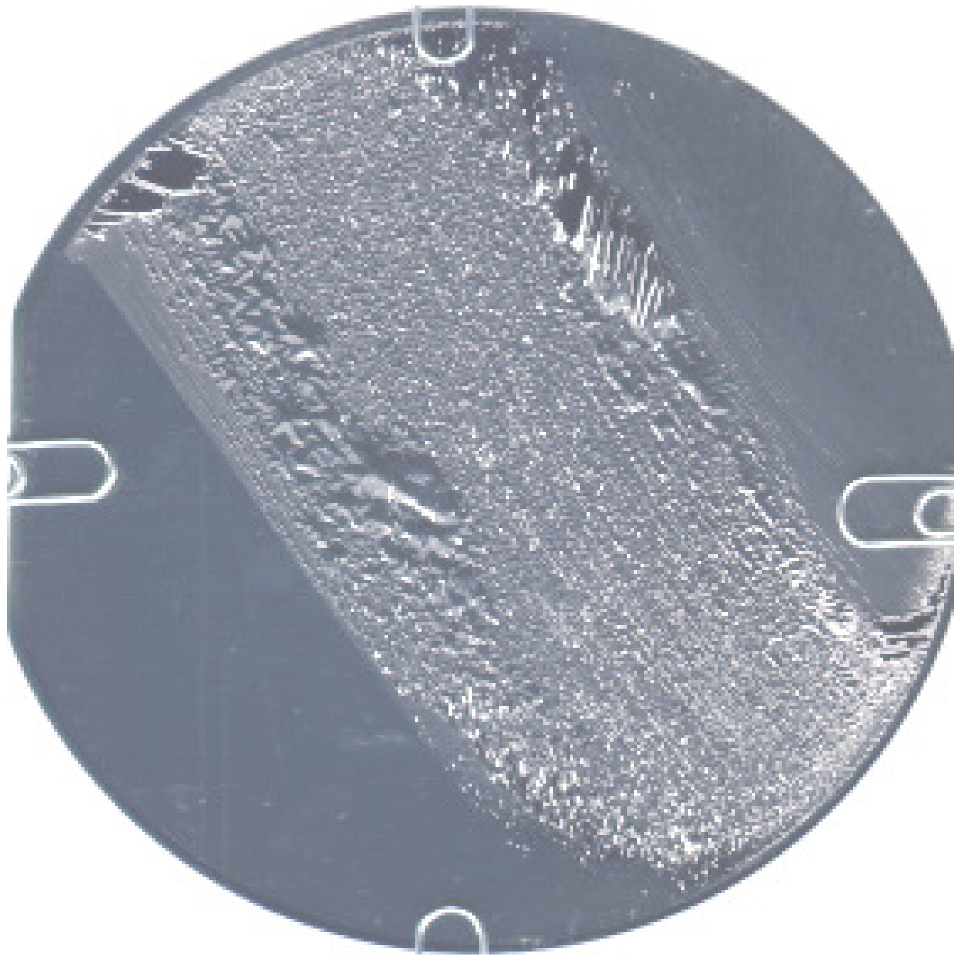
Tensile Stress

STRESS IN SPUTTERED TUNGSTEN FILMS

Tungsten

- CVC 601
- 4" Target
- 500 Watts
- 50 minutes
- 5 mTorr Argon
- Thickness ~ 0.8 µm

Compressive Stress



Picture from scanner in gowning

EXAMPLE CALCULATION

1. Compare the ADE measured stress for standard nitride to the Dr. Grande measured value.

    7.79E9 dynes/cm2 to 900 Mpa

    7790 E6 dynes/cm2

    But 10 dynes/cm2 = 1 Pascal

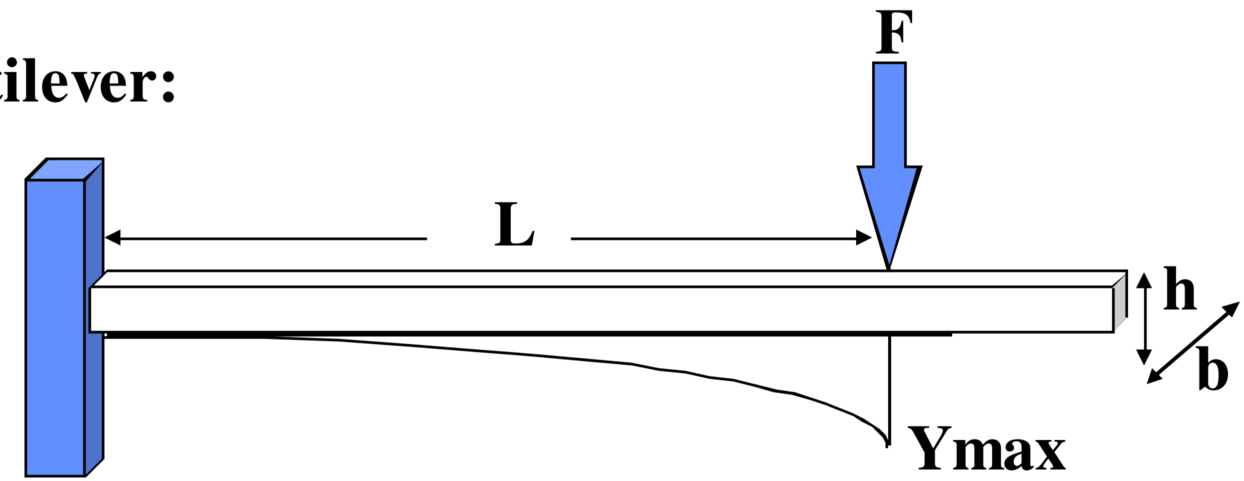
    So 779 Mpa = ~ 900 Mpa

2. Estimate the height difference for a 1mm length near the center of a wafer caused by the resulting curvature from stress for the standard 1500 A nitride deposition after removing nitride from one side of the wafer.

Answer ~ 10 um

SIMPLE CANTILEVER

Cantilever:



$$Ymax = F\,L^3/3EI$$

where E = Youngs Modulus
and $I = bh^3/12$, moment of inertia

Mechanics of Materials, by Ferdinand P. Beer, E. Russell Johnston, Jr., McGraw-Hill Book Co.1981

FORCE TO BEND CANTILEVER EXAMPLE

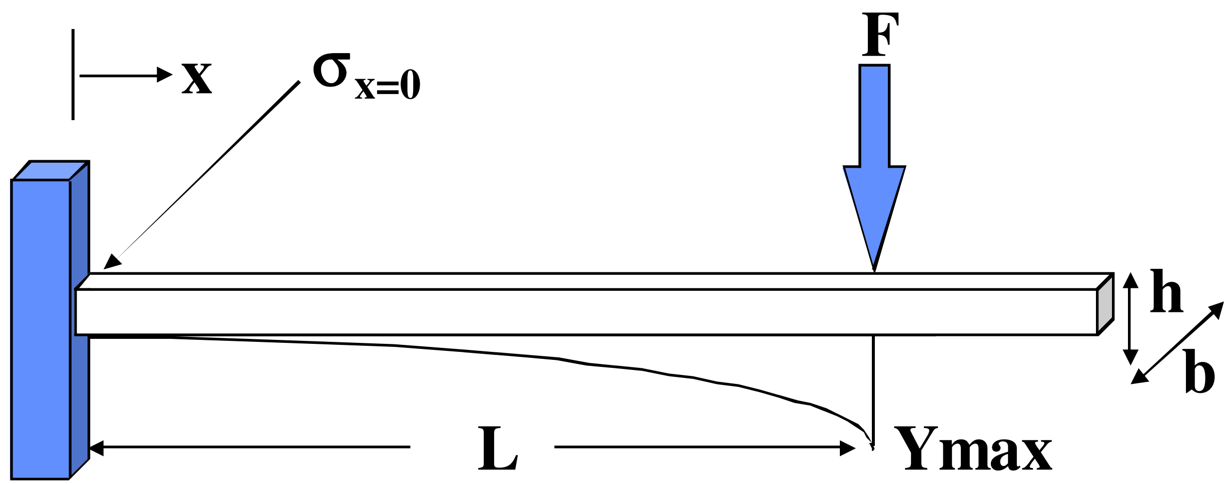
Example: Find the force needed to bend a simple polysilicon cantilever 1 µm, with the following parameters: b=4 µm, h=2µm, L=100 µm

$$F = \frac{Ymax\ 3\ E\ bh^3}{12L^3}$$

$$F = \frac{(1e\text{-}6)\ 3\ (\ 1.9e11)(4e\text{-}6)(2e\text{-}6)^3}{12\ (100e\text{-}6)^3}$$

F = 1.5e-6 newtons

STRESS IN A CANTILEVER BEAM

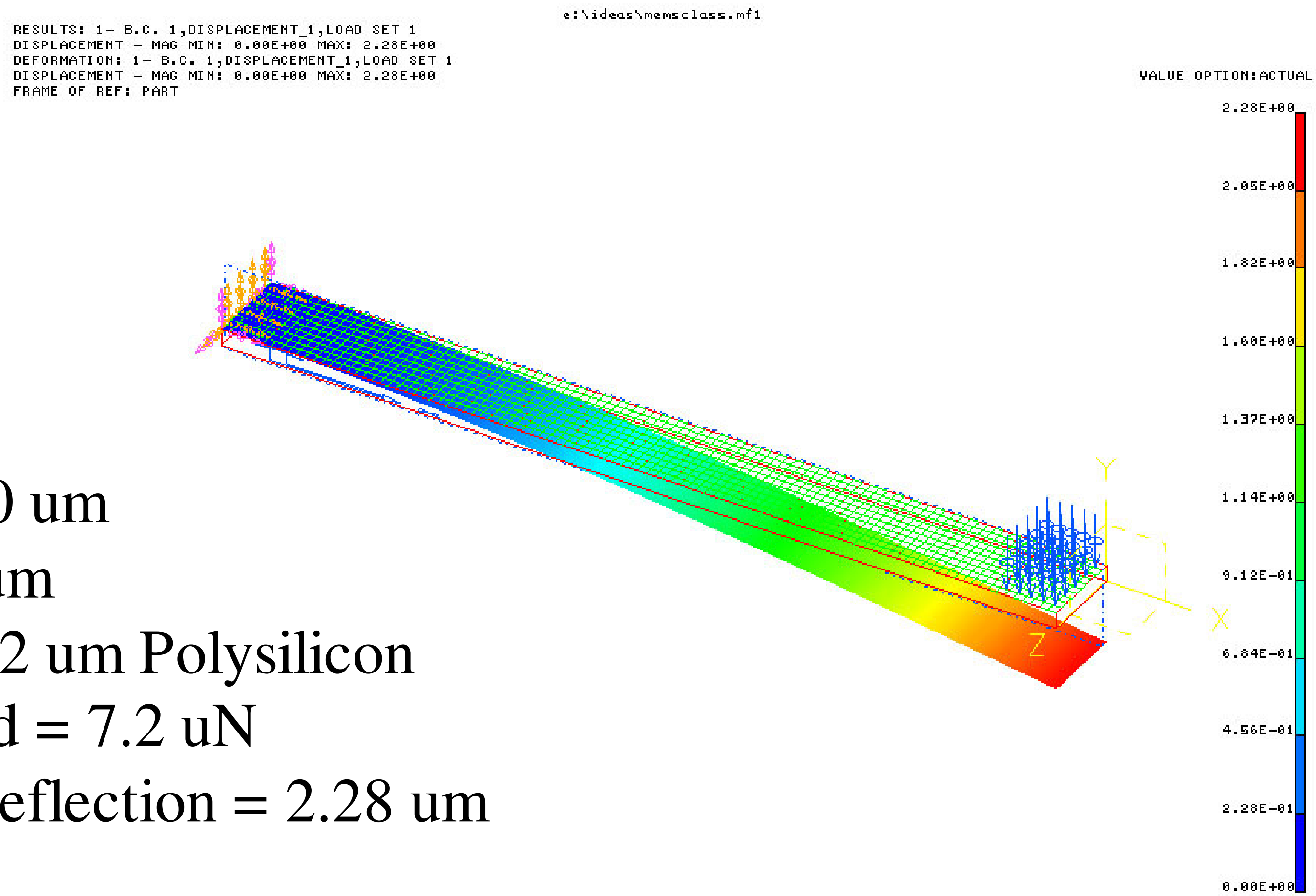


The maximum stress is at the top surface of the cantilever beam at the anchor where x=0

$\sigma_{x=0} = F\,Lh/2I$ where $I = bh^3/12$, moment of inertia

Mechanics of Materials, by Ferdinand P. Beer, E. Russell Johnston, Jr., McGraw-Hill Book Co.1981

FINITE ELEMENT ANALYSIS



IDEAS
Length = 100 um
Width = 10 um
Thickness = 2 um Polysilicon
Force applied = 7.2 uN
Maximum Deflection = 2.28 um

STRESS IN CANTILEVER BEAM EXAMPLE

Example: Find the maximum stress in a simple polysilicon cantilever with the following parameters. Ymax = 1 µm, b=4 µm, h=2µm, L=100 µm

