Frequency Response of the CE Amplifier

Dr. Lynn Fuller
Webpage: http://people.rit.edu/lffeee/
Microelectronic Engineering
Rochester Institute of Technology
82 Lomb Memorial Drive
Rochester, NY 14623-5604
Tel (585) 475-2035

Email: Lynn.Fuller@rit.edu
MicroE webpage: http://www.microe.rit.edu

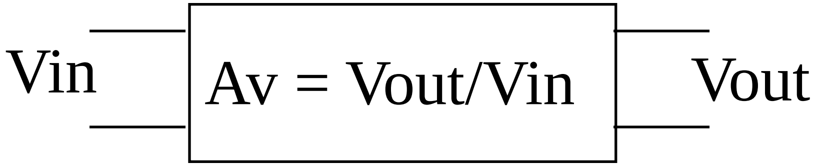
9-25-14 Frequency_Response.ppt

OUTLINE

Introduction
Gain Function and Bode Plots
Low Frequency Response of CE Amplifier
Millers Theorem
High Frequency Response of CE Amplifier
References
Homework Questions

INTRODUCTION

We will be interested in the voltage gain of an electronic circuit as a function of frequency.



Vin

Av = Vout/Vin

Vout

Decibel: the gain of some network can be expressed in logarithmic units. When this is done the overall gain of cascaded networks can be found by simple addition of the individual network gains.

The decibel is defined as:

$$A_p = 10 \log (P_o/P_{in}) \quad \text{dB}$$

where Ap is the power gain in decibels
Po is the power out and Pin is the power in

The decibel has also been used as a unit for voltage gain.

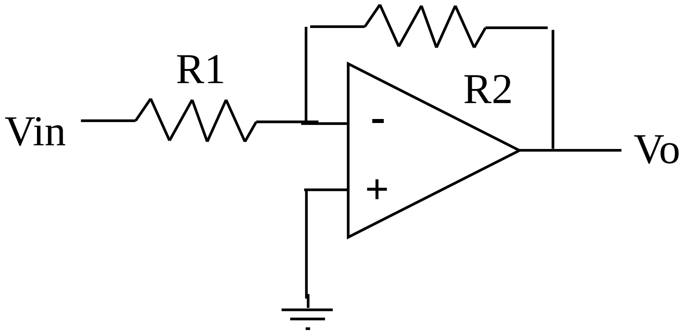
$$P_o = V_{out}^2/R_L \quad \text{and} \quad P_{in} = V_{in}^2/R_{in}$$

and if Rin=RL

$$A_p = 20 \log (V_{out}/V_{in}) \quad \text{dB}$$

INTRODUCTION

Thus the decibel is often used to express voltage gains. (Really only correct if RL=Rin but many people are not precise about this point)

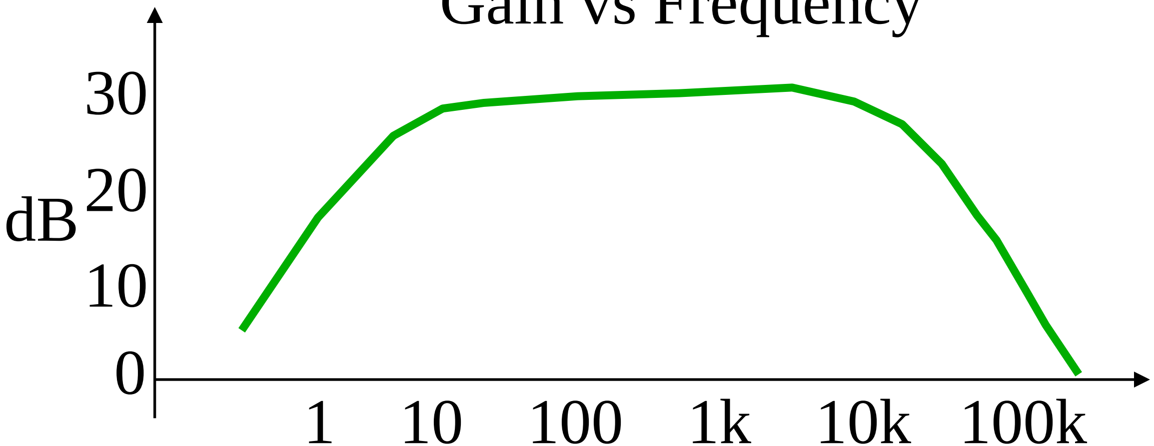


If R1=2K and R2=47K

Vo/Vin = - 47K/2K = -23.5

|Vo/Vin| = 23.5 or 27.4 dB

Gain vs Frequency



THE GAIN FUNCTION

The gain function, A(s): an expression for Vo/Vin which is found in a straight forward manor from the ac equivalent circuit.

Vo/Vin = A(s) or in particular $s=j\omega$ thus $A(j\omega)$

$$A(s) = \frac{a0 + a1\ s + a2\ s^2 + a3\ s^3\ ....}{b0 + b1\ s + b2\ s^2 + b3\ s^3\ ....}$$

$$A(s) = \frac{K\ (s-z1)(s-z2)(s-z3)\ldots}{(s-p1)(s-p2)(s-p3)\ldots}$$

Where z1, z2, z3 are zeros, p1, p2, p3 are poles

$$A(j\omega) = \frac{K\ (j\omega-z1)(j\omega-z2)(j\omega-z3)\ldots}{(j\omega-p1)(j\omega-p2)(j\omega-p3)\ldots}$$

$$A(j\omega) = \frac{A0\ (j\omega/\omega1)^N(j\omega/\omega3+1)(j\omega/\omega5+1)\ldots}{(j\omega/\omega2+1)(j\omega/\omega4+1)(j\omega/\omega6+1)\ldots}$$

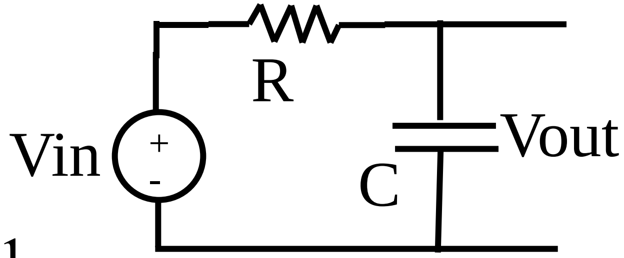
GOALS

1. Obtain the gain function from the ac equivalent circuit.
2. Predict the frequency response of the gain function.
3. Use graphical techniques to sketch the frequency response
3. Introduce a new model for transistors at high frequencies.
5. Analyze and predict the frequency response of a common emitter amplifier stage

GRAPHICAL TECHNIQUES AND BODE PLOTS

Gain Function:

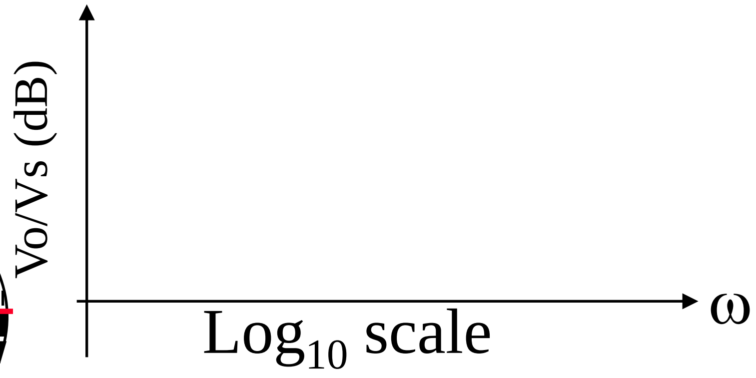
$$\text{Vo} = \text{Vin}\ \frac{1/sC}{R + 1/sC}$$

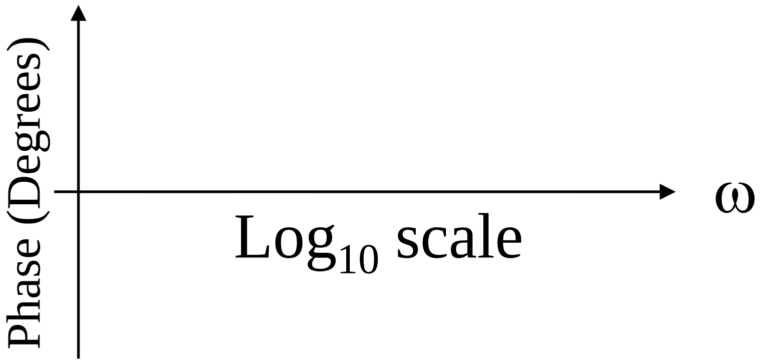


$$\text{Vo/Vin} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1} = \frac{1}{j\omega/\omega 1 + 1}$$

Where $\omega 1 = 1/RC$ and $f1 = 1 / 2 \pi RC$

Bode Plot: a plot of the gain function versus frequency (ω or f). Note: both magnitude and phase are a function of frequency. The Bode Plot plots this information separately.





CONTINUE PREVIOUS EXAMPLE 1

$$Av = Vo/Vin = \frac{1}{j\omega/\omega 1 + 1}$$

At low ω $\quad Vo/Vin = 1 \angle 0°$; $\left|Vo/Vin\right|dB = 0$ dB and $\Theta = 0°$

At high ω $\quad Vo/Vin = 1/(j\omega/\omega 1) \angle -90°$

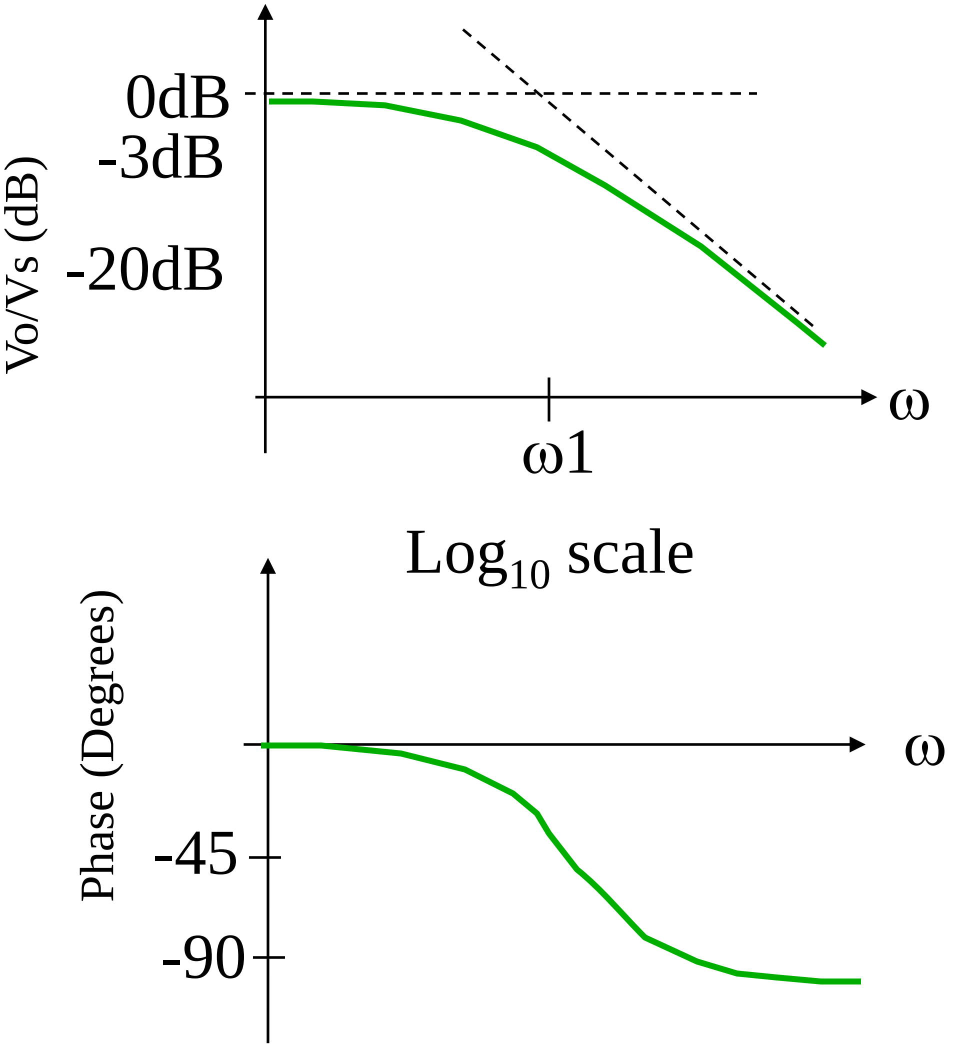
$\left|Vo/Vin\right|dB = \omega 1/\omega$ dB and $\Theta = -90°$

Note: at $\omega = 10\ \omega 1$ $\quad \left|Vo/Vin\right|dB = -20$ dB

Note: at $\omega = 100\ \omega 1$ $\quad \left|Vo/Vin\right|dB = -40$ dB

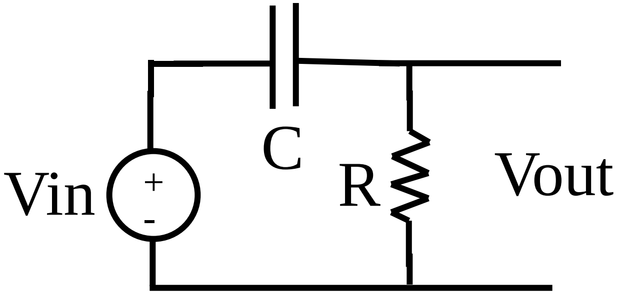
Thus we see at high frequencies the gain decreases by -20 dB / decade

CONTINUE EXAMPLE 1



EXAMPLE 2

Obtain the gain function for the network shown. Sketch the magnitude part of the Bode Plot.