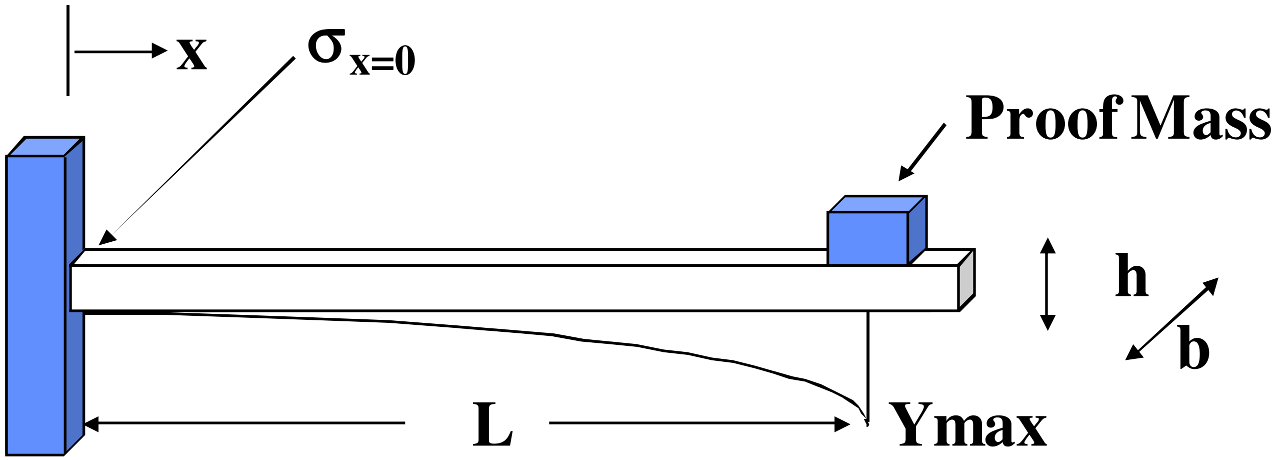
$$\sigma_{x=0} = \frac{12\ F\ L}{2b\ h^2}$$

$$\sigma_{x=0} = \frac{12\ (1.4e\text{-}6)\ (100e\text{-}6)}{2\ (4e\text{-}6)\ (2e\text{-}6)^2}$$

$$\sigma_{x=0} = 5.6e7 \text{ newton/m2} = 5.6e8 \text{ dyne/cm2}$$

Compare to the given table value for yield strength of 7e10 dyne/cm2

ACCELEROMETER - EXAMPLE



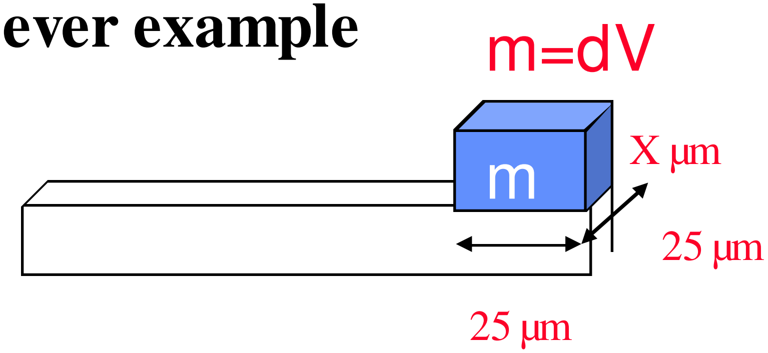
The proof mass will create a force in the presence of an acceleration, the strain can be measured with a piezoresistive device or the position can be measured with a capacitive measurement device

ACCELEROMETER-EXAMPLE

Example: from previous cantilever example

$$F = \frac{Ymax\ 3\ E\ bh^3}{12L^3}$$

F = 1.5e-6 newtons



Find height X for a proof mass of volume (V) of nickel (density (d)=8.91 gm/cm3) and maximum acceleration of 50G's

$$F = mA = m\ 50\ (9.8 m/s^2) = 1.5e\text{-}6 \text{ newtons}$$

and find m = 3.1e-9 Kgm = 3.1e-6 gm

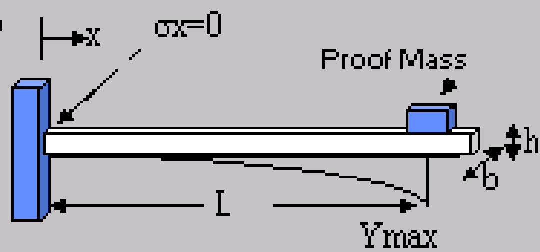
$$m = d\ V = 3.1e\text{-}6 = (8.91)(25e\text{-}4)(25e\text{-}4)(X) \text{ cm}$$

$$X = 0.055 \text{cm} = 550 \mu m$$

Approximately the thickness of a 4" wafer

CANTILEVER.XLS

Rochester Institute of Technology				12/7/2009	
Microelectronic Engineereing				Cantilever.xls	

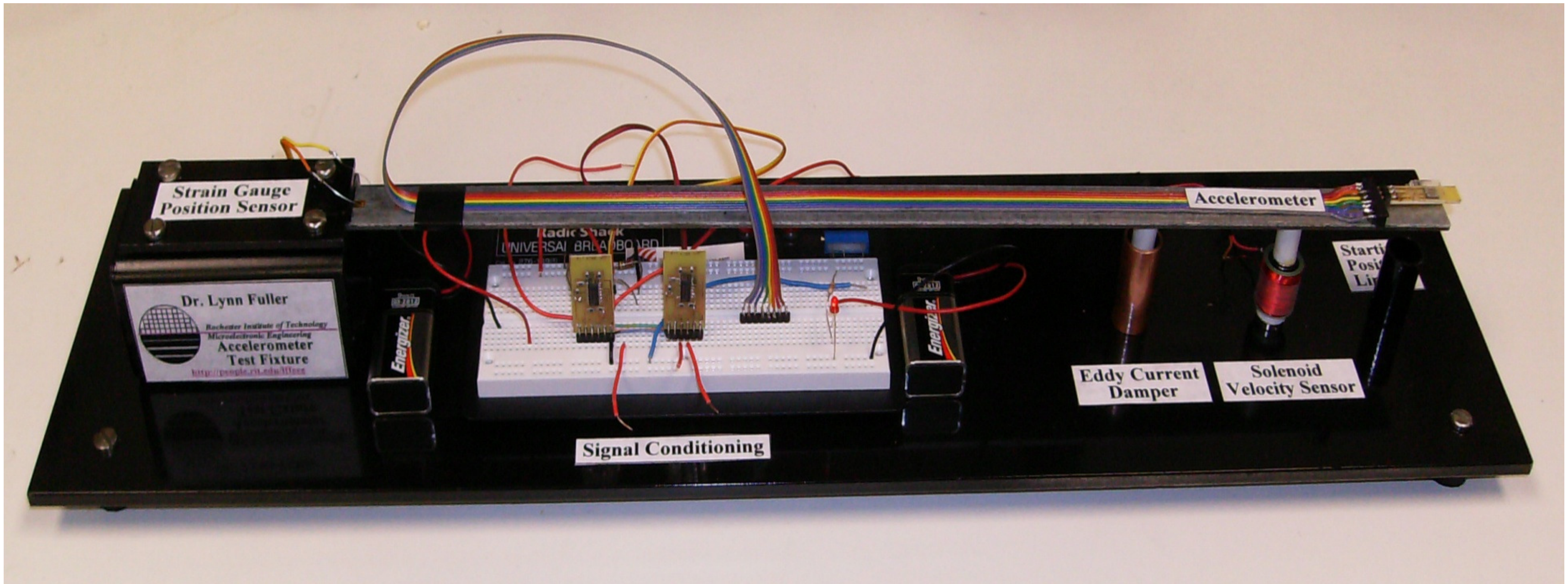


Length	L =	100	um
Width	b =	4	µm
Height	h =	2	µm
Force needed to cause given displacement			
Force	F =	1.52E-06	N
Desired Displacement	Ymax =	1	µm
Mass needed to give same Force at desired acceleration			
Mass	m =	3.10E-09	Kg
Acceleration in G's		50	G's
Acceleration due to gravety		9.8	m/s2
Proof Mass			
Volume needed to give that mass, m = d V	V =	3.48E-13	m3
Proof mass base, bm	bm =	25.00	µm
Proof mass width, wm	wm =	25.00	µm
Height of proof mass at end of beam, hm	hm =	*557.04*	µm

Material Properties

	Silicon	Oxide	Nitride		Aluminum	
Youngs Modulus	1.90E+11					N/m2
Density				8.91E+03		Kg/m3
Ultimate Strength						

ACCELEROMETER TEST SET UP



SPREAD SHEET FOR ACCELEROMETER CALCULATIONS

ROCHESTER INSTITUTE OF TECHNOLOGY 6/13/07

MICROELECTROMECHANICAL SYSTEMS accelerometer.xls

Dr. Lynn Fuller, Burak Baylav

To use this spreadsheet change the values in the white boxes. The rest of the sheet is protected and should not be changed unless you are sure of the consequences. The calculated results are shown in the purple boxes.

Calculations for Cantilever Beam (Test Fixture)

Frequency of Oscillation:

f0 = 1/2π {3EI / (L^3(m+0.236mB))}0.5

I = b h^3

The resonant frequency f0 of the cantilever beam	13.48	hz
Beam Length, L	0.4	m
Beam Width, b	0.025	m
Beam thickness, h	0.003	m
End Mass, mB	0	Kg
Density for Material Selected Below	2.33	gm/cm3
Beam Mass = density x volume, m	69.9	gm
Young's Modulus for Material Selected Below	1.90E+11	N/m2

Acceleration versus time:

a = d2X(t)/dt2 = Ao (2 π f0)^2 cos (2 π f0 t)

Initial Deflection	2	cm
Maximum Acceleration	143.34	m/s2
Maximum Acceleration, in g's	14.63	g's

Calculations for RIT Accelerometer

Force:

F = Fmax cos(2*p*f0*t)

Maximum force due to proof mass, Fmax = ma	8.35E-05	N
end mass on RIT accelerometer, m = d V	5.82E-04	gm
volume, V = 4/3 π r^3	6.54E-05	cm^3
radius, r =	250	µm

SPREAD SHEET FOR ACCELEROMETER CALCULATIONS

Accelerometer, cantilever beam bending

Ymax = F L^3/3EI

I = b h^3

σ max = F L h / 2I

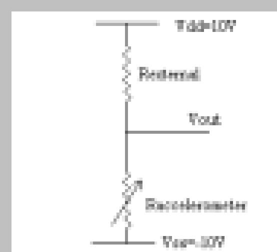
Ymax =	7.03E-01	µm
L =	2000	µm
b =	200	µm
h =	10	µm
Young's Modulus, E =	1.90E+11	N/m2
Max Stress, σ max =	5.01E+03	N/m2
Strain, ε = σ/E =	2.64E-08	µm/µm

Resistor Calculations

R = Rhos L/W

R' = R + Rhos (L+ L*ε)/W

Nominal Resistance, R =	5400	ohms
Measured Sheet Resistance, Rhos =	60	ohms
Resistor Length, L =	900	µm
Resistor Width, W =	10	µm
Resistance under Max Stress, R' =	5400.0001	ohms
Delta R = R' - R =	0.000142375	ohms
Vdd = Vss =	10	volts
Rexternal =	5400	ohms
Nominal Vout =	0	volts
Vout Max =	1.31829E-07	volts

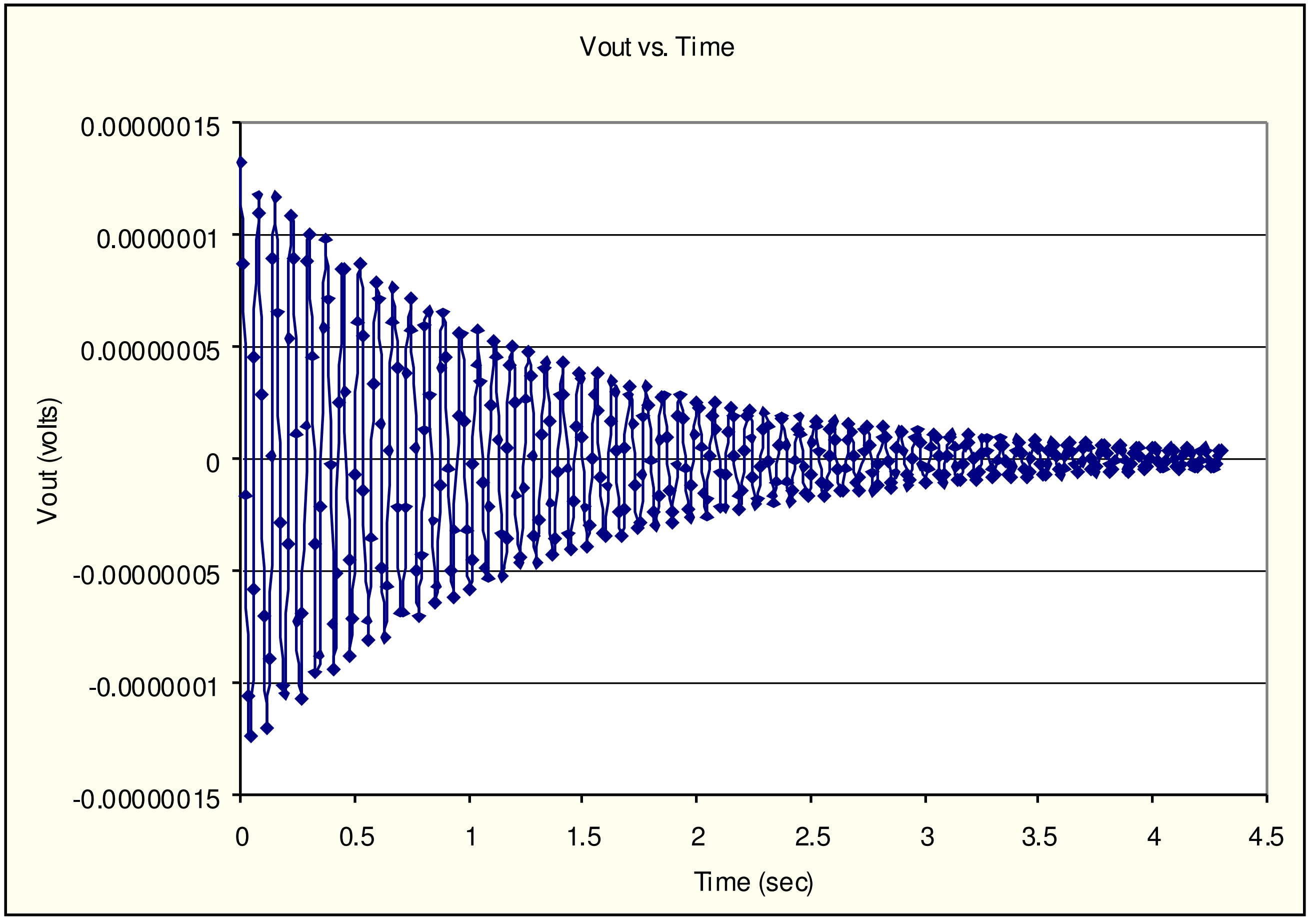


Constants:

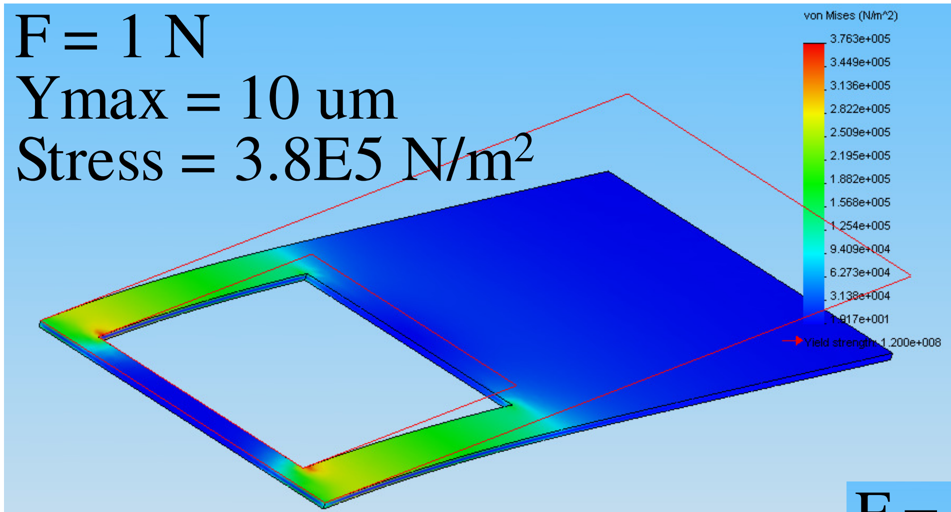
Select Material Using 1 or 0

	Youngs Modulus(N/m2)	**Density (gm/cm3)**	**Fixture**	**RIT Accerometer**
Silicon	1.9E+11	2.33	0	1
Silicondioxide	7.30E+10	2.19	0	0
Silicon Nitride	3.85E+11	3.44	0	0
Aluminum	6.80E+10	3.9	0	0
Steel	2.00E+11	7.8	1	0
Brass	1.10E+11	8.55	0	0
Lead		11.34		

CALCULATED PLOT OF VOUT VS. TIME

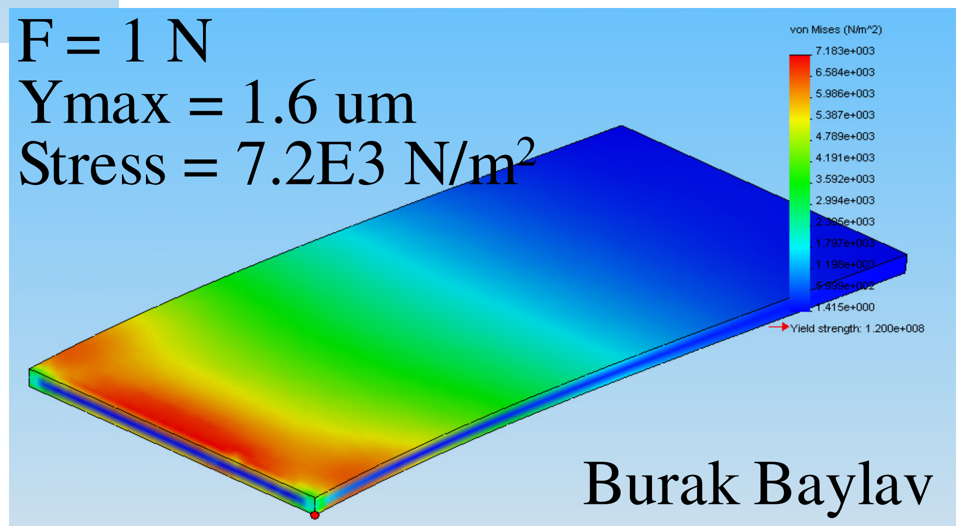


FINITE ELEMENT ANALYSIS (FEA) OF CANTILEVER



SolidWorks

Length = 1500 µm
Width = 600 µm
Thickness = 20 µm
Window ~ 300 x 300 µm



SPRINGS

F = k y

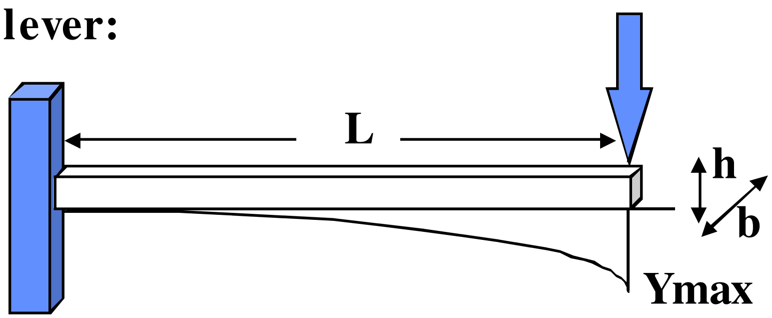
where k is the spring constant and y is the displacement

Analogy: I = G V

k is like conductance (G)

I is Force, V is potential

Cantilever:



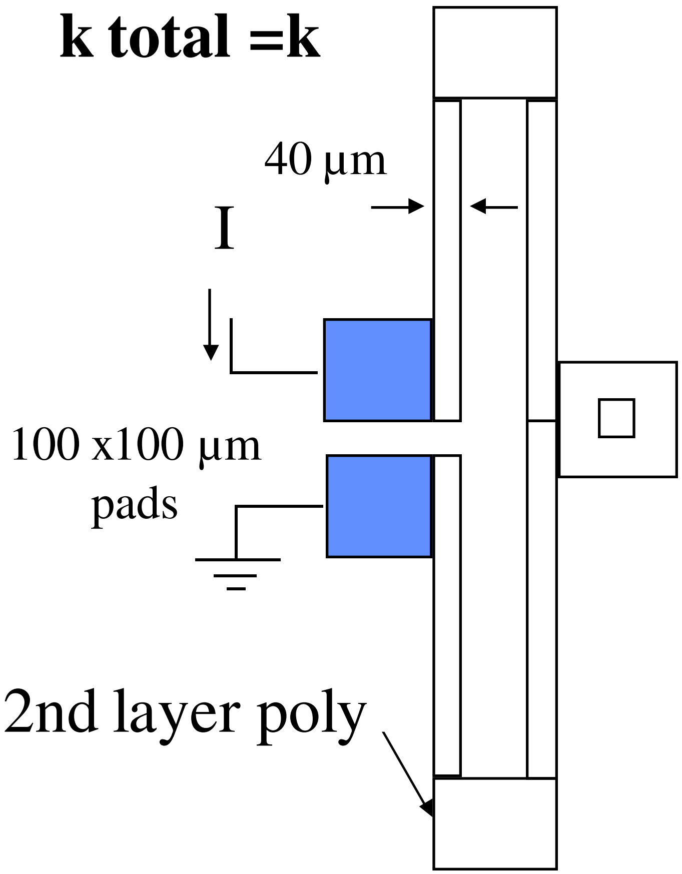
Ymax = F L³/3EI

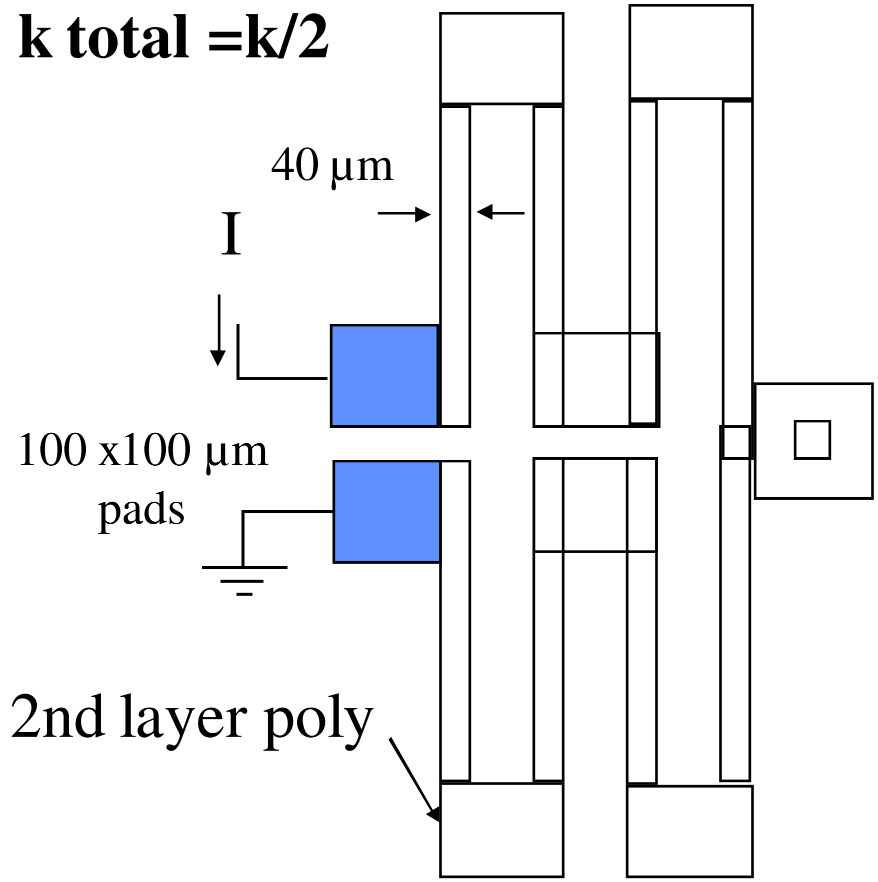
F = k y

F = (3E(bh³/12)/ L³) y

where E = Youngs Modulus
and I = bh³/12, moment of inertia

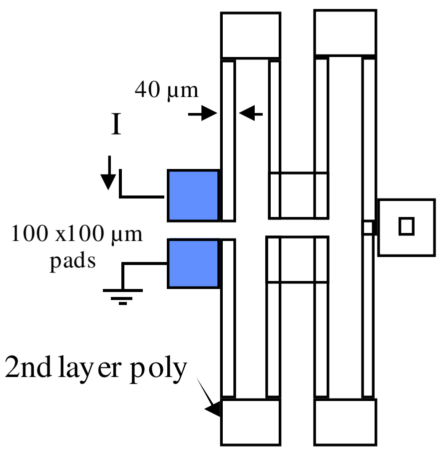
FOLDED SPRING



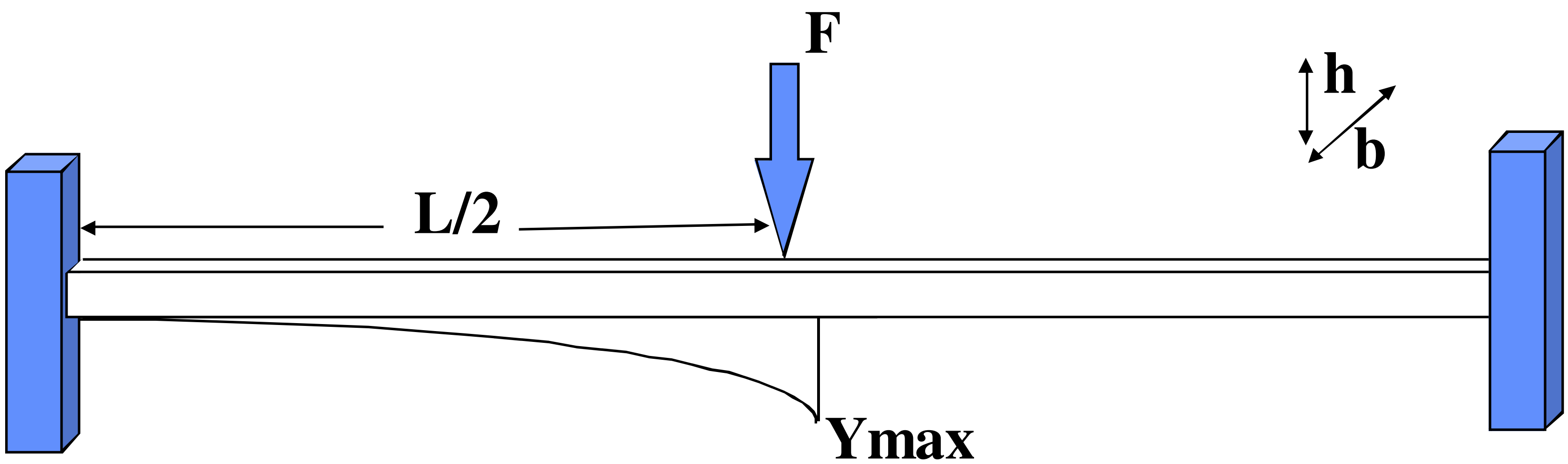


FOLDED SPRING EXAMPLE

Example: Calculate the force needed to elongate the spring shown by 50 µm. Assume each arm of the spring is 400 µm in length and the polysilicon thickness is 2 µm.