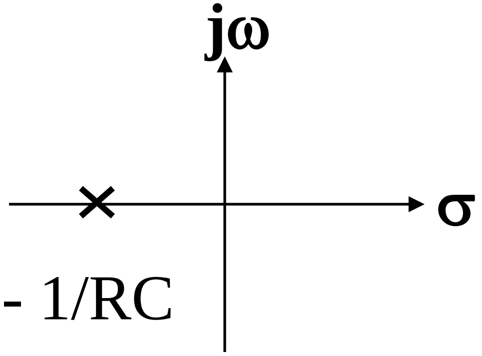


POLES AND ZEROS

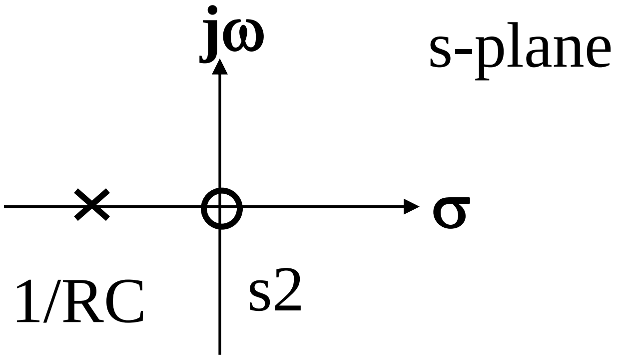
Poles and Zeros: the complex frequency at which the gain function goes to infinity in the case of poles or to zero in the case of zeros.

Example 1: $$Vo/Vin \qquad \frac{1}{sCR + 1}$$

Pole at s1 = - 1/RC



Example 2: $$Vo/Vin \qquad \frac{sCR}{sCR + 1}$$



Which has a Zero at zero and a Pole at s1 = - 1/RC

CORNER FREQUENCY

Corner frequencies: that frequency (f or ω) at which the real and imaginary parts of one term of the gain function are equal/

Example 1: $$Vo/Vin = \frac{1}{j\omega CR + 1}$$

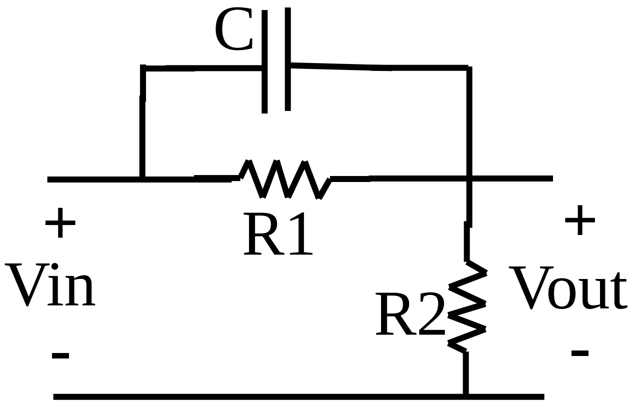
Has a corner at $\omega 1 = 1/RC$ or $f = 1/2\pi RC$

Example 2: $$Vo/Vin = \frac{j\omega CR}{j\omega CR + 1}$$

Has a corner $\omega 1 = 1/RC$

EXAMPLE 3

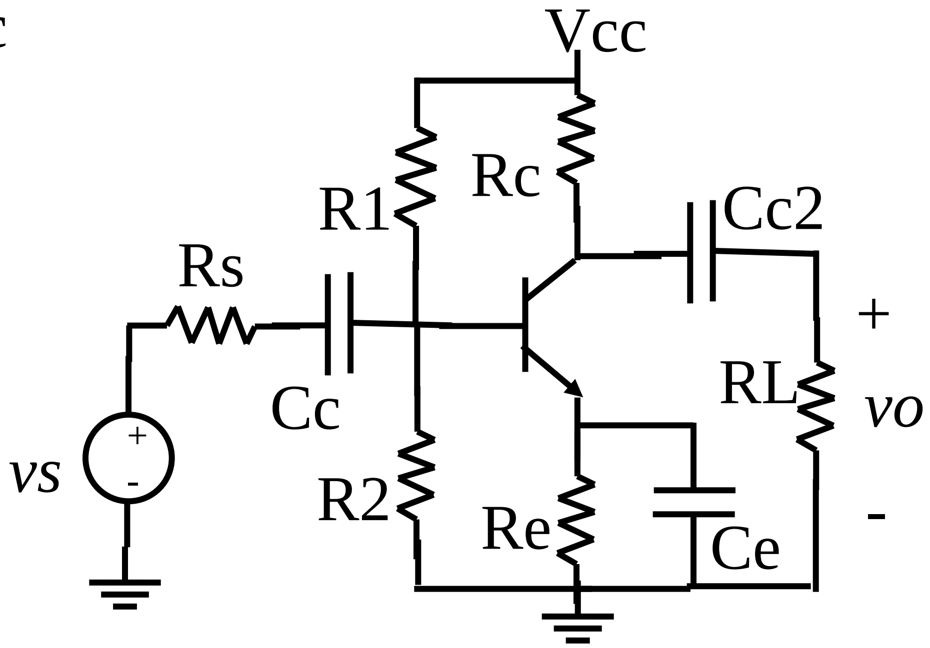
Find the gain function, poles, zeros and corner frequencies for the network shown, sketch the Bode plot.



LOW FREQUENCY MODEL OF CE AMPLIFIER

Effect of the Coupling Capacitor, Cc
assume Ce, and Cc2 act like a short.

Obtain the gain function from the ac equivalent circuit:



$$vo = -gm\ vbe\ Ry$$

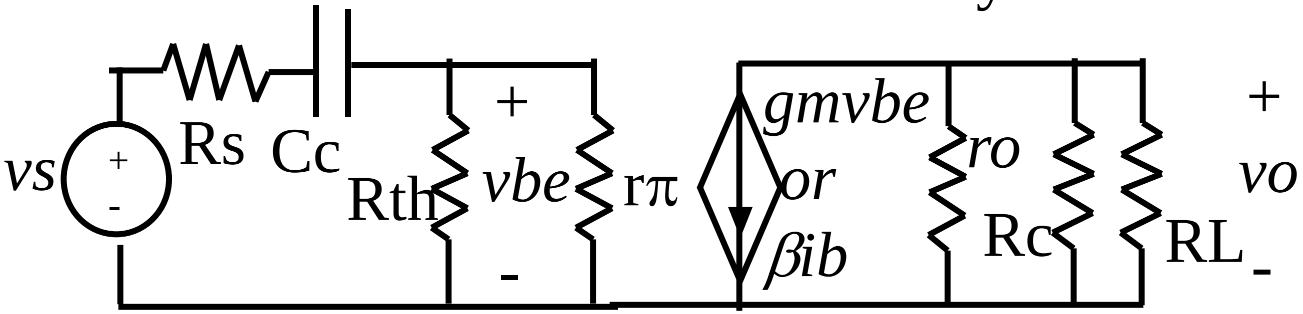
$$vbe = \frac{vs\ Rx}{(Rs+1/sCc + Rx)}$$

$$\boxed{vo/vs = \frac{-gm\ Rx\ Ry}{(Rs+1/sCc + Rx)}}$$

Rx = Rth//rπ

Ry = ro//Rc//RL

$s = j\omega$



EFFECT OF COUPLING CAPACITOR Cc

Manipulate the gain function until we have a form from which we can easily obtain the bode plot.

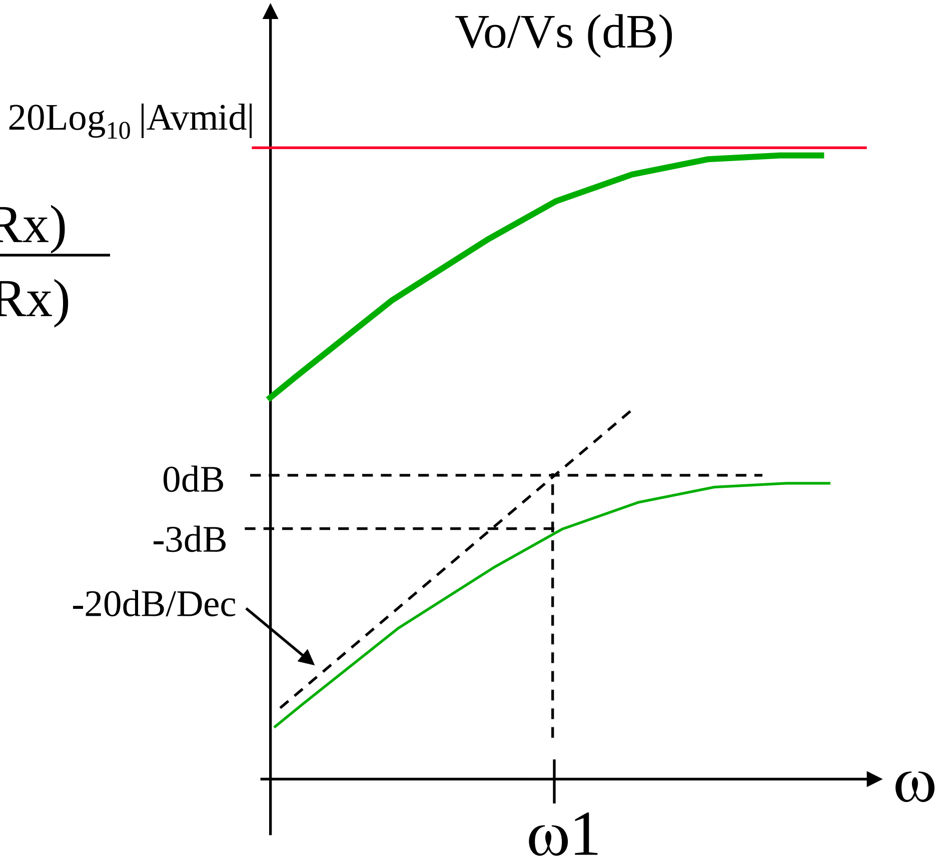
$$vo/vs = -gm\ RyRx \frac{s\ Cc}{(sCc(Rs+Rx)+1)}$$

$$vo/vs = -gm\ RyRx \frac{s\ Cc}{(sCc(Rs+Rx)+1)} \frac{(Rs+Rx)}{(Rs+Rx)}$$

$$vo/vs = \frac{-gm\ RyRx}{(Rs+Rx)} \frac{s\ Cc\ (Rs+Rx)}{(sCc(Rs+Rx)+1)}$$

$$\boxed{vo/vs = \text{Avmid} \frac{j\ \omega/\omega1}{(j\ \omega/\omega1+1)}}$$

Where ω1 = 1/Cc(Rs+Rx)



Log$_{10}$ scale

SUMMARY FOR EFFECT OF Cc

1. At low frequencies the coupling capacitor “opens” up and the voltage gain drops as the frequency decreases.

2. The corner frequency ω1 equals the inverse of the product ReqCc where Req is the resistance “seen” looking from the capacitor terminals with Vin = zero in the ac equivalent circuit.

3. At mid frequencies the voltage gain is the expected gain.

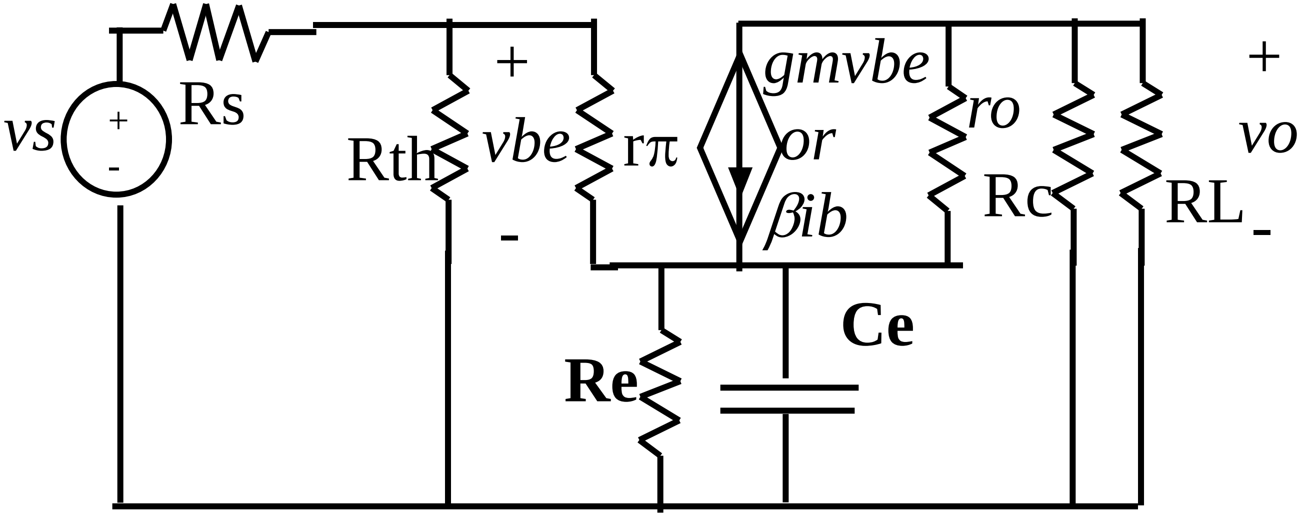
$$\text{Avmid} = \frac{-gm\ \text{RyRx}}{(\text{Rs+Rx})} = \frac{-\beta\ \text{RyRx}}{r\pi(\text{Rs+Rx})}$$

4. Summary 1, 2, and 3 above are true but the results are slightly different if the emitter bypass capacitor acts like an open near where Cc begins to open. (start with new ac equivalent circuit)

EFFECT OF Ce ON FREQUENCY RESPONSE

The ac equivalent circuit of the CE amplifier on page 14 above is shown. Here we assume Cc is a short (note: it is possible that Cc acts like an open rather than a short)

Let Rs = 0 and RL = *ro* = infinity to simplify the algebra



EFFECT OF Ce ON FREQUENCY RESPONSE

The gain function:

$$vo = -\ \beta\ ib\ Rc$$
$$vs = ib\ r\pi + (\beta+1)\ ib\ Re//(1/sCe)$$

$$\boxed{vo/vs = -\beta\ Rc\ \frac{1}{r\pi + (\beta+1)\ Re//(1/sCe)}}$$

Manipulate the gain function:

$$vo/vs = -\beta\ Rc\ \frac{1}{r\pi + \frac{(\beta+1)\ Re/sCe}{Re + 1/sCe}} = -\beta\ Rc\ \frac{1}{r\pi + \frac{(\beta+1)\ Re}{sCeRe + 1}}$$

EFECT OF Ce ON FREQUENCY RESPONSE

$$vo/vs \equiv -\beta\ Rc \frac{sCeRe + 1}{(sCeRe + 1)\ r\pi + (\beta+1)\ Re}$$

$$= -\beta\ Rc \frac{sCeRe + 1}{sCeRe\ r\pi + r\pi + (\beta+1)\ Re} = \frac{-\beta\ Rc}{r\pi + (\beta+1)\ Re} \frac{sCeRe + 1}{\frac{sCeRe\ r\pi}{r\pi + (\beta+1)\ Re} + 1}$$

$$\boxed{vo/vs = \underbrace{\frac{-\beta\ Rc}{r\pi + (\beta+1)\ Re}}_{k=Avlow} \frac{(j\omega/\omega e + 1)}{(j\omega/\omega e1 + 1)}}$$

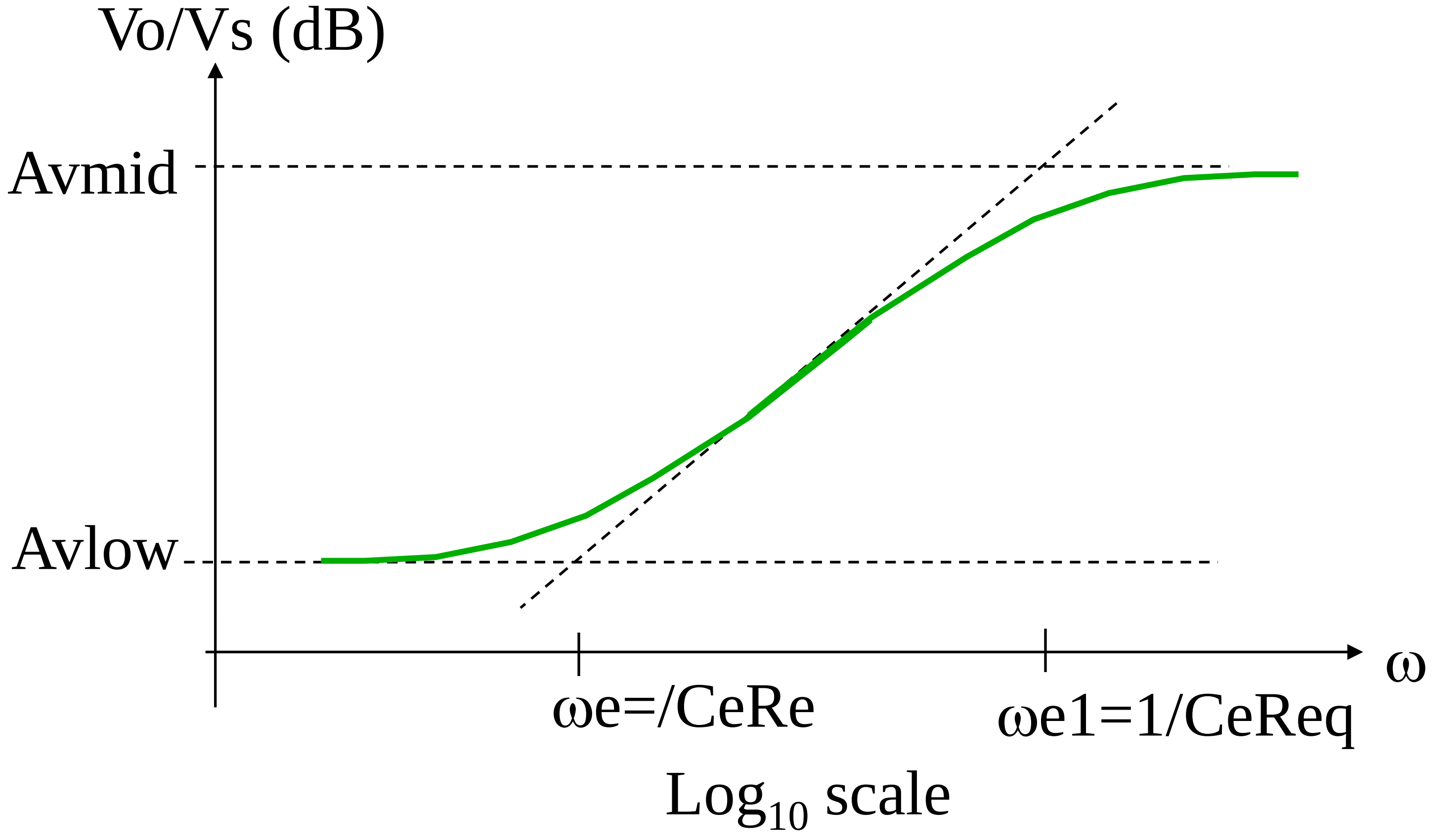
Where: $\omega e = 1/Ce\ Re$

$\omega e1 = 1/(Ce\ Re//(r\pi/(\beta+1)))$

Note: $\omega e1$ is always $> \omega e$

Note: $Avmid = Avlow\ \omega e1/\omega e$

EFFECT OF Ce ON FREQUENCY RESPONSE



SUMMARY FOR EFFECT OF Ce

1. At low frequencies the bypass capacitor, Ce, opens up and the voltage gain becomes that of an unbypassed CE amplifier, Avlow

2. At high frequencies the gain is Avmid

3. Because of 1 and 2 we see that there are two corner frequencies. They are:

$\omega e = 1/ReCe$

and $\omega e1 = 1/ReqCe$ where Req is the resistance seen from the terminals of Ce

$Req = Re//r\pi/(\beta+1)$ if Rs = 0 and Cc “short”

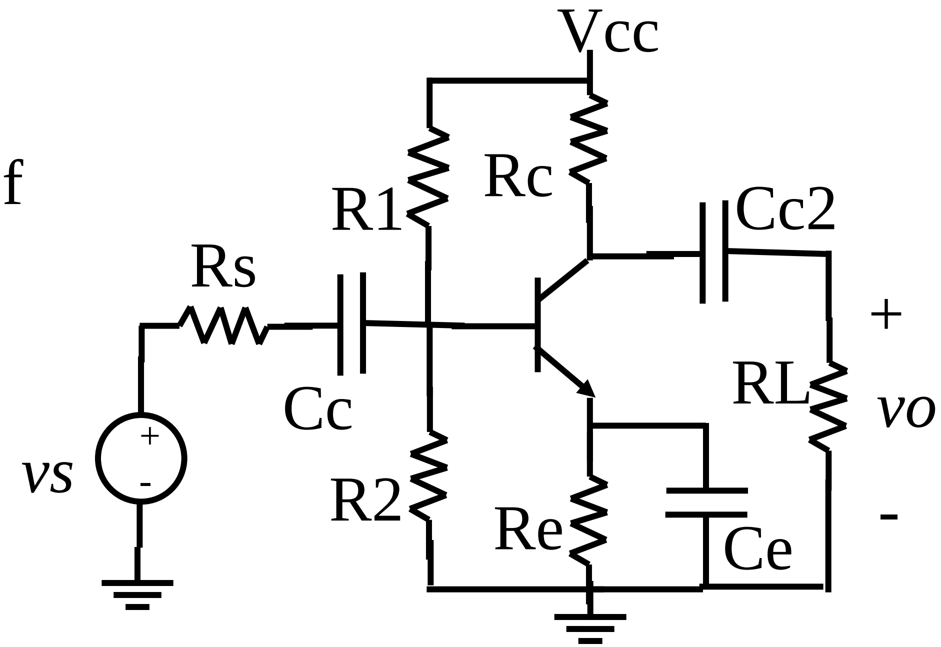
$Req = Re//(r\pi + R1//R2//Rs)/(\beta+1)$ if Rs not 0 and Cc “short”

$Req = Re//(r\pi + R1//R2)/(\beta+1)$ if Cc “open”

COMPLETE CE AMPLIFIER LOW FREQUENCY RESPONSE

Rs = 2K
R1 = 40K
R2 = 10K
RC = 4K
Re = 1K
RL = 2K

β = 100
Vcc = 20
Cc = Ce = Cc2 = 10µf



Find k, ω1, ω2, ω3, ω4

$$vo/vs = \frac{K\ (j\omega/\omega1)\ (j\omega/\omega2)\ (j\omega/\omega3+1)}{(j\omega/\omega1+1)\ (j\omega/\omega2+1)\ (j\omega/\omega4+1)}$$

EXAMPLE: SOLUTION

DC analysis:

Avmid = Voltage gain including RS and RL assume all C’s shorts

EXAMPLE: SOLUTION

$\omega1 = 1/Req\ Cc1$ assume Ce is open unless Ce is 10X Cc1

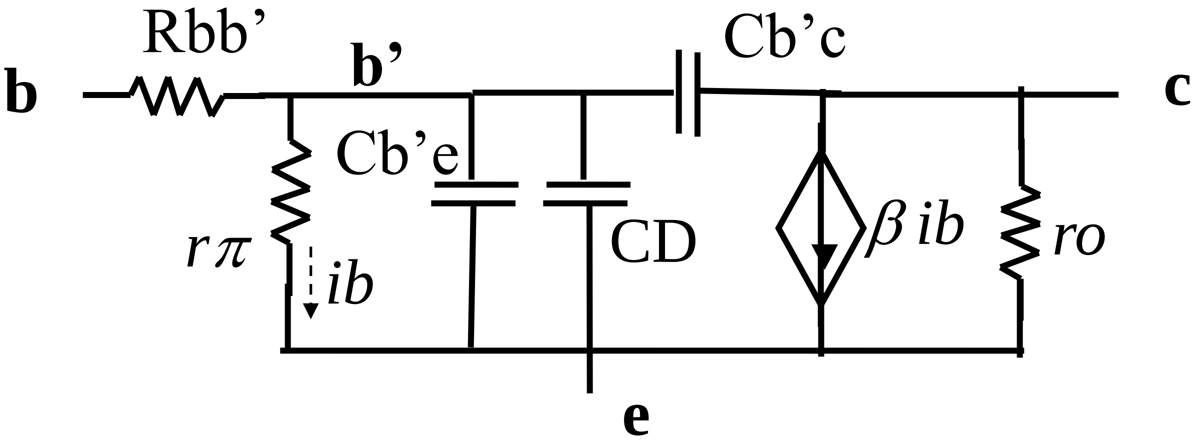
$\omega2 = 1/Req\ Cc2$

$\omega3 = 1/ReCe$

$\omega4 = 1/ReqCe$

K =Avlow =

HIGH FREQUENCY BJT TRANSISTOR MODEL



Rbb' is the series base resistance

rπ is the base emitter small signal junction resistance

Cb'e is the base emitter junction capacitance

Cb'c is the base collector junction capacitance

CD is the diffusion capacitance, represents the change in charge stored in the base caused by a change in base emitter voltage