BEAM ANCHORED AT BOTH ENDS

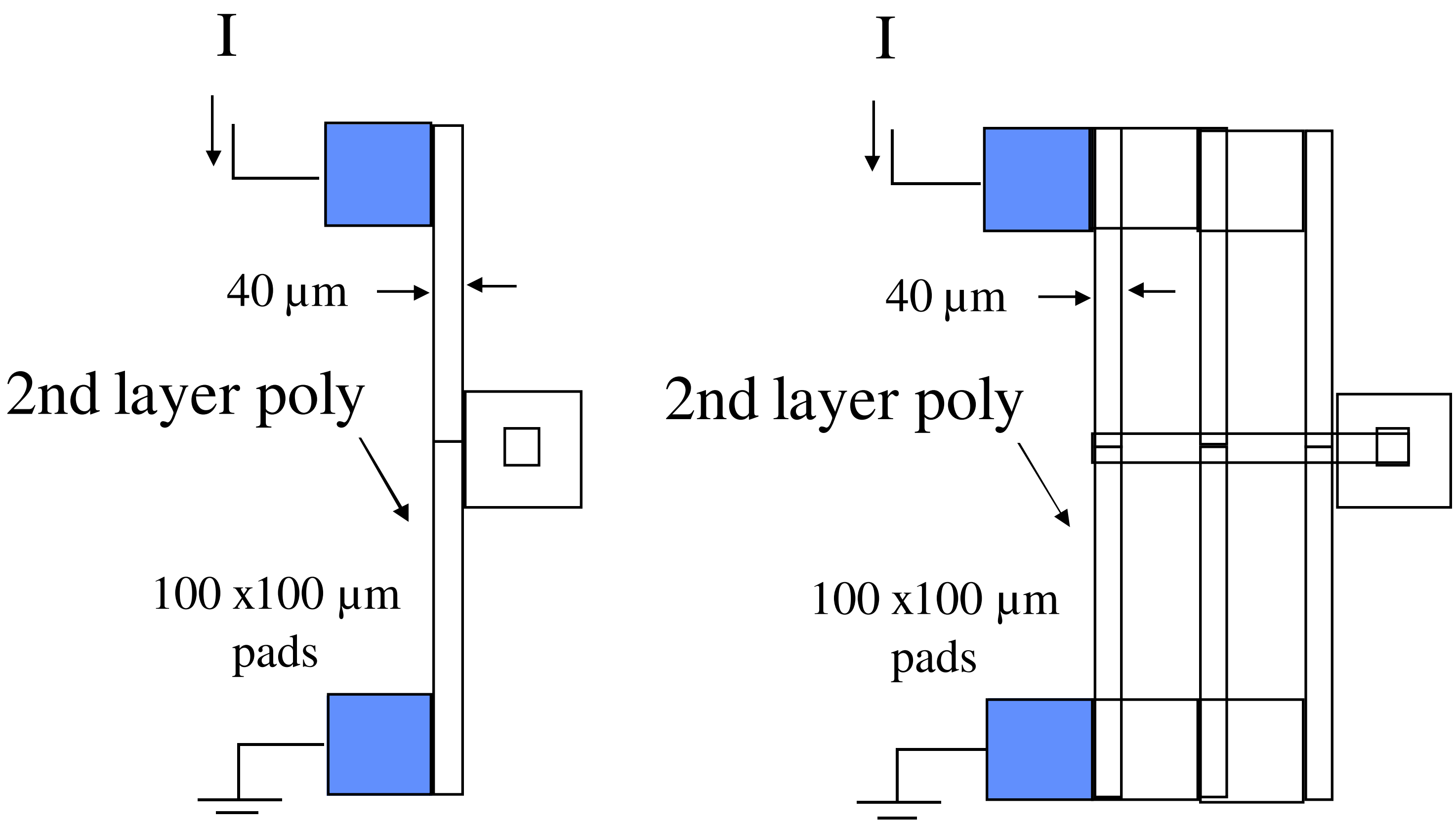


$$Ymax = F\,L^3/48EI$$

where E = Youngs Modulus
and $I = bh^3/12$, moment of inertia

Mechanics of Materials, by Ferdinand P. Beer,
E. Russell Johnston, Jr., McGraw-Hill Book Co.1981

SPRING ANCHORED AT BOTH ENDS



THERMAL STRAIN

The fractional change in length due to a change in temperature is given by:

$$\Delta L/L = \alpha\,(\Delta T)$$

where α is the coefficient of thermal expansion

This is also the thermal strain ε_T. In this case the strain does not cause a stress unless the material is confined in some way

BI METALIC ACTUATORS

Consider two materials bonded together forming a composite beam. Since the materials are bonded together the change in length for both materials is equal (at boundary)

- $\Delta L1 = \Delta L2$

each material experiences a thermal strain plus a strain due to the stress caused by the different thermal coefficients,

$$\Delta L/L = \alpha T + \sigma/E$$

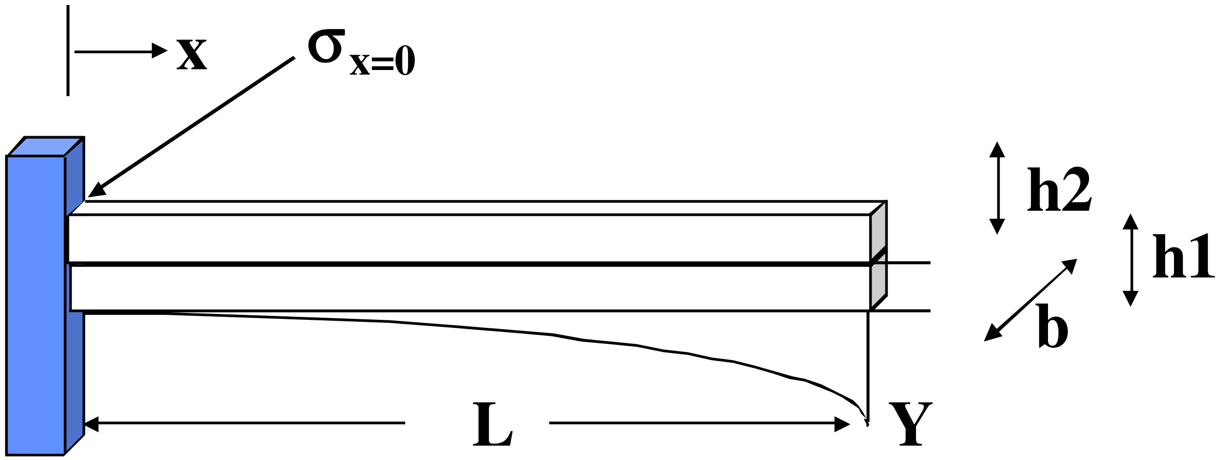
The material with the larger thermal coefficient will experience a compressive stress (-) while the other material will experience a tensile (+) stress.

This results in:

$$\alpha_1 T + \sigma_1/E_1 = \alpha_2 T + \sigma_2/E_2$$

$$\text{and} \quad \sigma_1 = -\sigma_2$$

BI METALIC ACTUATORS

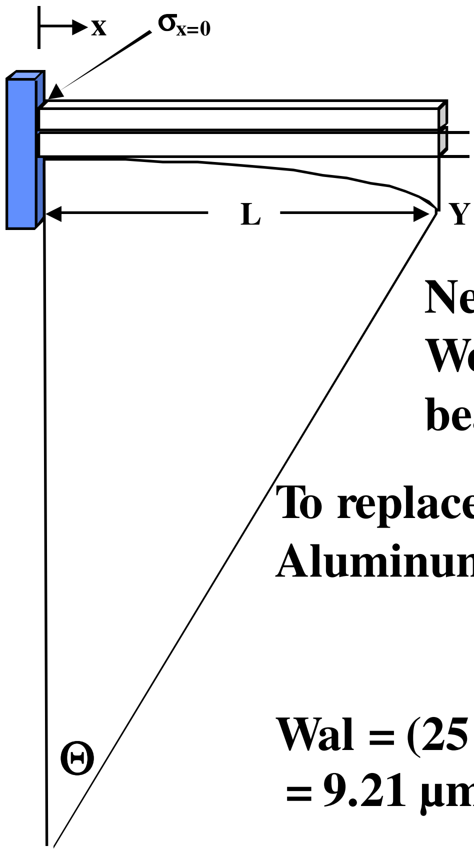


$$\sigma_1\, T - Y\,(3/2)\,(E_1^2\, h_1)/(2L^2) = \sigma_2\, T + Y\,(3/2)\,(E_2^2\, h_2)/(2L^2)$$

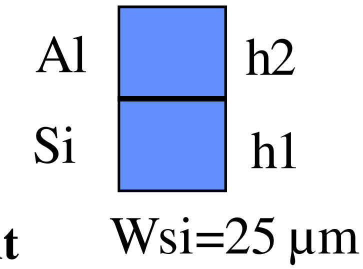
we know all quantities and can solve for Y

EXAMPLE - BI METALIC ACTUATORS

First find the radius of curvature to achieve a displacement of 1 µm



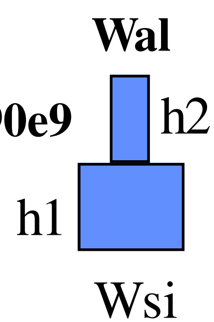
Tan Θ = Y/L and
the circumference C =2Πr
and the ratio of: L/C = Θ/360°
gives r=6.258 cm



Next we look at the composite structure
We need to find the centroid of an equivalent beam composed of only one of the two materials

To replace the Aluminum with silicon we multiply the width of the Aluminum by the ratio of the modulus of Al to the modulus of Si

Wal = (25 µm)70e9/190e9
= 9.21 µm



the centroid of this structure (with respect to the junction) is

$$\Psi = \frac{(Wsi/2)h1^2 - (Wal/2)\ h2^2}{(Wsi/2)h1 + (Wal/2)\ h2}$$

EXAMPLE - BI METALIC ACTUATORS

The centroid is the neutral axis of the composite beam. The length of the neutral axis never changes, $\sigma_{NA} = 0$, Y is the distance from the neutral axis to the junction thus Y and Ψ are equivalent.

Total Stress = Thermal Stress + Mechanical Stress

$$\text{Thermal Stress} = \alpha \Delta T\, E$$

$$\text{Mechanical Stress} = \varepsilon\, E \text{ and } \varepsilon = Y/r$$

Stress in Si must equal the Stress in Al

$$\alpha 1 \Delta T\, E1 + \varepsilon\, E1 = \alpha 2 \Delta T\, E2 + \varepsilon\, E2$$

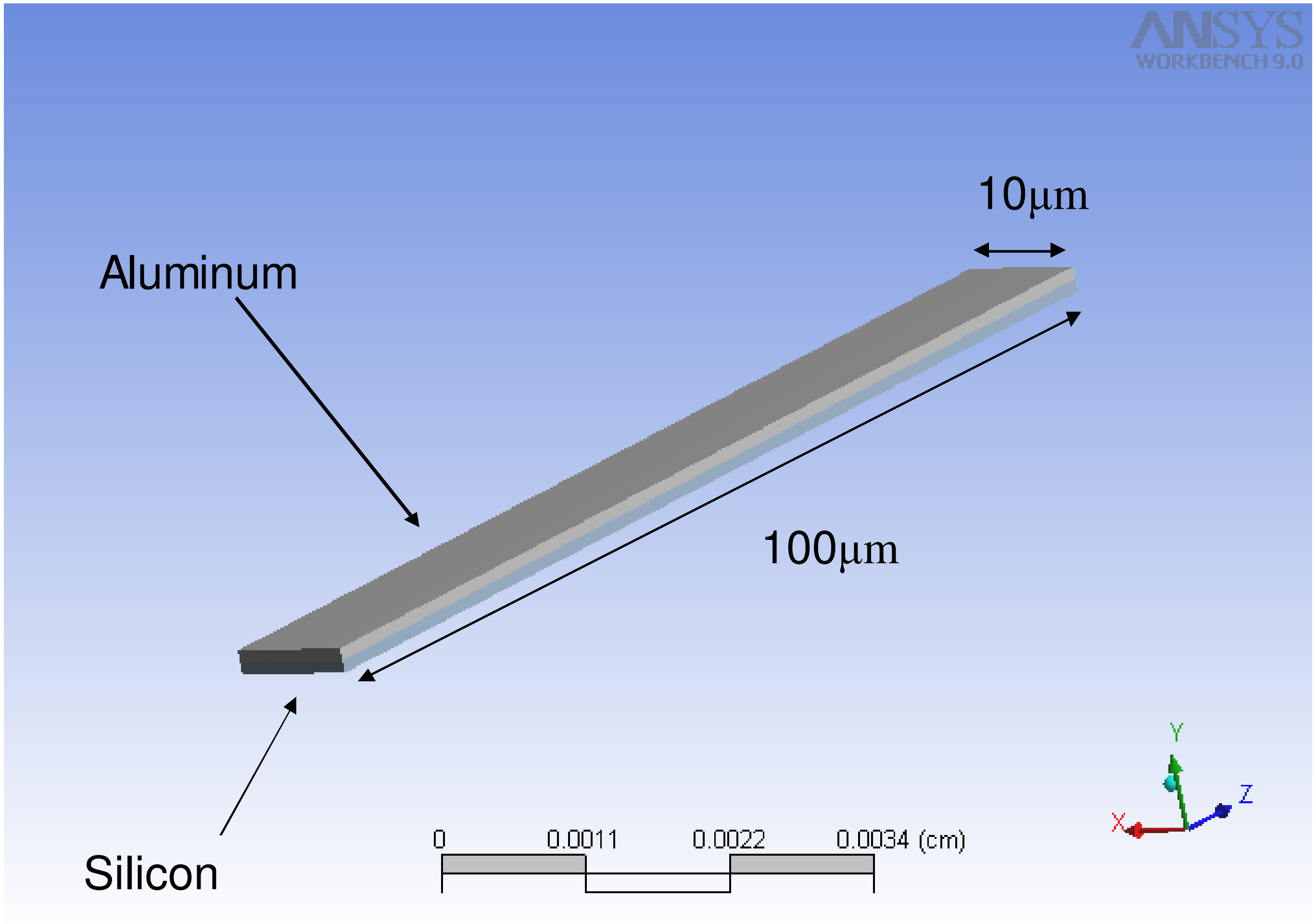
$$\alpha 1 \Delta T\, E1 + (Y/r)\, E1 = \alpha 2 \Delta T\, E2 + (Y/r)\, E2$$