ro is the small signal output resistance = VA/IC

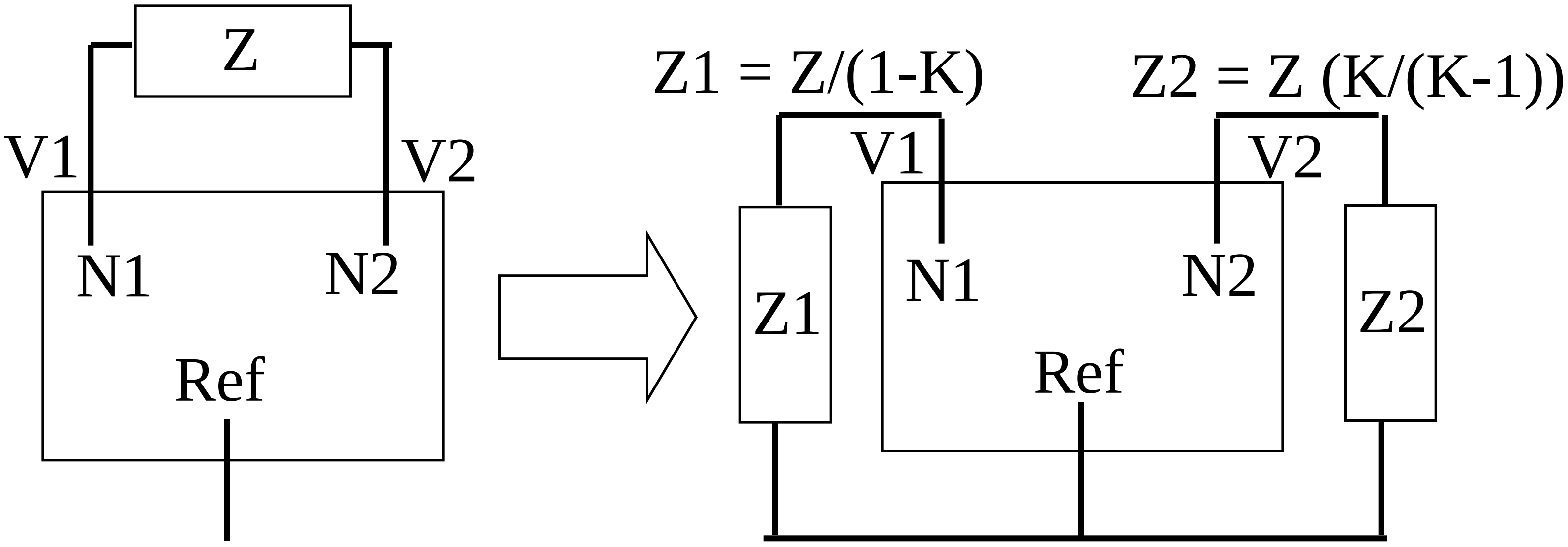
β is the short circuit common emitter current gain

MILLERS THEOREM

To predict the high frequency response of a common emitter amplifier we want to do some quick calculations. We would like to simplify the model given on the previous page. We can do this with the aid of Miller's theorem. The resulting model is approximate and might not give good results above the upper corner frequency where the voltage gain begins to fall off.

Millers Theorem: Consider a linear network with N nodes. An impedance, Z, between any two nodes, N1 and N2, can be removed and another impedance Z1 placed from N1 to reference and impedance Z2 placed from N2 to reference. If Z1 = Z/(1-K) and Z2 = ZK/(K-1) where K=V2/V1, then the nodal equations will not be changed and the resulting circuit will yield equivalent node voltages, V1, V2, etc.

MILLERS THEOREM



$Z1 = Z/(1-K)$ $Z2 = Z\ (K/(K-1))$

where $K=V2/V1$

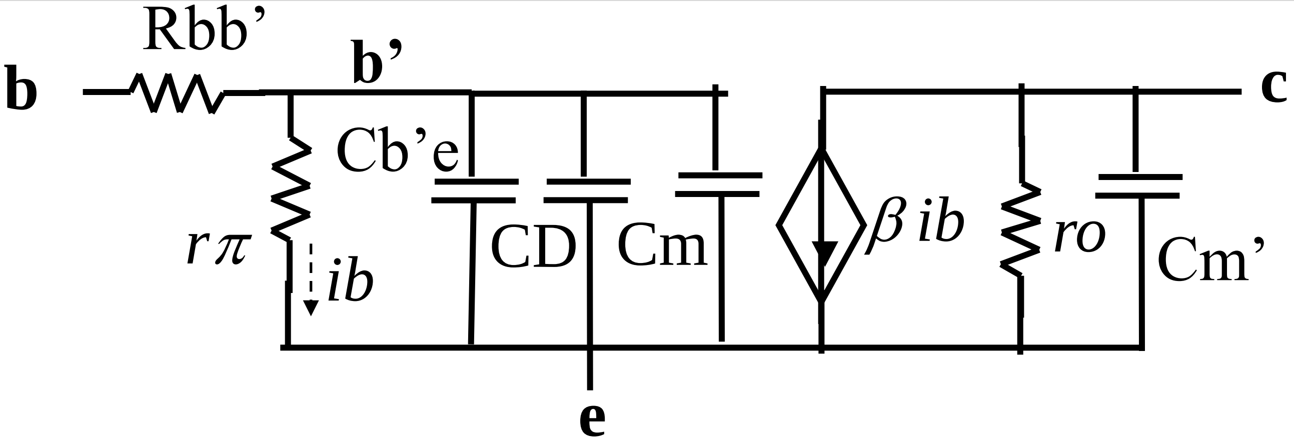
at N1 term $(V1-V2)/Z$

at N1 term $V1/Z1 = V1/(Z/(1-K))$

$= V1/(Z/(1-V2/V1))$

$= (V1-V2)/Z$

HIGH FREQUENCY MODEL OF CE AMPLIFIER



From: $Z1 = Z/(1\text{-}K)$ we have $1/sCm = \dfrac{1/sCb'c}{1\text{-}V2/V1}$

$V2 = -\beta\, ib\, ro$ and $V1 = ib\, r\pi$

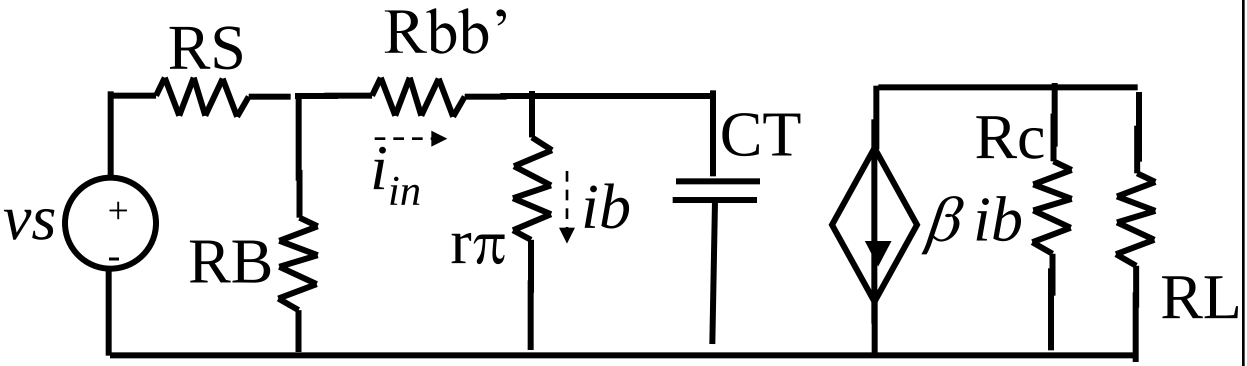
Therefore: $\boxed{Cm = Cb'c\,(1\text{-}\underbrace{-\ \beta\ ro/r\pi}_{\text{Voltage gain}})}$

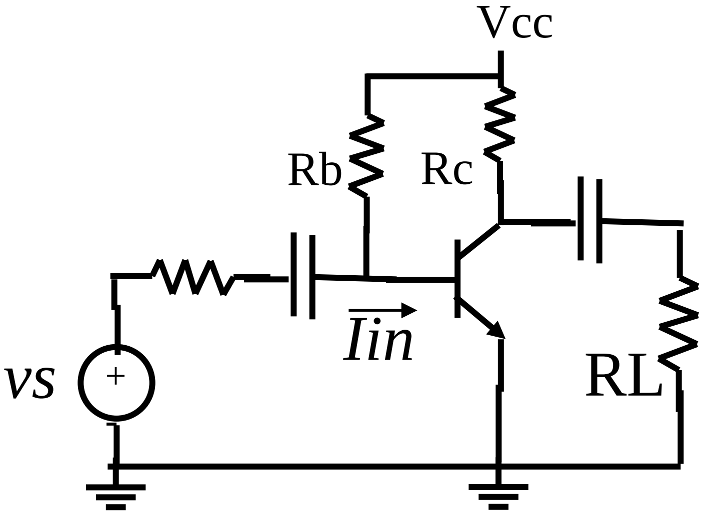
From: $Z2 = Z\,(K/(K\text{-}1))$

and : $\boxed{Cm' = \sim Cb'c}$

EXAMPLE: HIGH FREQUENCY CE AMPLIFIER

β = 100
rπ = 1K, Rb'b = 0
Cb'c = 20pf
Cb'e + CD = 20pf + 1000pf





Let CT = Cb'e + CD + Cm

and Cm = Cb'c(1- - β RL/rπ)

To find the gain function:

$$\left.\begin{array}{l} vo = -\beta\, ib\, Rc//RL \\ ib = Vb'e/r\pi \\ Vb'e = vs\,\dfrac{(RB//r\pi//(1/sCT)}{RS + (RB//r\pi//(1/sCT)} \end{array}\right\} \text{Next pg}$$

EXAMPLE: HIGH FREQUENCY CE AMPLIFIER

The gain function:

$$\boxed{vo/vs = \frac{-\beta\ Rc//RL}{r\pi}\ \frac{(RB//r\pi//(1/sCT)}{RS + (RB//r\pi//(1/sCT)}}$$

Manipulate the gain function: Let RB//rπ = R

$$vo/vs = \frac{-\beta\ Rc//RL}{r\pi}\ \frac{\frac{R(1/sCT)}{R+(1/sCT)}}{RS + \frac{R(1/sCT)}{R+(1/sCT)}}$$

$$vo/vs = \frac{-\beta\ RL}{r\pi}\ \frac{\frac{R}{sCT\ R+1}}{RS + \frac{R}{s\ CT\ R+1}}$$

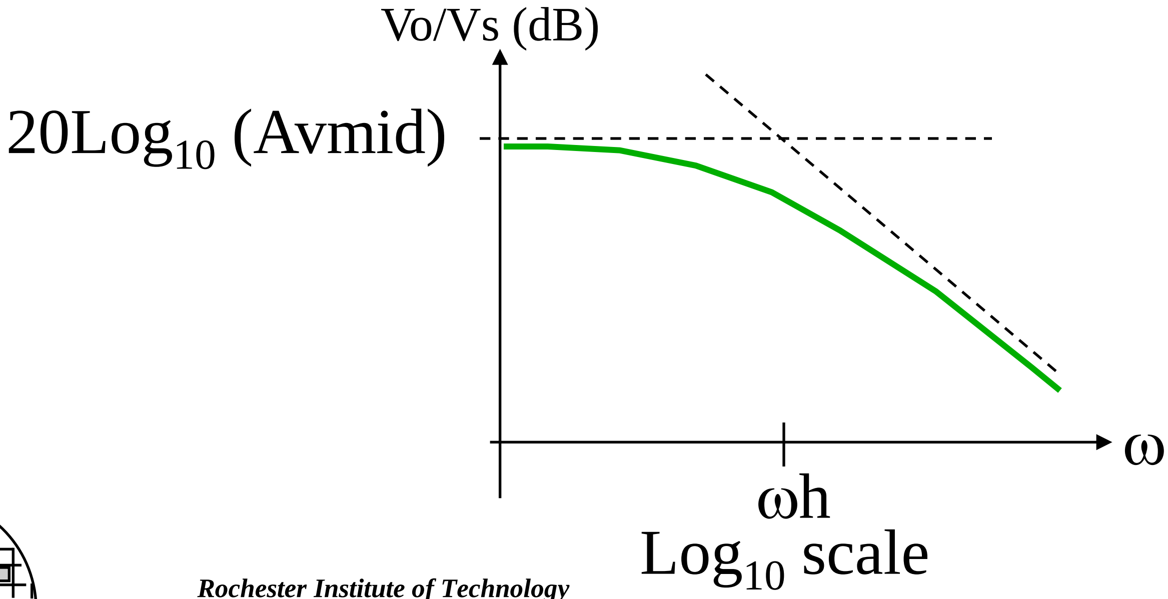
$$vo/vs = \frac{-\beta\ RL}{r\pi}\ \frac{R}{(s\ CT\ R+1)RS + R}$$

$$\boxed{vo/vs = \frac{-\beta\ Rc//RL}{r\pi}\ \frac{R}{(RS + R)}\ \frac{1}{\left(\frac{s\ CT\ R\ RS}{(RS + R)} + 1\right)}}$$

EXAMPLE: HIGH FREQUENCY CE AMPLIFIER

$$vo/vs = \underbrace{\frac{-\ \beta\ Rc//RL}{r\pi}\ \frac{R}{(RS + R)}}_{Avmid} \left(\frac{1}{j\omega\ \frac{CT\ R\ RS}{(RS + R)} + 1}\right)$$

$$\omega h = 1/\ CT\ (R//RS)$$



SUMMARY: HIGH FREQUENCY RESPONSE OF CE AMP

1. At high frequencies the internal capacitances in the transistor causes the voltage gain to decrease

2. At mid frequencies the gain is $\text{Avmid} = -\beta \dfrac{\text{Rc//RL}}{r\pi} \dfrac{\text{R}}{(\text{RS} + \text{R})}$

3. The corner frequency is $\omega\text{h} = 1/\ (\text{Req CT})$

where $\text{CT} = \text{Cb'e} + \text{CD} + \text{Cm}$

and Req = the equivalent resistance as “seen” from the terminals of the capacitor CT. (vs = zero)

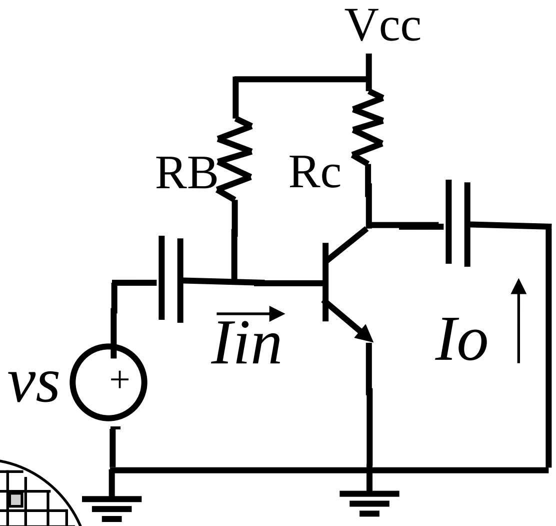
4. There is a second corner due to the miller capacitance Cm’. Since ωh occurs first we are not normally interested in the corner due to Cm’

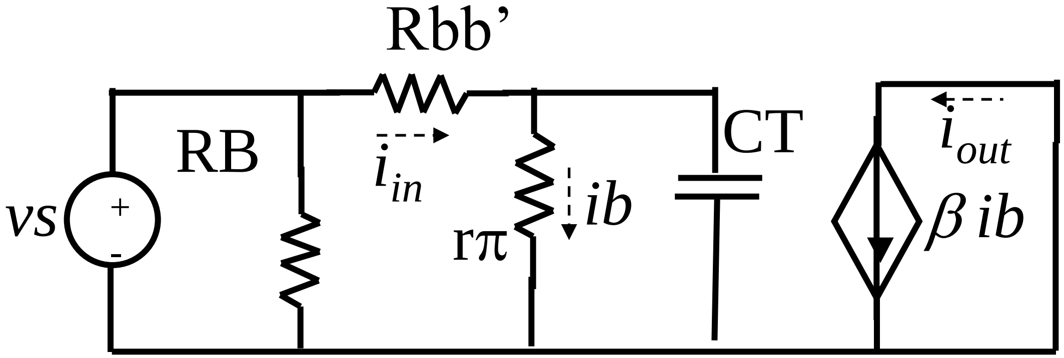
HOW DOES MANUFACTURER SPECIFY CD, Cbe, Cbc

Cb’c is usually given by the manufacturer as the common base output capacitance which it is.

Cb’e and CD are given indirectly by the manufacturers specification of the transition frequency fT

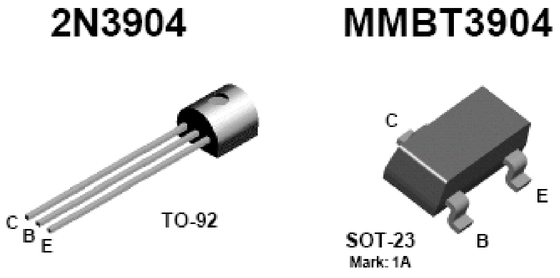
fT is the frequency at which the CE short circuit current gain goes to 1





2N3904

FAIRCHILD
SEMICONDUCTOR ™



SMALL SIGNAL CHARACTERISTICS

f_T	Current Gain - Bandwidth Product	I_C = 10 mA, V_{CE} = 20 V, f = 100 MHz	300		MHz
C_{obo}	Output Capacitance	V_{CB} = 5.0 V, I_E = 0, f = 1.0 MHz		4.0	pF
C_{ibo}	Input Capacitance	V_{EB} = 0.5 V, I_C = 0, f = 1.0 MHz		8.0	pF
NF	Noise Figure	I_C = 100 μA, V_{CE} = 5.0 V, R_S =1.0kΩ,f=10 Hz to 15.7kHz		5.0	dB

NPN General Purpose Amplifier

This device is designed as a general purpose amplifier and switch. The useful dynamic range extends to 100 mA as a switch and to 100 MHz as an amplifier.

Absolute Maximum Ratings* T_A = 25°C unless otherwise noted

Symbol	Parameter	Value	Units
V_{CEO}	Collector-Emitter Voltage	40	V
V_{CBO}	Collector-Base Voltage	60	V
V_{EBO}	Emitter-Base Voltage	6.0	V
I_C	Collector Current - Continuous	200	mA
T_J, T_{stg}	Operating and Storage Junction Temperature Range	-55 to +150	°C

*These ratings are limiting values above which the se

Rb = 10 ohms

Spice Model

NPN (Is=6.734f Xti=3 Eg=1.11 Vaf=74.03 Bf=416.4 Ne=1.259 Ise=6.734 Ikf=66.78m Xtb=1.5 Br=.7371 Nc=2 Isc=0 Ikr=0 Rc=1 Cjc=3.638p Mjc=.3085 Vjc=.75 Fc=.5 Cje=4.493p Mje=.2593 Vje=.75 Tr=239.5n Tf=301.2p Itf=.4 Vtf=4 Xtf=2 Rb=10)

ANALYSIS OF SHORT CIRCUIT CURRENT GAIN TO EXTRACT Cb'e + CD

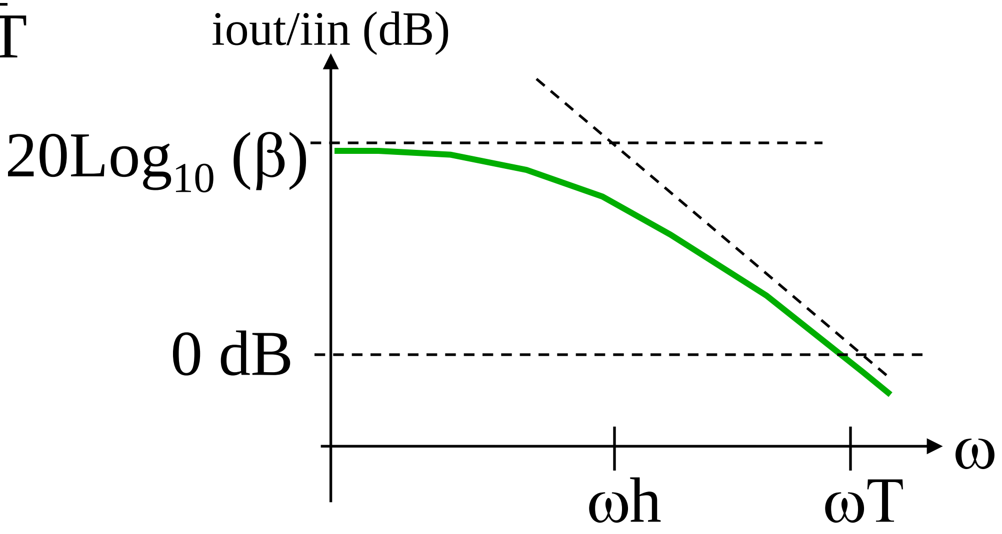
$$iout = \beta\ ib$$

$$ib = iin\ \frac{1/sCT}{r\pi + 1/sCT}$$

$$iout/iin = \frac{\beta}{sCT\ r\pi + 1}$$

$$iout/iin = \frac{\beta}{j\omega CT\ r\pi + 1}$$

$$\boxed{iout/iin = \frac{\beta}{j\omega/\omega b + 1}}$$



$\omega b = 1/(CT\ r\pi)$

$\omega T = 2\pi\ fT$ = transition freq in radians/s

ANALYSIS OF SHORT CIRCUIT CURRENT GAIN TO EXTRACT Cb'e + CD

at ωT, $\text{iout/in} = 1 = \sim \dfrac{\beta}{j\omega/\omega b}$

$$2\,\pi\,fT = \frac{\beta}{CT\,r\pi}$$

So $\quad CT = \dfrac{\beta}{2\,\pi\,fT\,r\pi} = Cb'e + CD + Cm$

and Cm = Cb'c since Av = zero

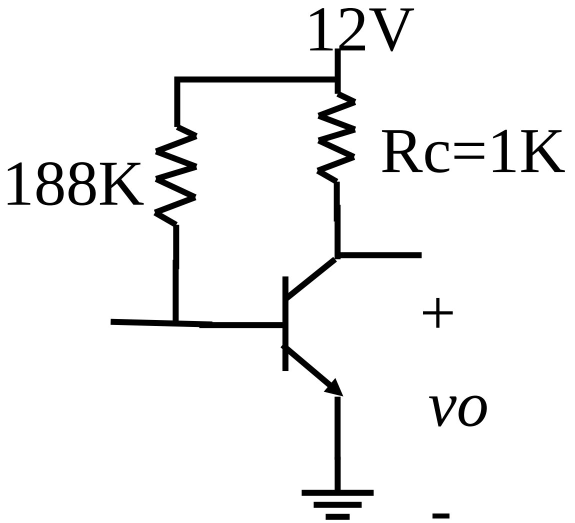
Finally

$$\boxed{Cb'e + CD = \frac{\beta}{2\,\pi\,fT\,r\pi} - Cb'c}$$

EXAMPLE: DETERMINATION OF Cb’c, Cb’e +CD

Given:
1. Common-Base Open Circuit Output Capacitace of 12 pf is measured at f = 1 Mhz, VCB = 10V and IE = zero.
2. A transition frequency of 100 Mhz is measured using the following test conditions, VCE = 2V, IC = 50mA, β response with frequency is extrapolated at -20 dB/Dec to fT at which β = 1 from f = 20Mhz where β =100