we can find Y

$$Y = \Psi = \frac{(Wsi/2)h1^2 - (Wal/2)\, h2^2}{(Wsi/2)h1 + (Wal/2)\, h2}$$

for a given h1 we can find h2, let h1=2µm we find h2 = 8 µm

BIMETALIC CANTILEVER BEAM



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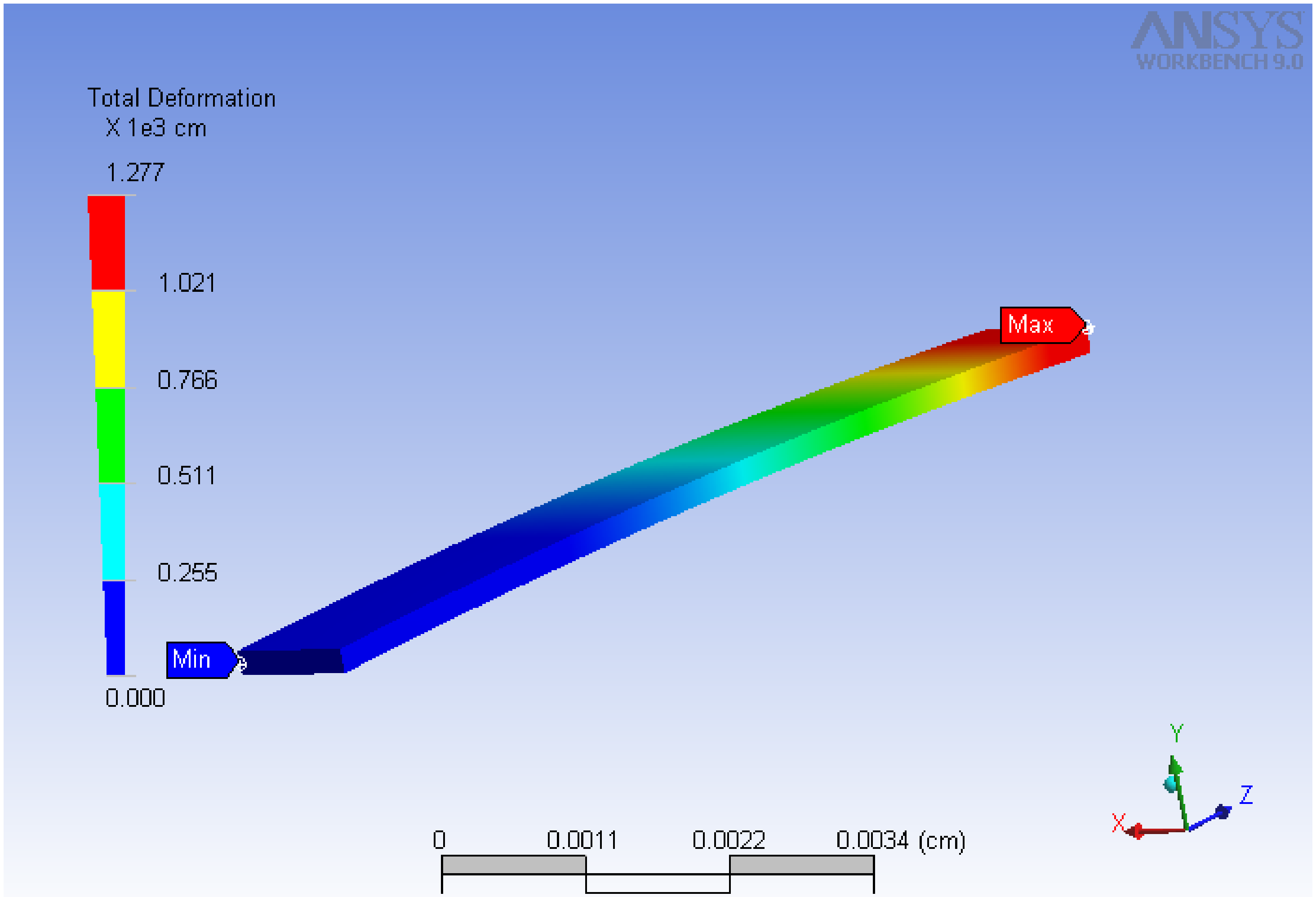
RAISE TEMPERATURE FROM 20 TO 200 C



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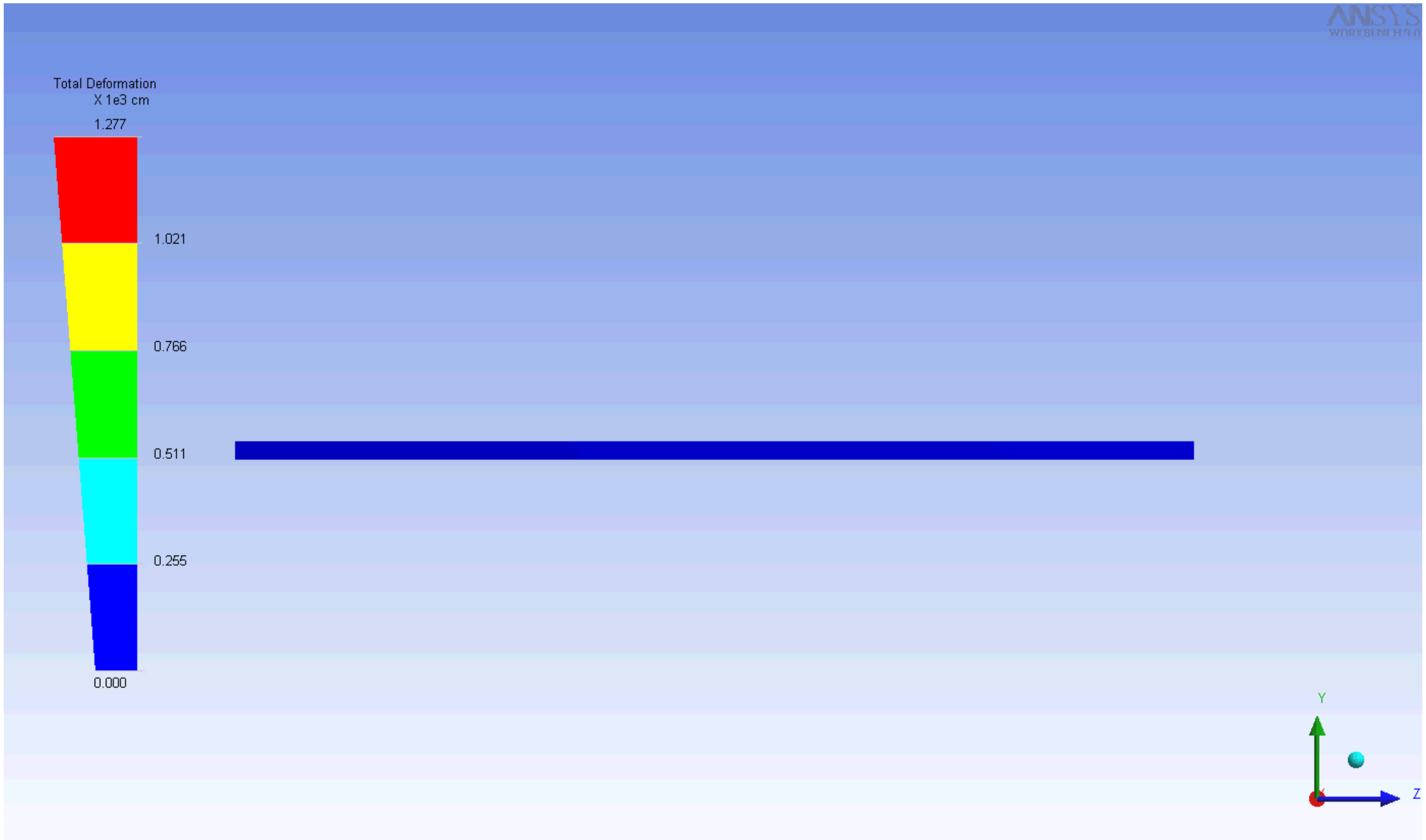
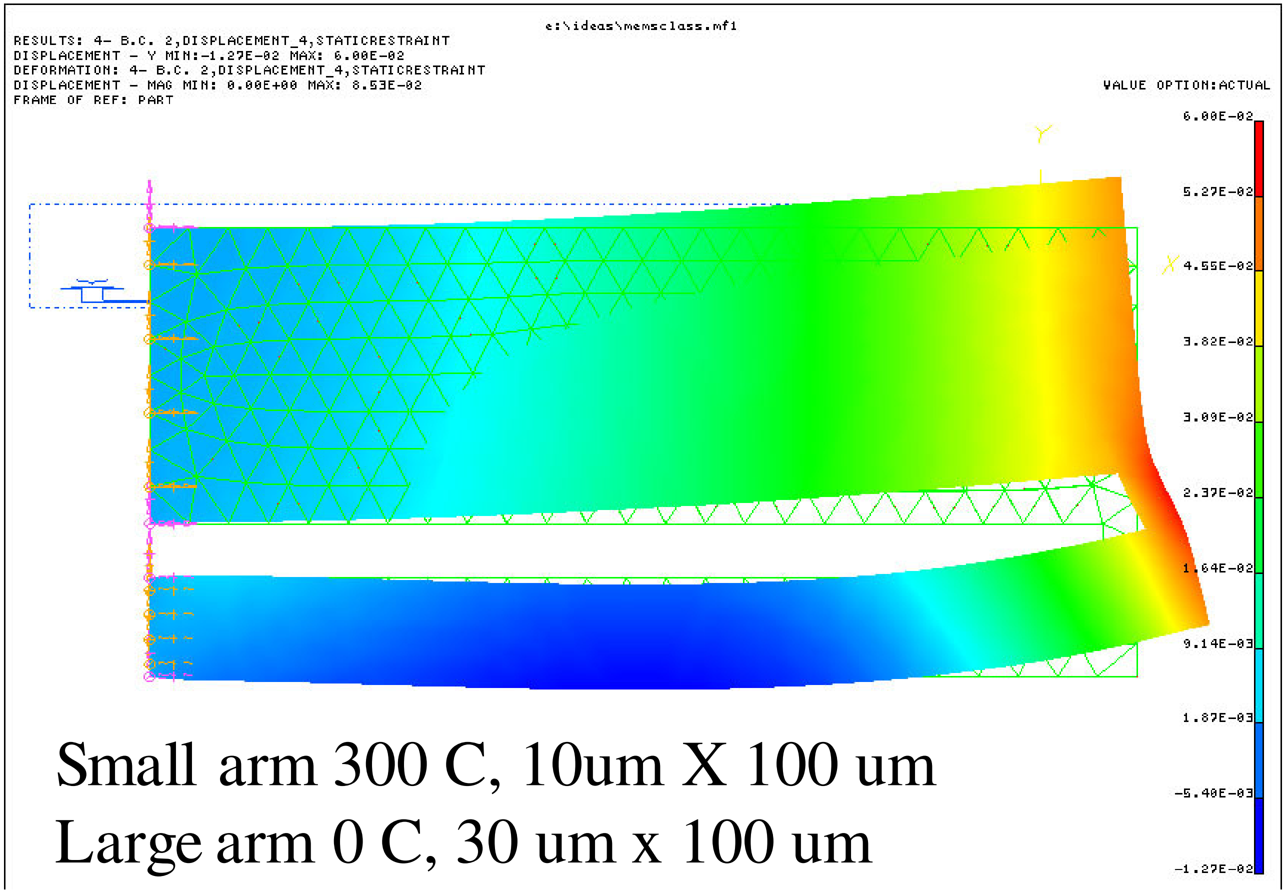
MOVIE OF TOTAL DEFORMATION



FINITE ELEMENT ANALYSIS OF THERMAL BENDING



Small arm 300 C, 10um X 100 um
Large arm 0 C, 30 um x 100 um
Maximum Displacement = 6 um

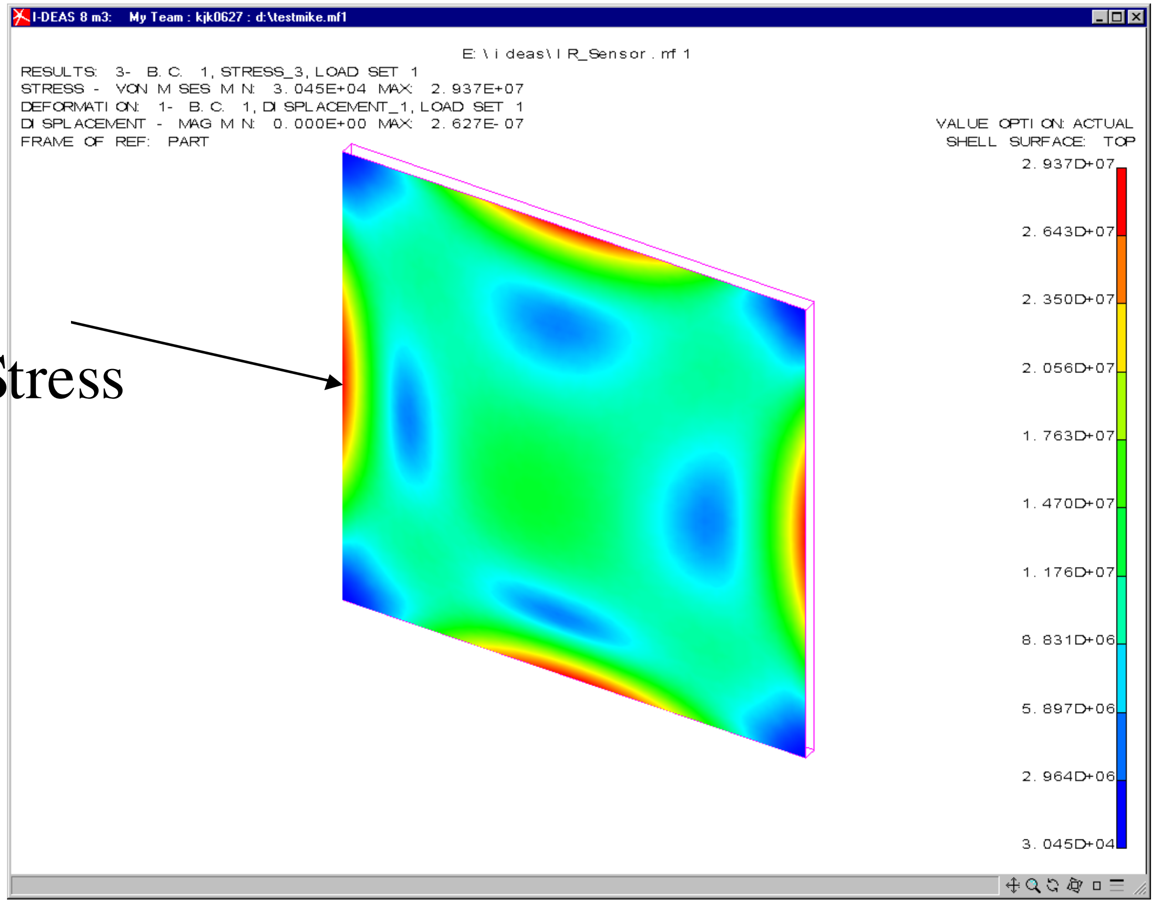
IDEAS – Andrew Randall

EXCELL SPREADSHEET FOR DIAPHRAGM CALCULATIONS

Rochester Institute of Technology				11-Jun-07
Dr. Lynn Fuller	Microelectronic Engineering, 82 Lomb Memorial Dr., Rochester, NY 14623			
To use this spread sheet enter values in the white boxes. The rest of the sheet is protected and should not be				
changed unless you are sure of the consequences. The results are displayed in the purple boxes.				
Diaphragm				
Deflection Ymax = 0.0151 P $L^4(1-Nu^2)/EH^3$		Ymax =	12.45	µm
	P = Pressure	P =	5000	lbs/in2
	L = Length of side of square diaphragm	L =	1500	µm
	E = Youngs Modulus	E =	1.90E+11	N/m2
	Nu = Poissons Ratio	Nu =	0.32	
	H = Diaphragm Thickness	H =	100	µm
		P =	3.45E+07	Pascal
Diaphragm				
Stress = 0.3 P $(L/H)^2$ (at center of each edge)		Stress =	2.33E+09	Pascal
	P = Pressure	Yield Strength =	1.20E+10	Pascal
	L = Square Diaphragm Side Length			
	H = Diaphragm Thickness			
Two Parallel Plates				
Capacitance = eoer Area/d		C =	7.97E-11	F
	eo = Permitivitty of free space = 8.85E-14 F/cm			