Find Cb’c,. Cb’e + CD



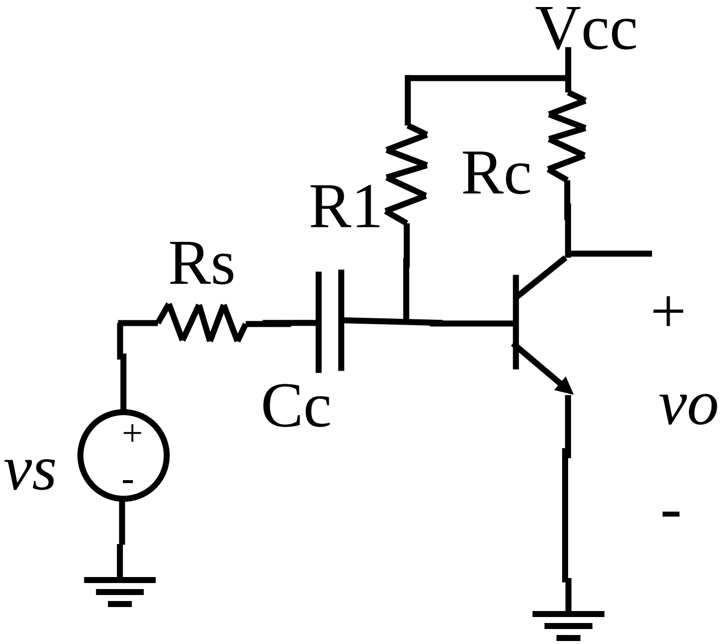
SOLUTION TO EXAMPLE ON PREVIOUS PAGE

ANOTHER EXAMPLE

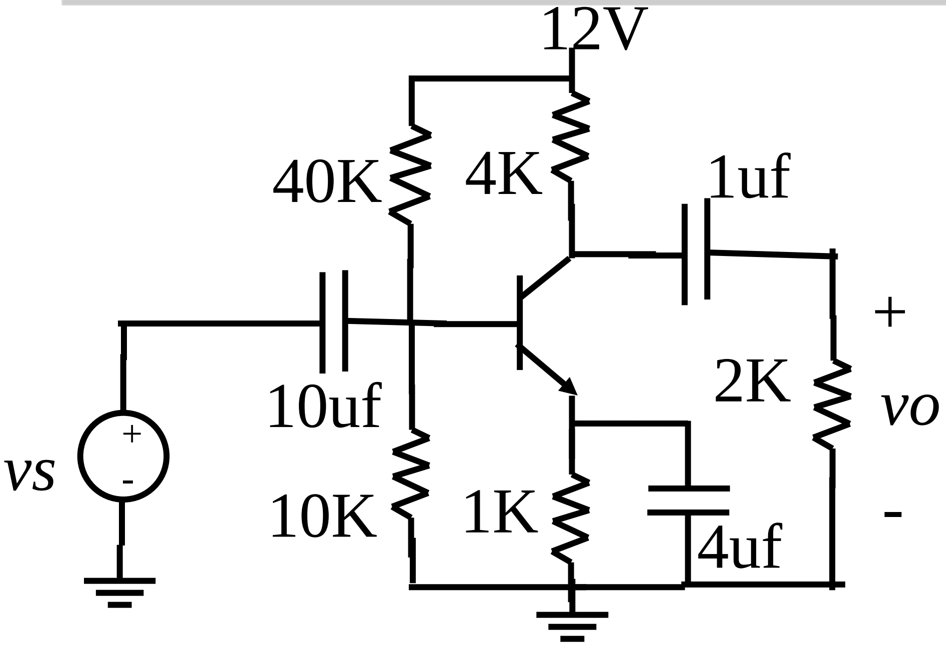
Vcc = 15
Rs = 100
R1 = 150K
RC = 500

β = 100
VA = infinity
Rb'b = 100
Cb'c = 20pf at Vcb = 5
fT = 100 Mhz

Find rπ, Cm, CT and ωh



EXAMPLE FROM OLD EXAM



Assume $\beta = 150$
VA = 100
Cb'c = 10pF @ 10 V
fT = 200 MHz
Rb'b = 100 ohms

Find k, ω1, ω2, ω3, ω4, each 5 pts and ωh 10 pts

$$vo/vs = \frac{k \;(j\omega/\omega1)\;(j\omega/\omega2)\;(j\omega/\omega3+1)}{(j\omega/\omega1+1)\;(j\omega/\omega2+1)\;(j\omega/\omega4+1)\;(j\omega/\omega h+1)}$$

REFERENCES

1. Sedra and Smith, chapter 5.
2. Device Electronics for Integrated Circuits, 2nd Edition, Kamins and Muller, John Wiley and Sons, 1986.
3. The Bipolar Junction Transistor, 2nd Edition, Gerald Neudeck, Addison-Wesley, 1989.
4. Data sheets for 2N3904

HOMEWORK – FREQUENCY RESPONSE OF CE AMP

1. For the circuit on page 22 find Avmid, k, ω1, ω2, ω3, ω4 given:

Rs = 1K
R1 = 50K
R2 = 10K
RC = 5K
Re = 1K
RL = 5K

β = 150
Vcc = 24
Cc = 1uf
Ce = 2uf
Cc2 = 10μf

2. Create a spread sheet to analyze CE circuits like that in problem 1 to find Avlow, Avmid, k, low frequency corners.
Extra points if you also do high frequency analysis?

3. If Cb’c is measured at Vcb = 5 what is it at Vcb=10?

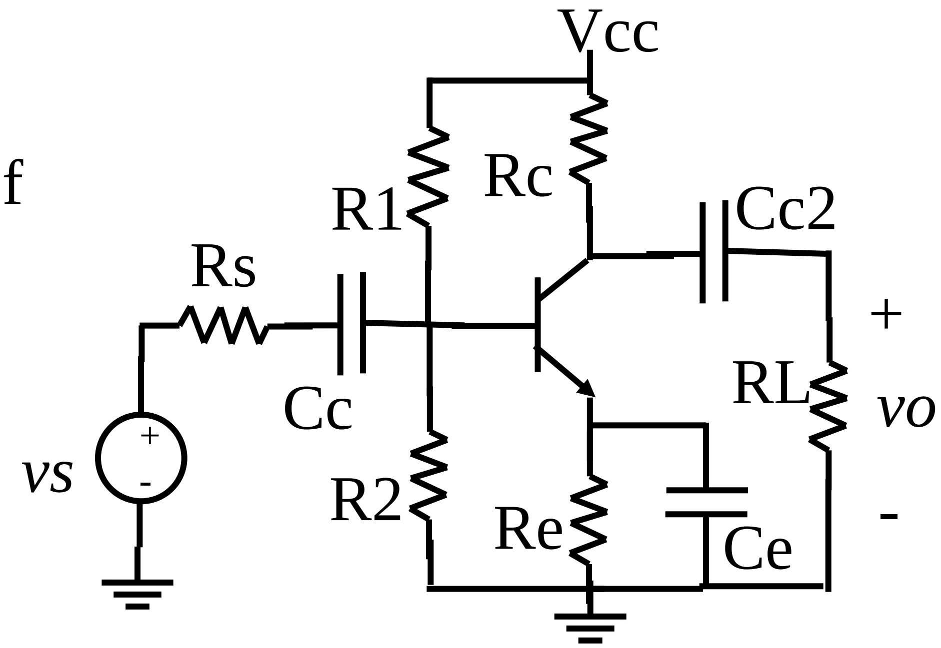
4. Find Cb’e + CD for fT = 200 Mhz and IC = 5mA, β = 150 and Cb’c = 10pf

5. Create a spread sheet to calculate and graph the magnitude part of the Bode Plot given k, ω1, ω2, ω3, ω4 and ωh

EXAMPLE PROBLEM FROM PAGE 22

Rs = 2K
R1 = 40K
R2 = 10K
RC = 4K
Re = 1K
RL = 2K

$\beta = 100$
Vcc = 20
Cc = Ce = Cc2 = 10µf



Find k, ω1, ω2, ω3, ω4

$$vo/vs = \frac{K\ (j\omega/\omega1)\ (j\omega/\omega2)\ (j\omega/\omega3+1)}{(j\omega/\omega1+1)\ (j\omega/\omega2+1)\ (j\omega/\omega4+1)}$$

EXAMPLE pg 22: SOLUTION

DC analysis: Rth = R1//R2 = (10)(40)/(10+40) = 8K

Vth= Vcc R2/(R1+R2) = 20 (10)/(10+40) = 4V

KVL: IB Rth +0.7 +(B+1)IB Re – Vth = 0

IB = 4 - 0.7 / (Rth +101K) = 30.3uA

IC = B IB = 100 (30.3uA) = 3.03 mA

gm = IC/VT = 3.03/0.026 = 117 mS

rπ = Vt/IB = 0.026/30.3uA = 858 ohms

ro = VA/IC = assume large

Avmid = Voltage gain including RS and RL assume all C's shorts

Vo = gm Rc//RL Vin

Vin = Vs Rin/(Rin + Rs)

Vo/Vs = Vo/Vin x Vin/Vs = -(gmRC//RL){Rin/(Rin+Rs)}

Vo/Vs = -117m (4K//2K) (Rth//rπ)/((Rth//rπ)+2K)

= - 43.6

EXAMPLE pg 22: SOLUTION(continued)

w1 = 1/Req Cc1 assume Ce is open unless it is 10X Cc1

Req = Rs+Rth// (rπ +(B+1)Re) = 2K+ 8K // (0.858K +101K)
= 9.42K

w1 = 1/ (9.42K 10 uF) = 10.6 r/s or 1.69 Hz

w2 = 1/Req Cc2

Req = RL + Rc = 6K

w2 = 1 / (6K 10uF) = 16.7 r/s or 2.65 Hz

w3 = 1/ReCe = 1/(1K 10uF) = 100 r/s or 15.9 Hz

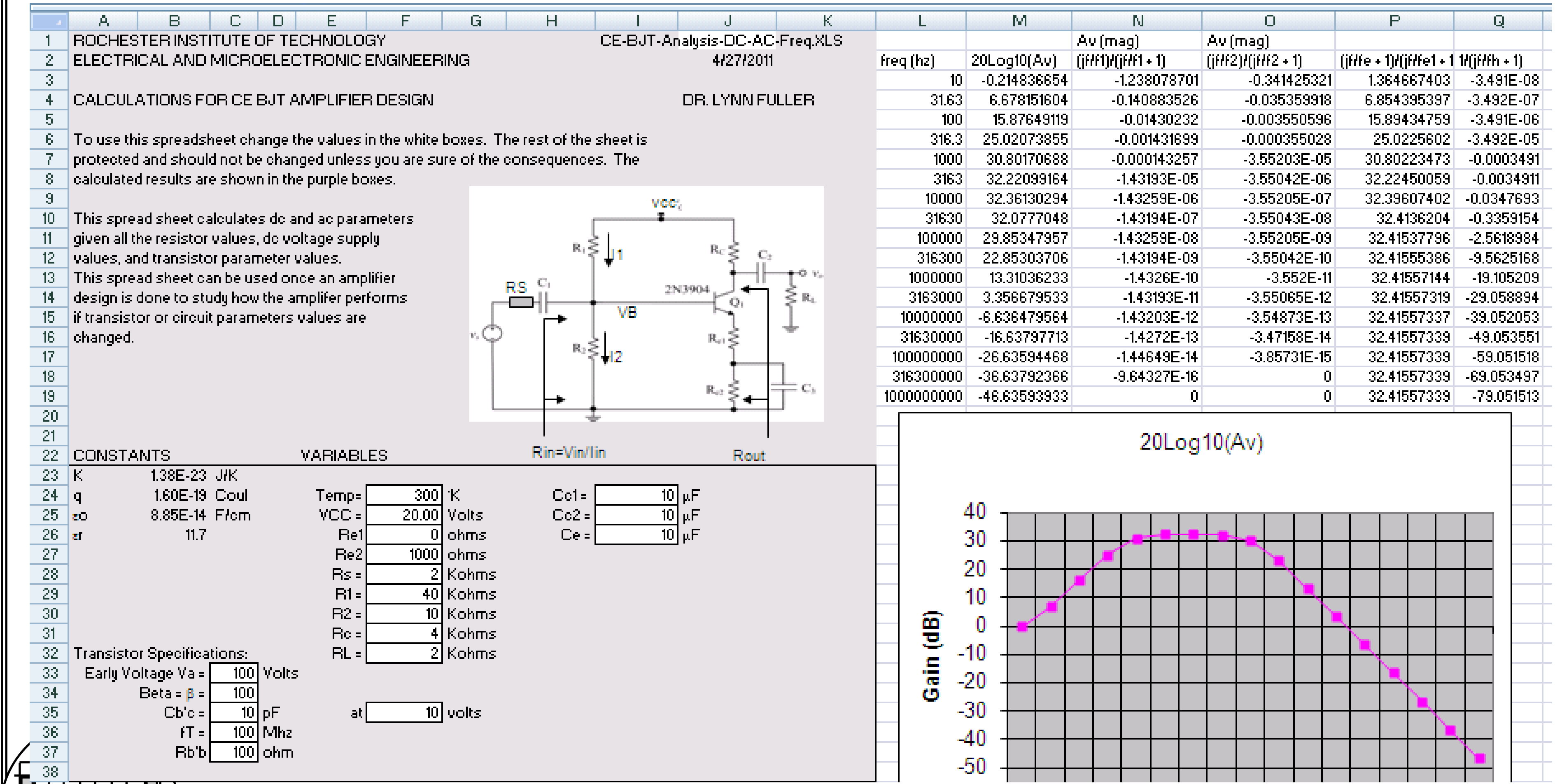
w4 =1/ReqCe

Req = Re//((rπ+Rth//Rs)/(B+1)) = 1K//((0.858K+8K//2K)/101)
= 24.3 ohms

w4 = 1/(24.3 10uF) = 4214 r/s or 671 Hz

K =Avlow = Avmid w3/w4 = -42 (100/4214) = -0.99

SPREAD SHEET SOLUTION



ROCHESTER INSTITUTE OF TECHNOLOGY CE-BJT-Analysis-DC-AC-Freq.XLS
ELECTRICAL AND MICROELECTRONIC ENGINEERING 4/27/2011

CALCULATIONS FOR CE BJT AMPLIFIER DESIGN DR. LYNN FULLER

To use this spreadsheet change the values in the white boxes. The rest of the sheet is protected and should not be changed unless you are sure of the consequences. The calculated results are shown in the purple boxes.

This spread sheet calculates dc and ac parameters given all the resistor values, dc voltage supply values, and transistor parameter values.
This spread sheet can be used once an amplifier design is done to study how the amplifer performs if transistor or circuit parameters values are changed.

		Av (mag)	Av (mag)		
freq (hz)	20Log10(Av)	(jf/f1)/(jf/f1 + 1)	(jf/f2)/(jf/f2 + 1)	(jf/fe + 1)/(jf/fe1 + 1	1/(jf/fh + 1)
10	-0.214836654	-1.238078701	-0.341425321	1.364667403	-3.491E-08
31.63	6.678151604	-0.140883526	-0.035359918	6.854395397	-3.492E-07
100	15.87649119	-0.01430232	-0.003550596	15.89434759	-3.491E-06
316.3	25.02073855	-0.001431699	-0.000355028	25.0225602	-3.492E-05
1000	30.80170688	-0.000143257	-3.55203E-05	30.80223473	-0.0003491
3163	32.22099164	-1.43193E-05	-3.55042E-06	32.22450059	-0.0034911
10000	32.36130294	-1.43259E-06	-3.55205E-07	32.39607402	-0.0347693
31630	32.0777048	-1.43194E-07	-3.55043E-08	32.4136204	-0.3359154
100000	29.85347957	-1.43259E-08	-3.55205E-09	32.41537796	-2.5618984
316300	22.85303706	-1.43194E-09	-3.55042E-10	32.41555386	-9.5625168
1000000	13.31036233	-1.4326E-10	-3.552E-11	32.41557144	-19.105209
3163000	3.356679533	-1.43193E-11	-3.55065E-12	32.41557319	-29.058894
10000000	-6.636479564	-1.43203E-12	-3.54873E-13	32.41557337	-39.052053
31630000	-16.63797713	-1.4272E-13	-3.47158E-14	32.41557339	-49.053551
100000000	-26.63594468	-1.44649E-14	-3.85731E-15	32.41557339	-59.051518
316300000	-36.63792366	-9.64327E-16	0	32.41557339	-69.053497
1000000000	-46.63593933	0	0	32.41557339	-79.051513

CONSTANTS

K	1.38E-23	J/K
q	1.60E-19	Coul
εo	8.85E-14	F/cm
εr	11.7	

VARIABLES

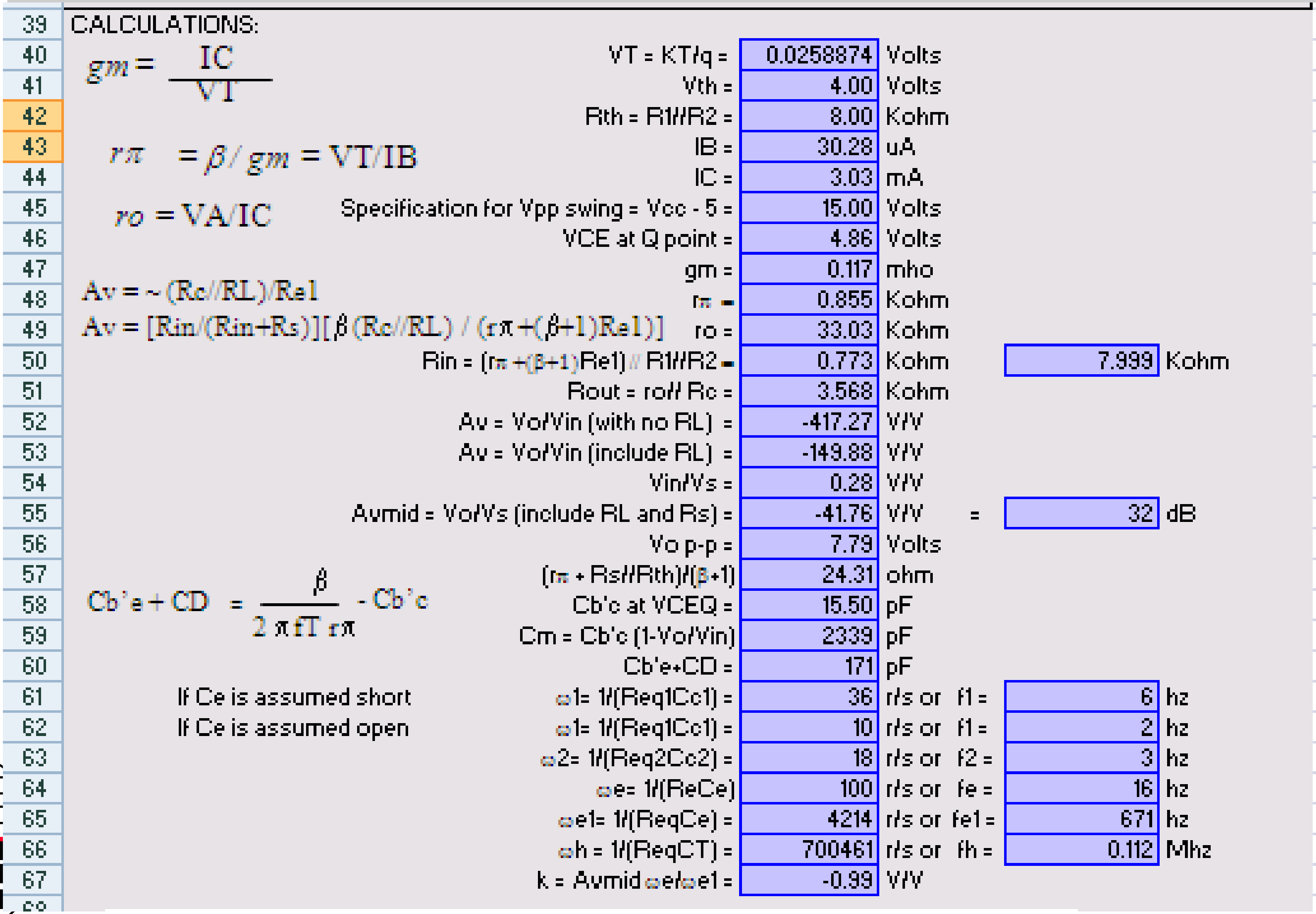
Temp=	300	°K
VCC =	20.00	Volts
Re1	0	ohms
Re2	1000	ohms
Rs =	2	Kohms
R1 =	40	Kohms
R2 =	10	Kohms
Rc =	4	Kohms
RL =	2	Kohms

Cc1 =	10	µF
Cc2 =	10	µF
Ce =	10	µF

Transistor Specifications:

Early Voltage Va =	100	Volts
Beta = β =	100	
Cb'c =	10	pF at 10 volts
fT =	100	Mhz
Rb'b	100	ohm

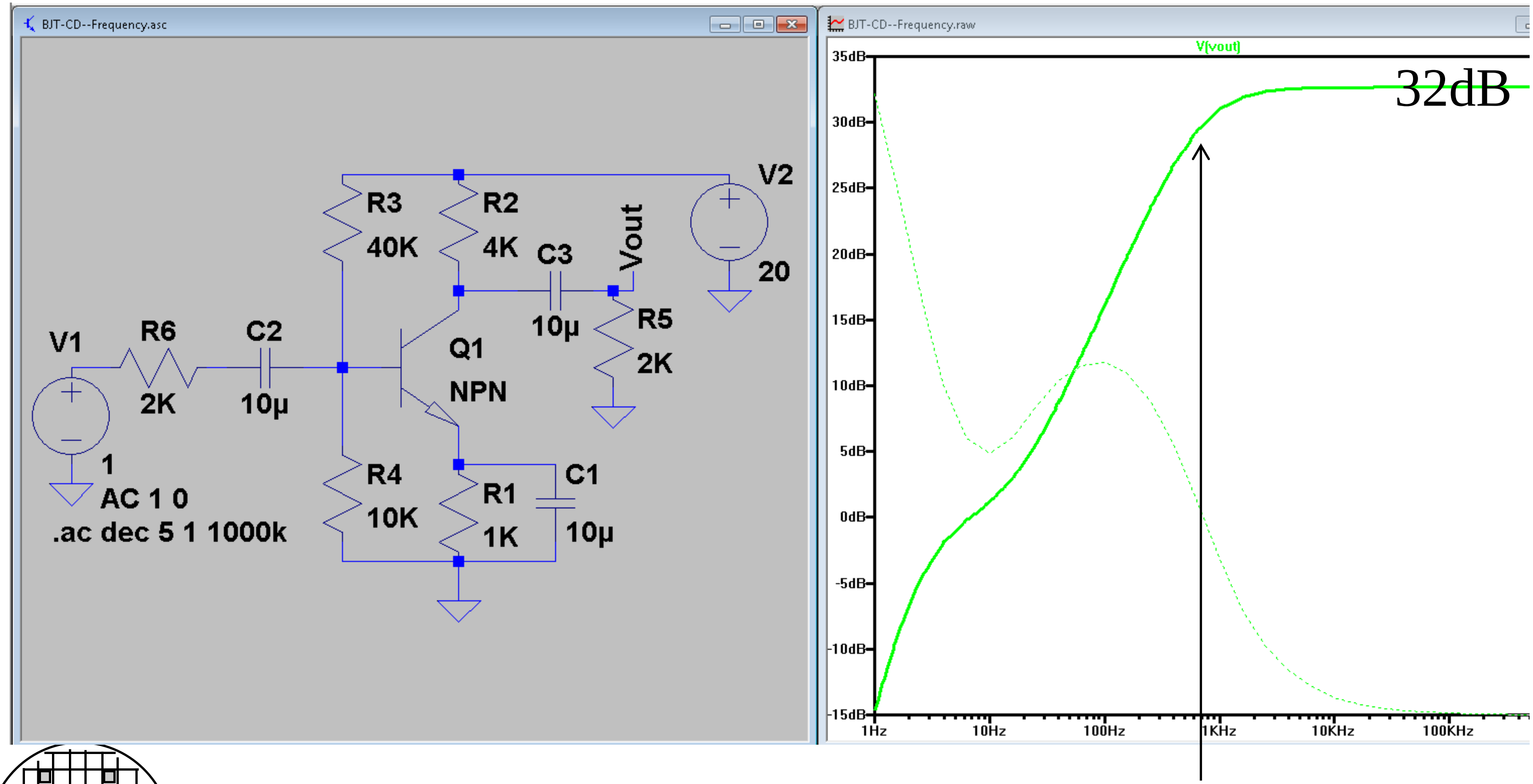
SPREAD SHEET SOLUTION



39	CALCULATIONS:						
40	$gm = \frac{IC}{VT}$	VT = KT/q =	0.0258874	Volts			
41		Vth =	4.00	Volts			
42		Rth = R1//R2 =	8.00	Kohm			
43	$r\pi = \beta / gm = VT/IB$	IB =	30.28	uA			
44		IC =	3.03	mA			
45	$ro = VA/IC$	Specification for Vpp swing = Vcc - 5 =	15.00	Volts			
46		VCE at Q point =	4.86	Volts			
47		gm =	0.117	mho			
48	$Av = \sim (Rc//RL)/Re1$	rπ =	0.855	Kohm			
49	$Av = [Rin/(Rin+Rs)][\beta(Rc//RL) / (r\pi+(\beta+1)Re1)]$	ro =	33.03	Kohm			
50		Rin = (rπ +(β+1)Re1) // R1//R2 =	0.773	Kohm		7.999	Kohm
51		Rout = ro// Rc =	3.568	Kohm			
52		Av = Vo/Vin (with no RL) =	-417.27	V/V			
53		Av = Vo/Vin (include RL) =	-149.88	V/V			
54		Vin/Vs =	0.28	V/V			
55		Avmid = Vo/Vs (include RL and Rs) =	-41.76	V/V	=	32	dB
56		Vo p-p =	7.79	Volts			
57		(rπ + Rs//Rth)/(β+1)	24.31	ohm			
58	$Cb'e + CD = \frac{\beta}{2\pi fT r\pi} - Cb'c$	Cb'c at VCEQ =	15.50	pF			
59		Cm = Cb'c (1-Vo/Vin)	2339	pF			
60		Cb'e+CD =	171	pF			
61	If Ce is assumed short	ω1= 1/(Req1Cc1) =	36	r/s or	f1 =	6	hz
62	If Ce is assumed open	ω1= 1/(Req1Cc1) =	10	r/s or	f1 =	2	hz
63		ω2= 1/(Req2Cc2) =	18	r/s or	f2 =	3	hz
64		ωe= 1/(ReCe)	100	r/s or	fe =	16	hz
65		ωe1= 1/(ReqCe) =	4214	r/s or	fe1 =	671	hz
66		ωh = 1/(ReqCT) =	700461	r/s or	fh =	0.112	Mhz
67		k = Avmid ωe/ωe1 =	-0.99	V/V			

Frequency Response

LTSPICE SOLUTION



```
.ac dec 5 1 1000k
```

32dB

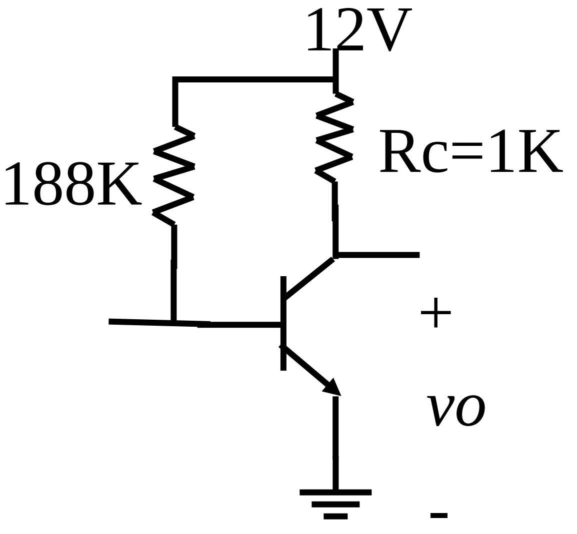
700Hz

Rochester Institute of Technology
Microelectronic Engineering

SOLUTION

Given:
1. Common-Base Open Circuit Output Capacitace of 12 pf is measured at f = 1 Mhz, VCB = 10V and IE = zero.
2. A transition frequency of 100 Mhz is measured using the following test conditions, VCE = 2V, IC = 50mA, β response with frequency is extrapolated at -20 dB/Dec to fT at which β = 1 from f = 20Mhz where β =100