FINITE ELEMENT ANALYSIS

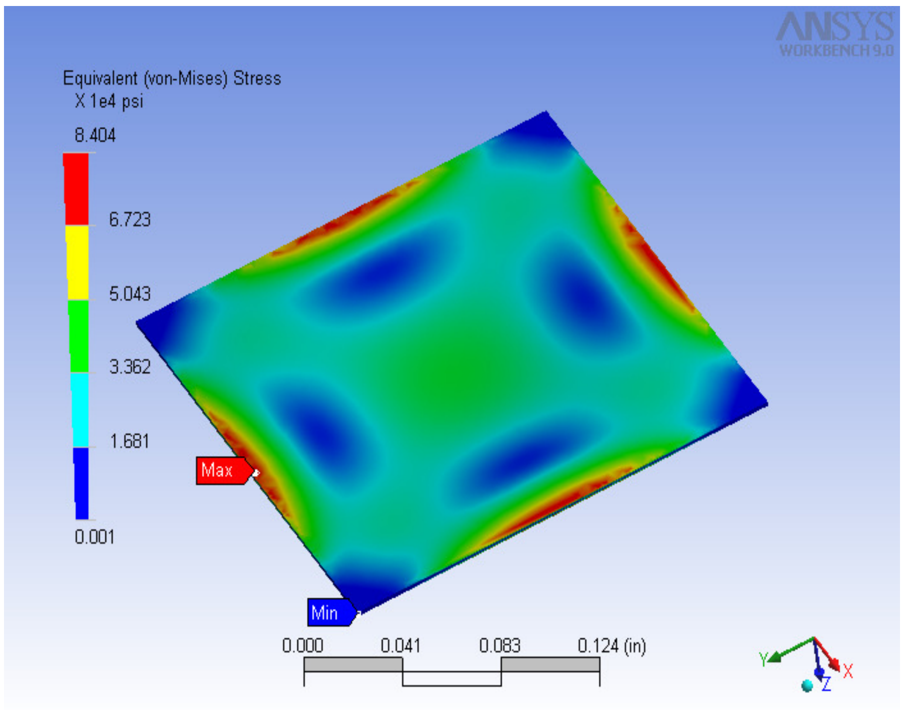
Points of
Maximum Stress



IDEAS – Rob Manley

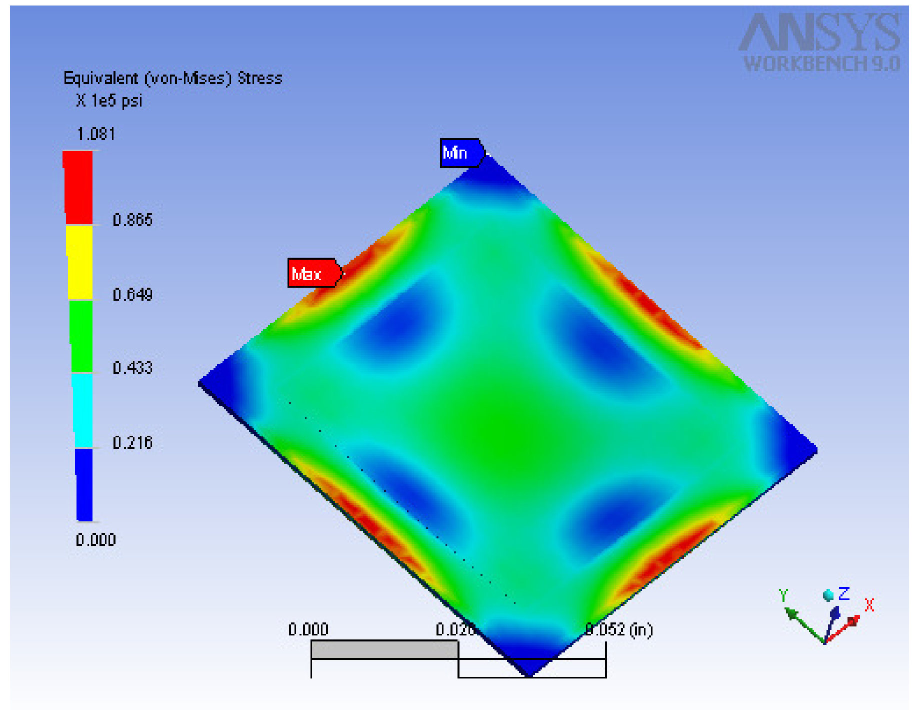
ANSYS FINITE ELEMENT ANALYSIS

Regular Si Diaphragm



Corrugated Diaphragm

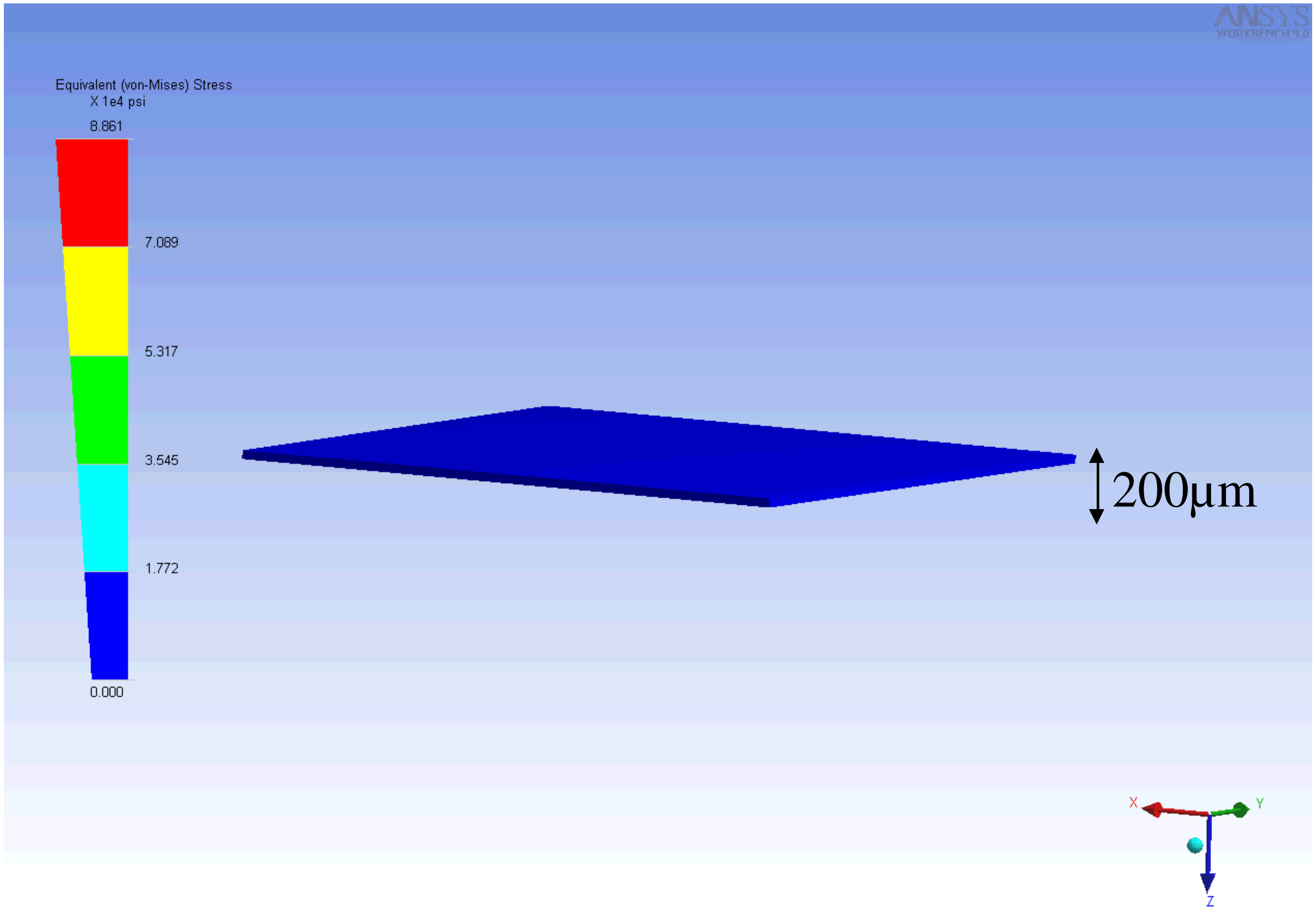
Layer 2: 1.5mm x 1.5mm Polysilicon 1µm thick