Find Cb'c,. Cb'e + CD



First do DC analysis to find IC and VCB

KVL IB 188K + 0.7 = 12 = 0
IB = (12-0.7)/188K = 60uA
IC = Beta IB = 100 60uA = 6mA
VCB = 12-Rc 6mA – 0.7 = 5.3 volts

SOLUTION TO EXAMPLE ON PREVIOUS PAGE

Find $r\pi = 0.026/IB = 0.026/0.060 \text{ mA} = 433 \text{ ohms}$

Find Cb'c at the VCB of 5.3 volts $= 12\text{pF} \sqrt{\dfrac{10 \text{ volts}}{5.3 \text{ volts}}} = 16.5 \text{ pF}$

$$\boxed{Cb'e + CD = \frac{\beta}{2\ \pi\ fT\ r\pi} - Cb'c}$$

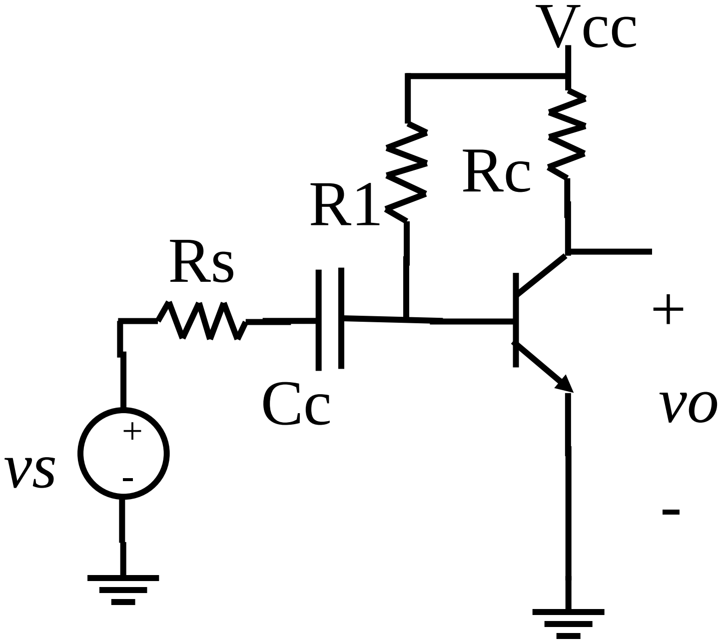
Beta = 100 and fT = 100 MEG

Cb'e + CD = 368 pF – 16.5 pF = 352 pF

SOLUTION

Vcc = 15
Rs = 100
R1 = 150K
RC = 500

β = 100
VA = infinity
Rb’b = 100
Cb’c = 20pf at Vcb = 5
fT = 100 Mhz



Find rπ, Cm, CT and ωh

First do DC analysis to find IC and VCB

KVL IB 150K + 0.7 = 15 = 0
IB = (15-0.7)/150K = 95.3uA
IC = Beta IB = 100 95.3uA = 9.53mA
VCB = 15-Rc 9.53mA – 0.7 = 9.54 volts

SOLUTION TO EXAMPLE ON PREVIOUS PAGE

Find $r_\pi = 0.026/I_B = 0.026/0.0953\ \text{mA} = 273$ ohms

Find $C_{b'c}$ at the V_{CB} of 9.54 volts $= 20\text{pF}\sqrt{\dfrac{5\ \text{volts}}{9.54\ \text{volts}}} = 14.5\ \text{pF}$

$$\boxed{C_{b'e} + C_D = \frac{\beta}{2\ \pi\ f_T\ r_\pi} - C_{b'c}}$$

Beta = 100 and fT = 100 MEG

$C_{b'e} + C_D = 583\ \text{pF} - 14.5\ \text{pF} = 569\ \text{pF}$

SOLUTION TO EXAMPLE ON PREVIOUS PAGE

Voltage gain *Vo / Vb'e* at mid frequencies is used for miller capacitance Calculations.

Vo = -B ib RC//RL//ro = -100 ib 500
ib = Vb'e / rπ = Vb'e / 273

Vo / Vb'e = -183

Cm = Cb'c (1- - 183) = 14.5 pF x 184 = 2668 pF

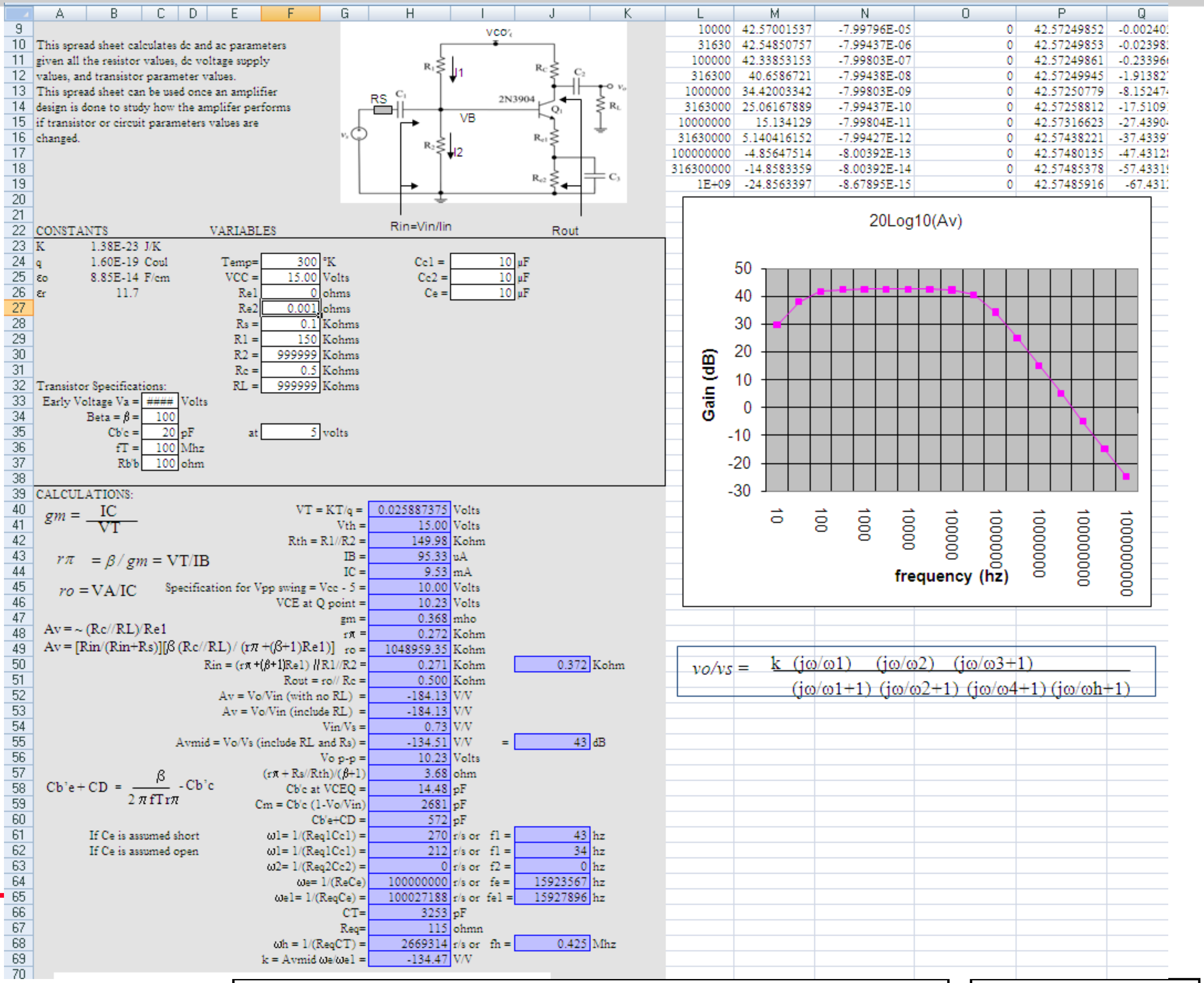
CT = Cb'e + CD + Cm = 569 pF + 2668 pF = 3237 pF

ωh = 1 / Req CT

Req = ((RS // Rth) + Rbb') // rπ = 199.9 // 273 = 115

ωh = 1 / Req CT = 1 / (115 x 3237 pF) = 2.69M r/s = 0.428 MHz

COMPARE TO SPREAD SHEET



This spread sheet calculates dc and ac parameters given all the resistor values, dc voltage supply values, and transistor parameter values.
This spread sheet can be used once an amplifier design is done to study how the amplifer performs if transistor or circuit parameters values are changed.

L	M	N	O	P	Q
10000	42.57001537	-7.99796E-05	0	42.57249852	-0.00240
31630	42.54850757	-7.99437E-06	0	42.57249853	-0.02398
100000	42.33853153	-7.99803E-07	0	42.57249861	-0.23396
316300	40.6586721	-7.99438E-08	0	42.57249945	-1.91382
1000000	34.42003342	-7.99803E-09	0	42.57250779	-8.15247
3163000	25.06167889	-7.99437E-10	0	42.57258812	-17.5109
10000000	15.134129	-7.99804E-11	0	42.57316623	-27.4390
31630000	5.140416152	-7.99427E-12	0	42.57438221	-37.4339
100000000	-4.85647514	-8.00392E-13	0	42.57480135	-47.4312
316300000	-14.8583359	-8.00392E-14	0	42.57485378	-57.4331
1E+09	-24.8563397	-8.67895E-15	0	42.57485916	-67.431

CONSTANTS

K 1.38E-23 J/K
q 1.60E-19 Coul
εo 8.85E-14 F/cm
εr 11.7

VARIABLES

Temp= 300 °K
VCC = 15.00 Volts
Re1 0 ohms
Re2 0.001 ohms
Rs = 0.1 Kohms
R1 = 150 Kohms
R2 = 999999 Kohms
Rc = 0.5 Kohms
RL = 999999 Kohms

Cc1 = 10 µF
Cc2 = 10 µF
Ce = 10 µF

Transistor Specifications:
Early Voltage Va = #### Volts
Beta = β = 100
Cb'c = 20 pF at 5 volts
fT = 100 Mhz
Rb'b 100 ohm

CALCULATIONS:

$$gm = \frac{IC}{VT}$$

$$r\pi \;\; = \beta / gm = VT/IB$$

$$ro = VA/IC$$

Av = ~ (Rc//RL)/Re1
Av = [Rin/(Rin+Rs)][β (Rc//RL)/ (rπ +(β+1)Re1)]

$$Cb'e + CD \;=\; \frac{\beta}{2\,\pi\, fT\, r\pi} - Cb'c$$

If Ce is assumed short
If Ce is assumed open

VT = KT/q = 0.025887375 Volts
Vth = 15.00 Volts
Rth = R1//R2 = 149.98 Kohm
IB = 95.33 uA
IC = 9.53 mA
Specification for Vpp swing = Vcc - 5 = 10.00 Volts
VCE at Q point = 10.23 Volts
gm = 0.368 mho
rπ = 0.272 Kohm
ro = 1048959.35 Kohm
Rin = (rπ +(β+1)Re1) //R1//R2 = 0.271 Kohm 0.372 Kohm
Rout = ro// Rc = 0.500 Kohm
Av = Vo/Vin (with no RL) = -184.13 V/V
Av = Vo/Vin (include RL) = -184.13 V/V
Vin/Vs = 0.73 V/V
Avmid = Vo/Vs (include RL and Rs) = -134.51 V/V = 43 dB
Vo p-p = 10.23 Volts
(rπ + Rs//Rth)/(β+1) 3.68 ohm
Cb'c at VCEQ = 14.48 pF
Cm = Cb'c (1-Vo/Vin) 2681 pF
Cb'e+CD = 572 pF
ω1= 1/(Req1Cc1) = 270 r/s or f1 = 43 hz
ω1= 1/(Req1Cc1) = 212 r/s or f1 = 34 hz
ω2= 1/(Req2Cc2) = 0 r/s or f2 = 0 hz
ωe= 1/(ReCe) 100000000 r/s or fe = 15923567 hz
ωe1= 1/(ReqCe) = 100027188 r/s or fe1 = 15927896 hz
CT= 3253 pF
Req= 115 ohmn
ωh = 1/(ReqCT) = 2669314 r/s or fh = 0.425 Mhz
k = Avmid ωe/ωe1 = -134.47 V/V

$$vo/vs = \frac{k\;\; (j\omega/\omega1)\quad (j\omega/\omega2)\quad (j\omega/\omega3+1)}{(j\omega/\omega1+1)\;(j\omega/\omega2+1)\;(j\omega/\omega4+1)\;(j\omega/\omega h+1)}$$