2mm x 2mm diaphragm 30µm thick, 50 psi applied

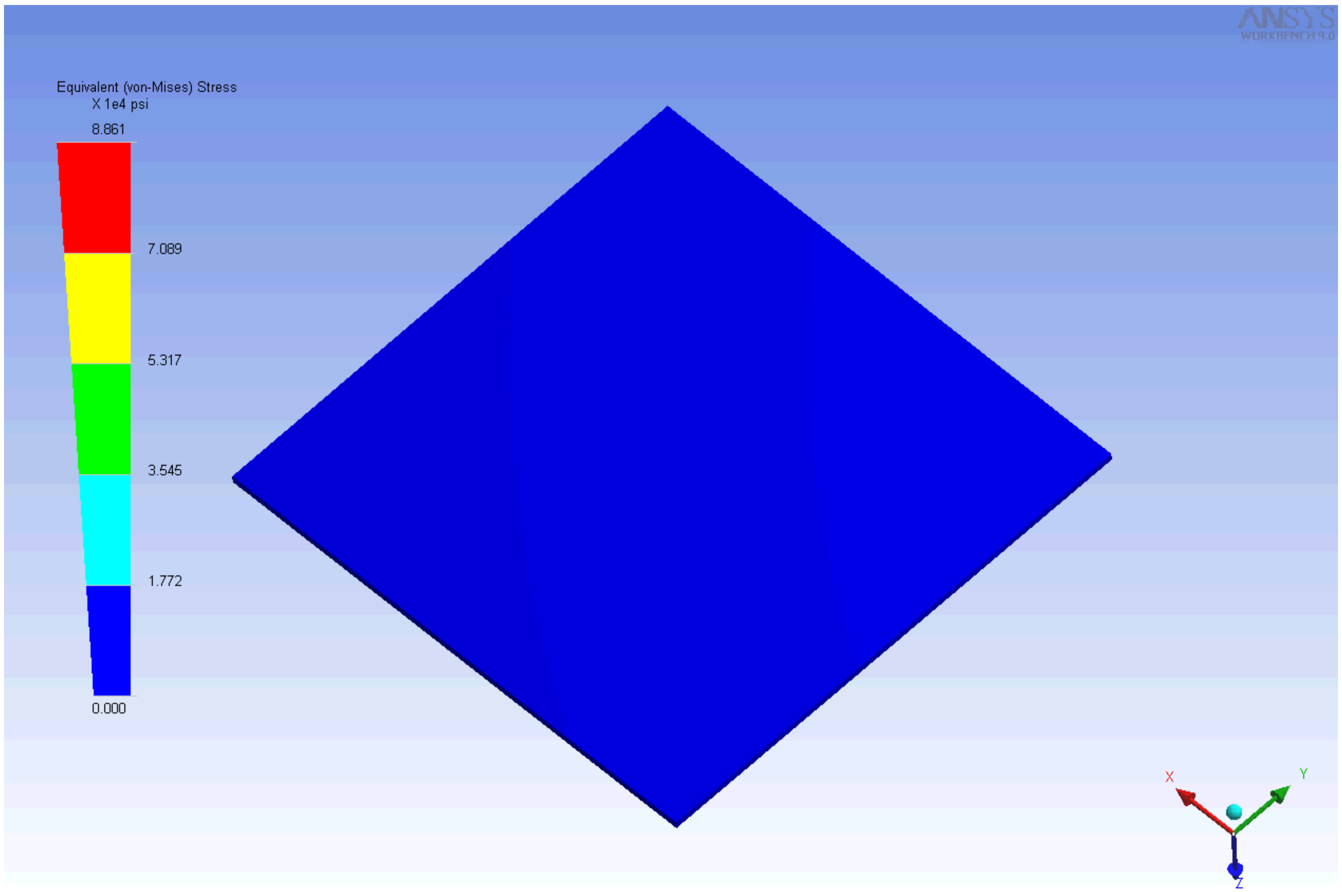
Rob Manley, 2005

DIAPHRAGM DEFORMATION MOVIE



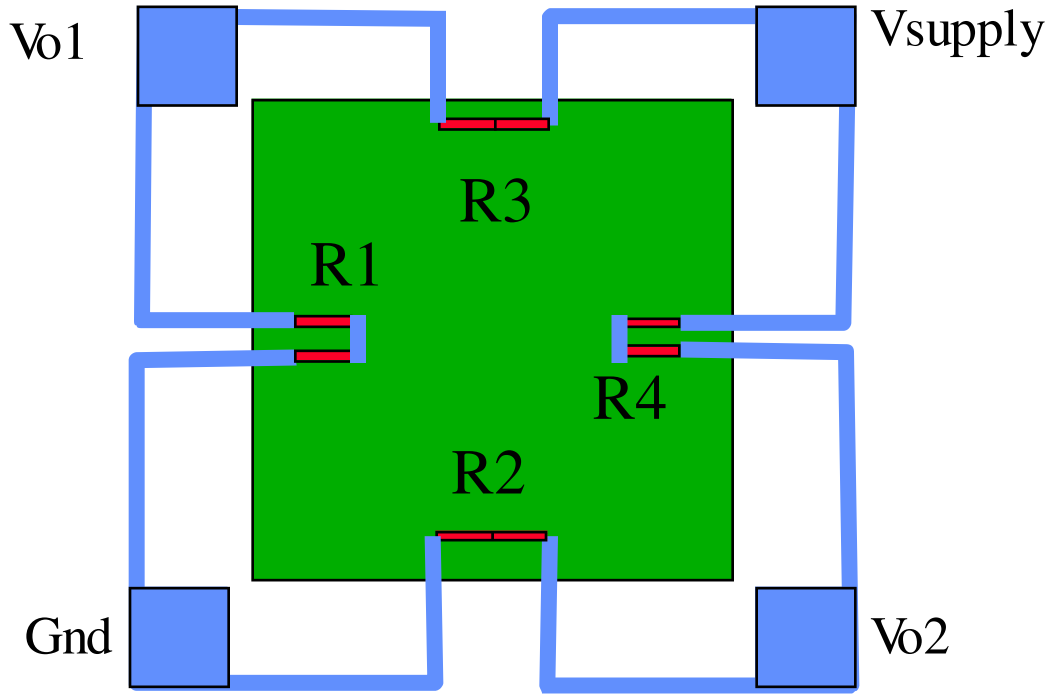
Rob Manley, 2005

DIAPHRAGM STRESS MOVIE



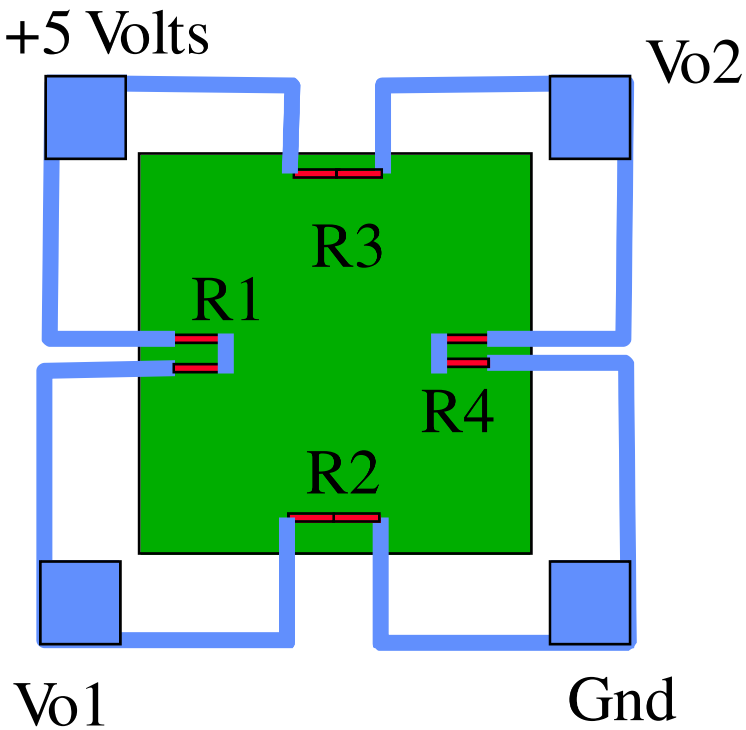
Rob Manley, 2005

RESISTOR LAYOUT



Two resistors parallel to edge near region of maximum stress and two resistors perpendicular to the edge arranged in a full bridge circuit. If all resistors are of equal value then Vout = Vo1-Vo2 = zero with no pressure applied.

CALCULATION OF EXPECTED OUTPUT VOLTAGE



The equation for stress at the center edge of a square diaphragm (S.K. Clark and K.Wise, 1979)

Stress = $0.3\ P(L/H)^2$ where P is pressure, L is length of diaphragm edge, H is diaphragm thickness

For a 3000µm opening on the back of the wafer the diaphragm edge length L is 3000 – 2 (500/Tan 54.74°) = 2246 µm

CALCULATION OF EXPECTED OUTPUT VOLTAGE

$$\text{Stress} = 0.3\ P\ (L/H)^2$$

If we apply vacuum to the back of the wafer that is equivalent to and applied pressure of 14.7 psi or 103 N/m^2

P = 103 N/m^2

L= 2246 µm

H= 25 µm

Stress = 2.49E8 N/m^2

Hooke's Law: Stress = E Strain where E is Young's Modulus

$$\sigma = E\ \varepsilon$$

Young's Modulus of silicon is 1.9E11 N/m^2

Thus the strain = 1.31E-3 or .131%

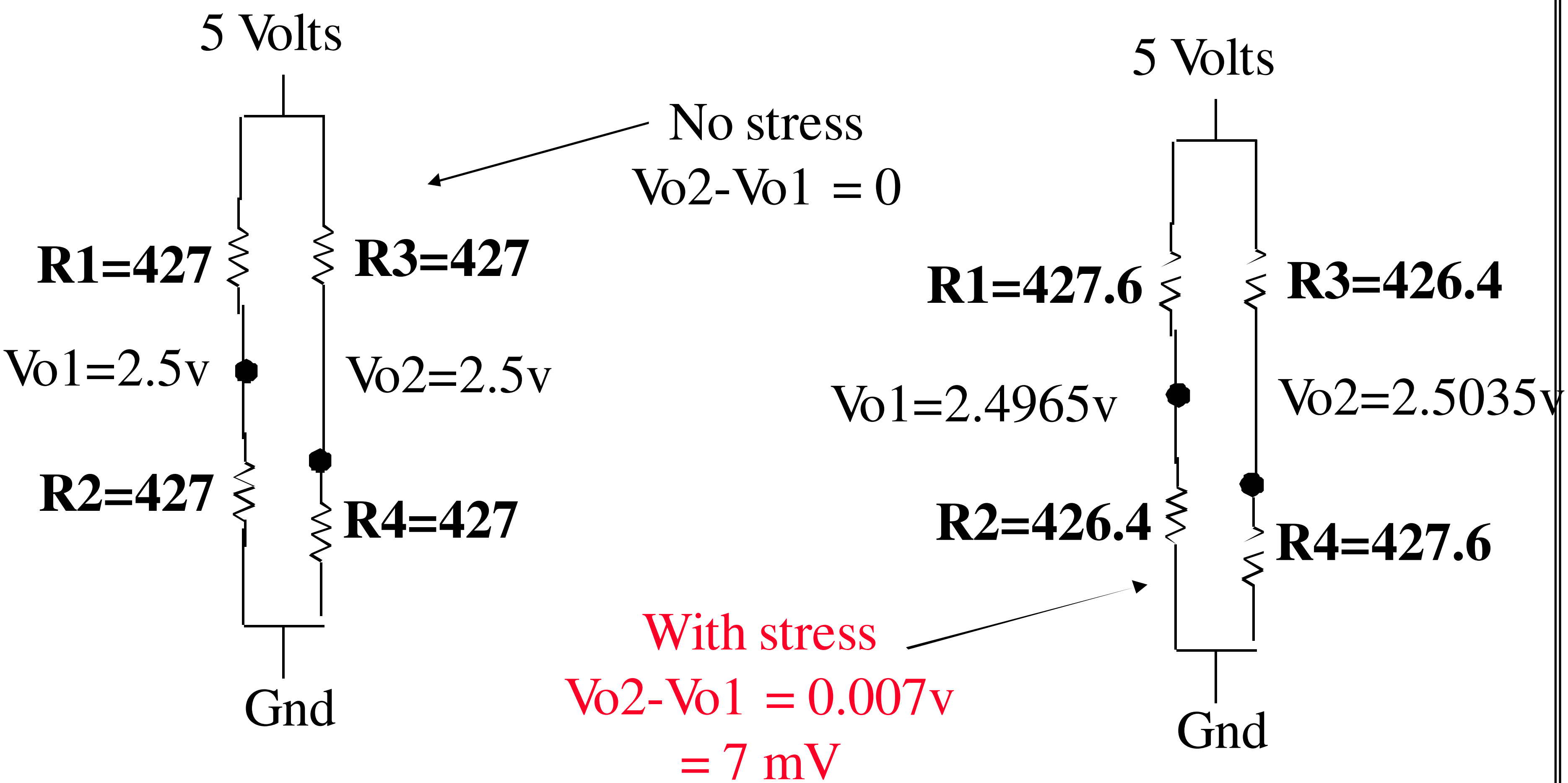
CALCULATION OF EXPECTED OUTPUT VOLTAGE

The sheet resistance (Rhos) from 4 point probe is 61 ohms/sq
The resistance is R = Rhos L/W
For a resistor R3 of L=350 µm and W=50 µm we find:
R3 = 61 (350/50) = 427.0 ohms

R3 and R2 decrease as W increases due to the strain
assume L is does not change, W' becomes 50+50x0.131%
W' = 50.0655 µm
R3' = Rhos L/W' = 61 (350/50.0655) = 426.4 ohms

R1 and R4 increase as L increases due to the strain
assume W does not change, L' becomes 350 + 350x0.131%
R1' = Rhos L'/W = 61 (350.459/50) = 427.6 ohms

CALCULATION OF EXPECTED OUTPUT VOLTAGE



IF RESISTORS ARE SINGLE CRYSTAL SILICON

In addition to the effects of strain on the resistance if the resistor is made of single crystal silicon there is also a significant piezoresistive effect on the resistor value. Strain effects the mobility of holes and electrons in silicon. The resistors on the diaphragm of the pressure sensor drawn above have current flow longitudinal (R1 and R4) and transverse (R2 and R3) to the strain. The strain is tensile on the top surface of the diaphragm where the resistors are located if positive pressure is applied to the top of the diaphragm. The peizoresistive coefficient for R1 and R4 is 71.8 and for R2 and R3 is -66.3 E-11/Pa. The calculations above give the stress as 2.49E8 Pa thus the hole mobility will decrease in R1 and R4 (R increases in value) by 2.49E8 x 71.8e-11 = 17.9% while R2 and R3 (decrease in value) because the mobility increases by 2.49E8 x 66.3E-11 = 16.5%, thus the overall effect will be dominated by the piezoresistance rather than the effect of strain on the dimensions.

EXPRESSION FOR RESISTANCE

$$R = Ro\,[\,1 + \pi_L \sigma_{xx} + \pi_T(\sigma_{yy} + \sigma_{zz})]$$

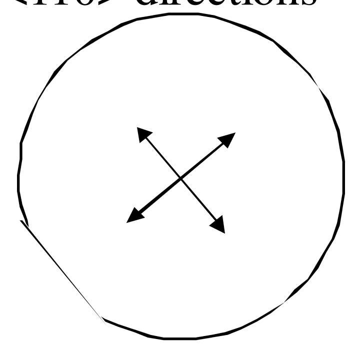
where $Ro = (L/W)(1/(q\mu(N,T)\ Dose))$

π_L is longitudinal piezoresistive coefficient
π_T is transverse piezoresistive coefficient
σ_{xx} is the x directed stress, same direction as current
σ_{yy} is the y directed stress, transverse to current flow
σ_{zz} is the z directed stress, transverse to current flow

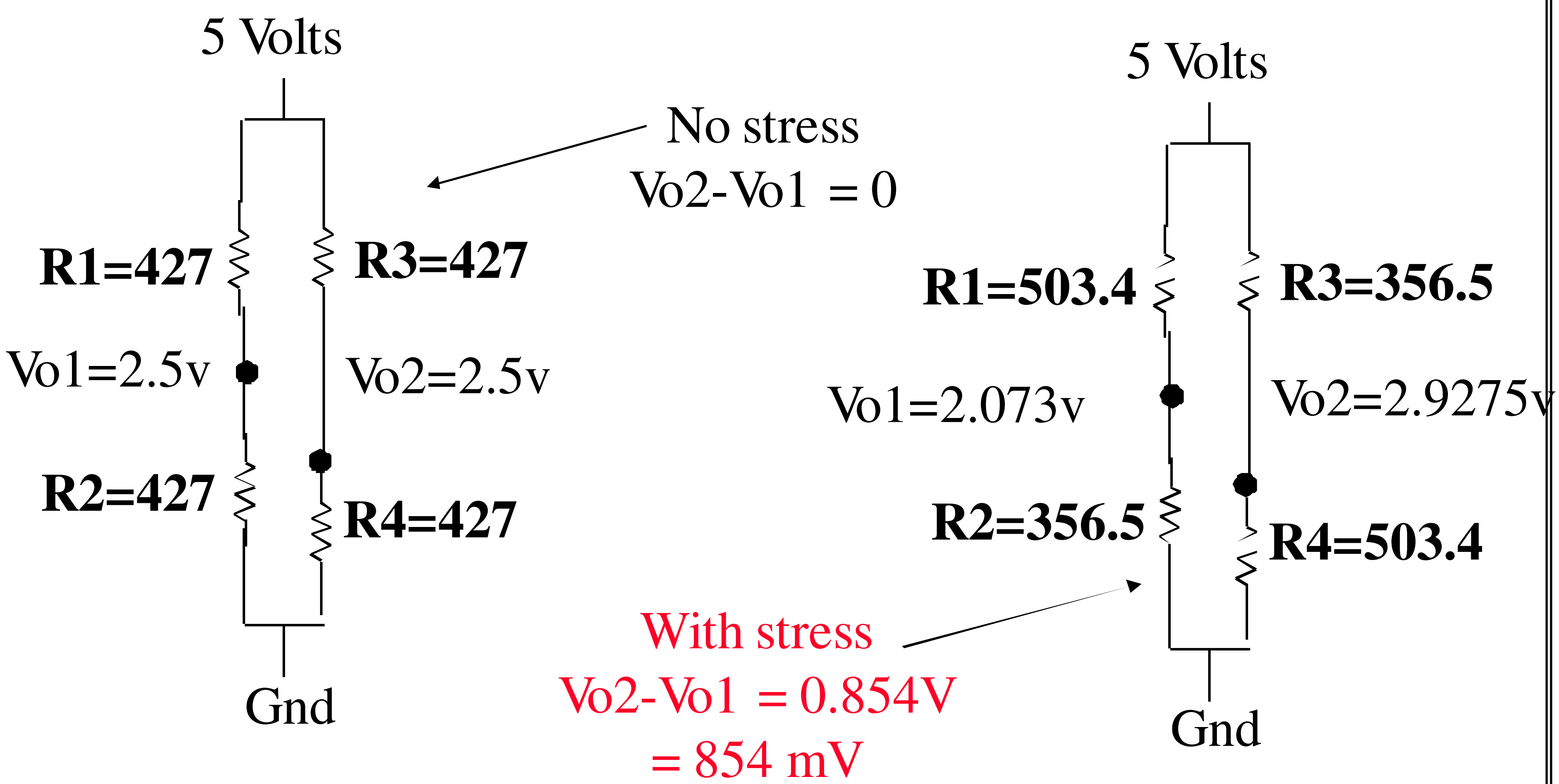
In the <110> direction

	$\pi_L (E^{-11}/Pa)$	$\pi_T (E^{-11}/Pa)$
Electrons	**-31.6**	**-17.6**
holes	**71.8**	**-66.3**

(100) wafer
<110> directions

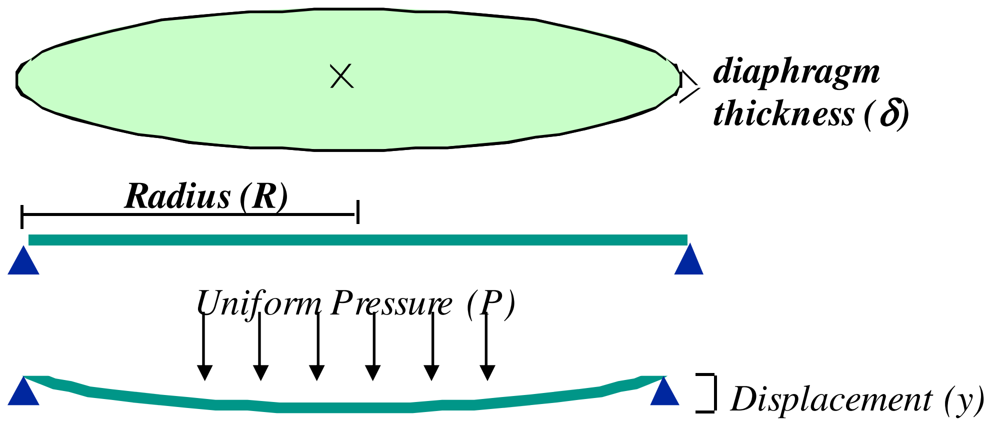


CALCULATION OF EXPECTED OUTPUT VOLTAGE FOR SINGLE CRYSTAL RESISTORS



DIAPHRAGM

Diaphragm: Displacement



Equation for deflection at center of diaphragm

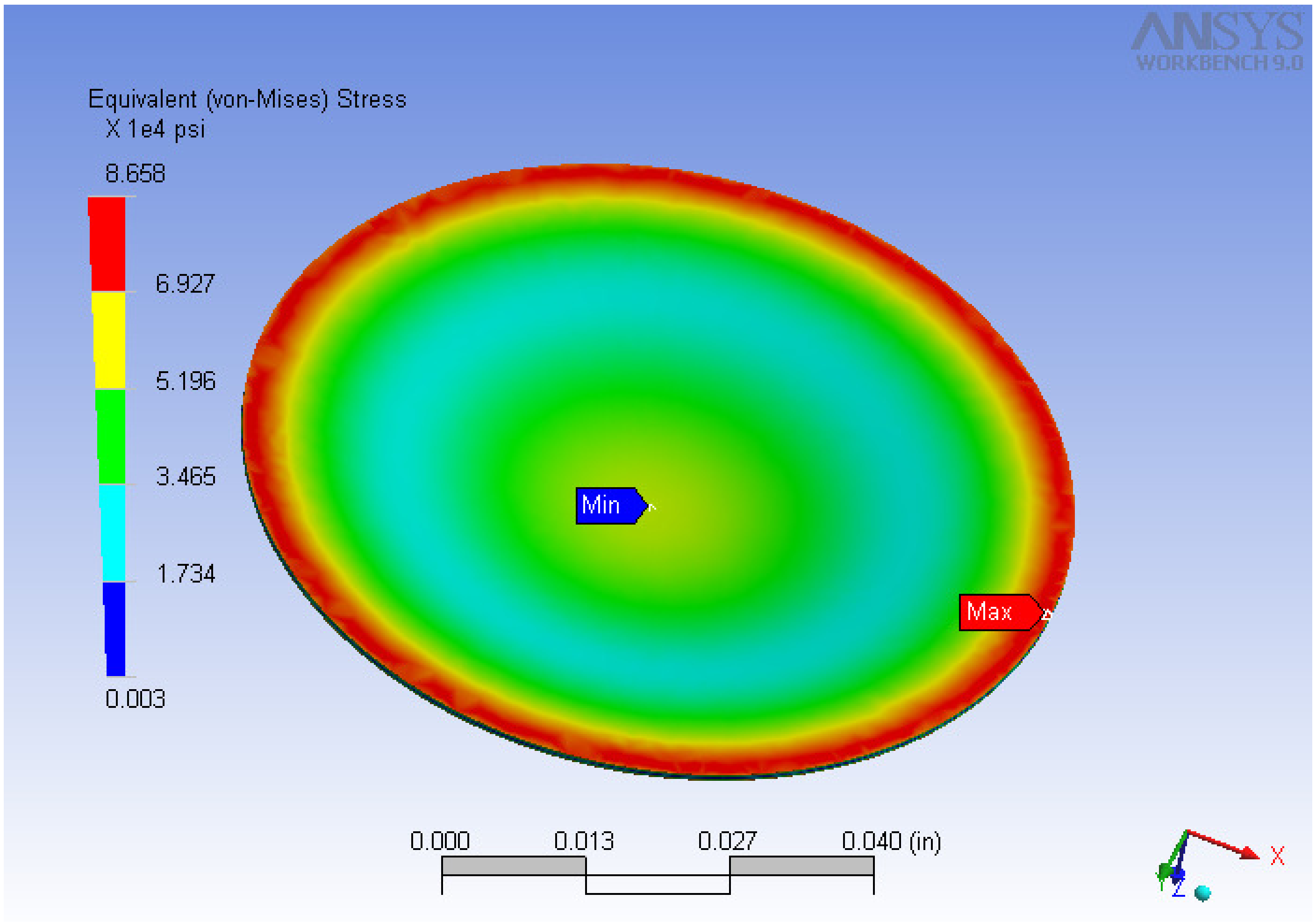
$$y = \frac{3PR^4[(1/\nu)^2-1]}{16E(1/\nu)^2\delta^3} = \frac{(249.979)PR^4[(1/\nu)^2-1]}{E(1/\nu)^2\delta^3}$$

E = Young's Modulus, ν = Poisson's Ratio

for Aluminum ν =0.35

*The second equation corrects all units assuming that pressure is mmHg, radius and diaphragm is μm, Young's Modulus is dynes/cm^2, and the calculated displacement found is μm.

CIRCULAR DIAPHRAGM FINITE ELEMENT ANALYSIS0



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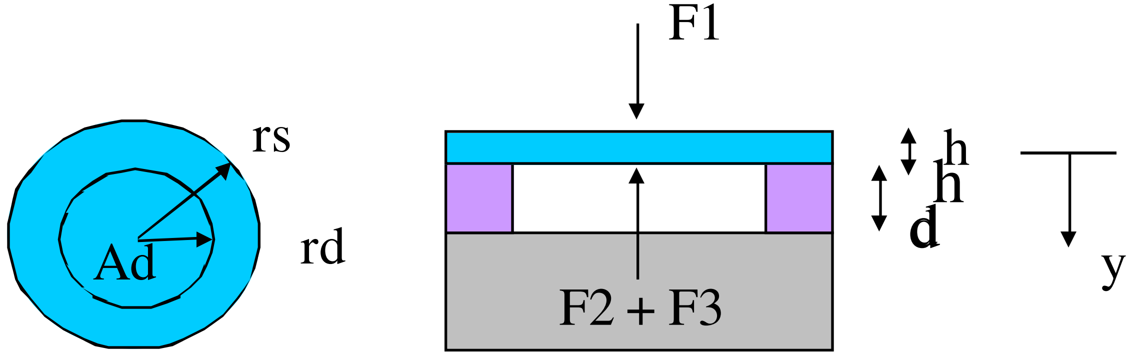
CAPTURED VOLUME

$$PV = nRT$$

F1 = force on diaphragm = external pressure times area of diaphragm
F2 = force due to captured volume of air under the diaphragm
F3 = force to mechanically deform the diaphragm



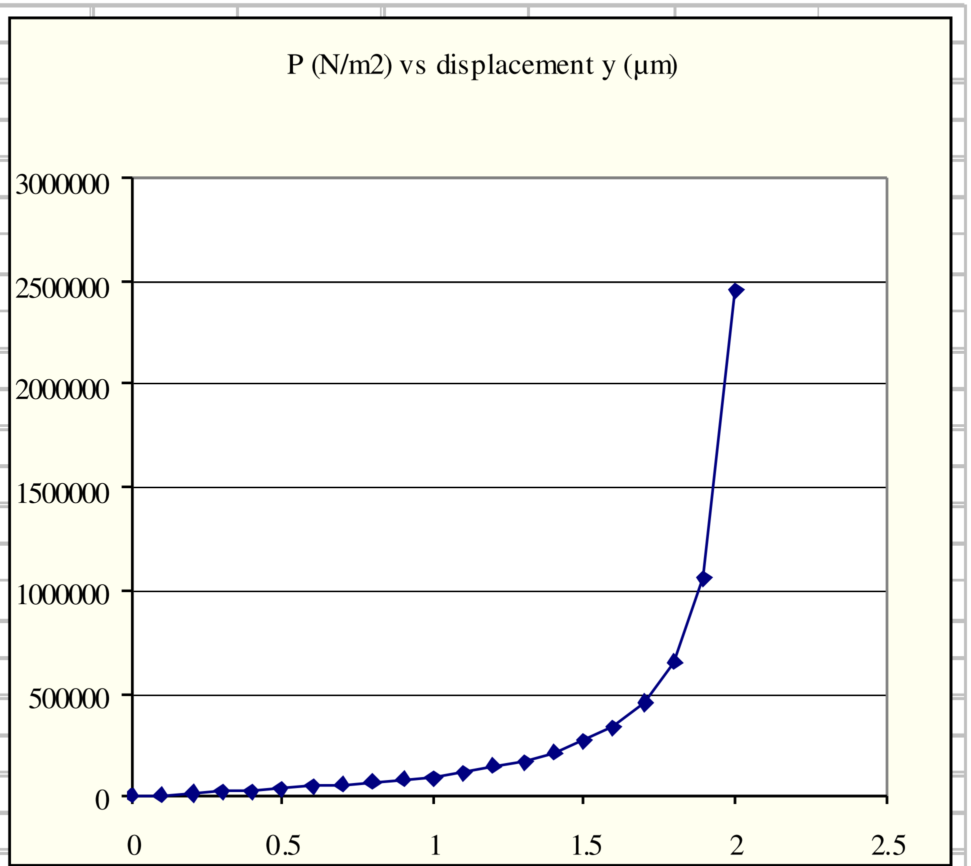
F1= F2 + F3 F1 = P x Ad F2 = nRT Ad / (Vd + Vs)

where Vd = Ad (d-y) and Vs = G1 Pi ($rs^2 – rd^2$)(d) where G1 is the % of spacer that is not oxide

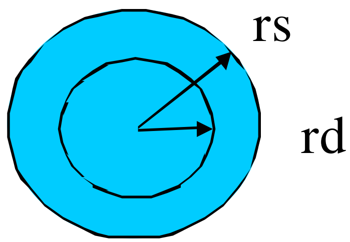
$$F3 = (16\ E\ (1/\nu)^2\ h^3\ y)/(3\ rd^4[(1/\nu)^2-1])$$

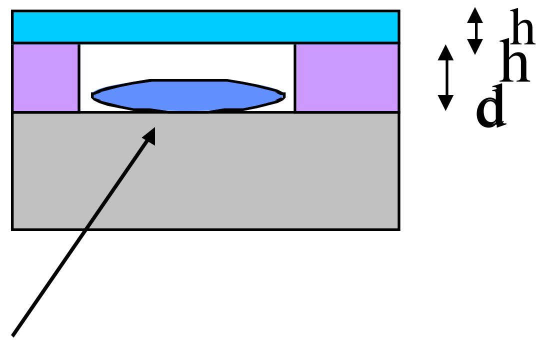
DIAPHRAGM WITH CAPTURED VOLUME

y (µm)	P (N/m2)
0	0
0.1	5103.44
0.2	10748.09
0.3	17025.03
0.4	24047
0.5	31955.23
0.6	40929.06
0.7	51199.69
0.8	63070.47
0.9	76947.31
1	93386.12
1.1	113169.1
1.2	137433.1
1.3	167895.7
1.4	207281.3
1.5	260184.8
1.6	335009.7
1.7	448945.4
1.8	643513.4
1.9	1051199
2	2446196



CAPTURED VOLUME & PHASE CHANGE ACTUATORS





Liquid that is heated enough to go to gas state, expands and deflects the diaphragm.

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HOMEWORK – MECHANICAL FUNDAMENTALS

1. For a simple cantilever beam of length 1000um, width of 100um and thickness of 25um. Calculate the force needed to cause the end of the beam to move 10um. Calculate the stress at the anchor when this force is applied.
2. If a p-type diffused resistor on (100) silicon cantilever is placed on a simple (110) cantilever at the anchor where the stress is maximum and is oriented parallel to cantilever beam what is the expected piezoresistance coefficient. Write and expression for the resistor that includes effects of strain, piezoresistance and force at the free end of the cantilever. Include variables for cantilever L, W, H, etc.
3. Calculate the maximum deflection for a circular polysilicon diaphragm, 500 µm in diameter, 2 µm in thickness, with a uniform pressure of 2 lbs/in^2
4. Make an excel spreadsheet for doing calculations associated with cantilever beam of different materials, sizes, mass and force at end, etc. Show example results. Make practical assumptions.