Diode Sensors Theory

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4-16-11 Diode_Sensors_Theory.ppt

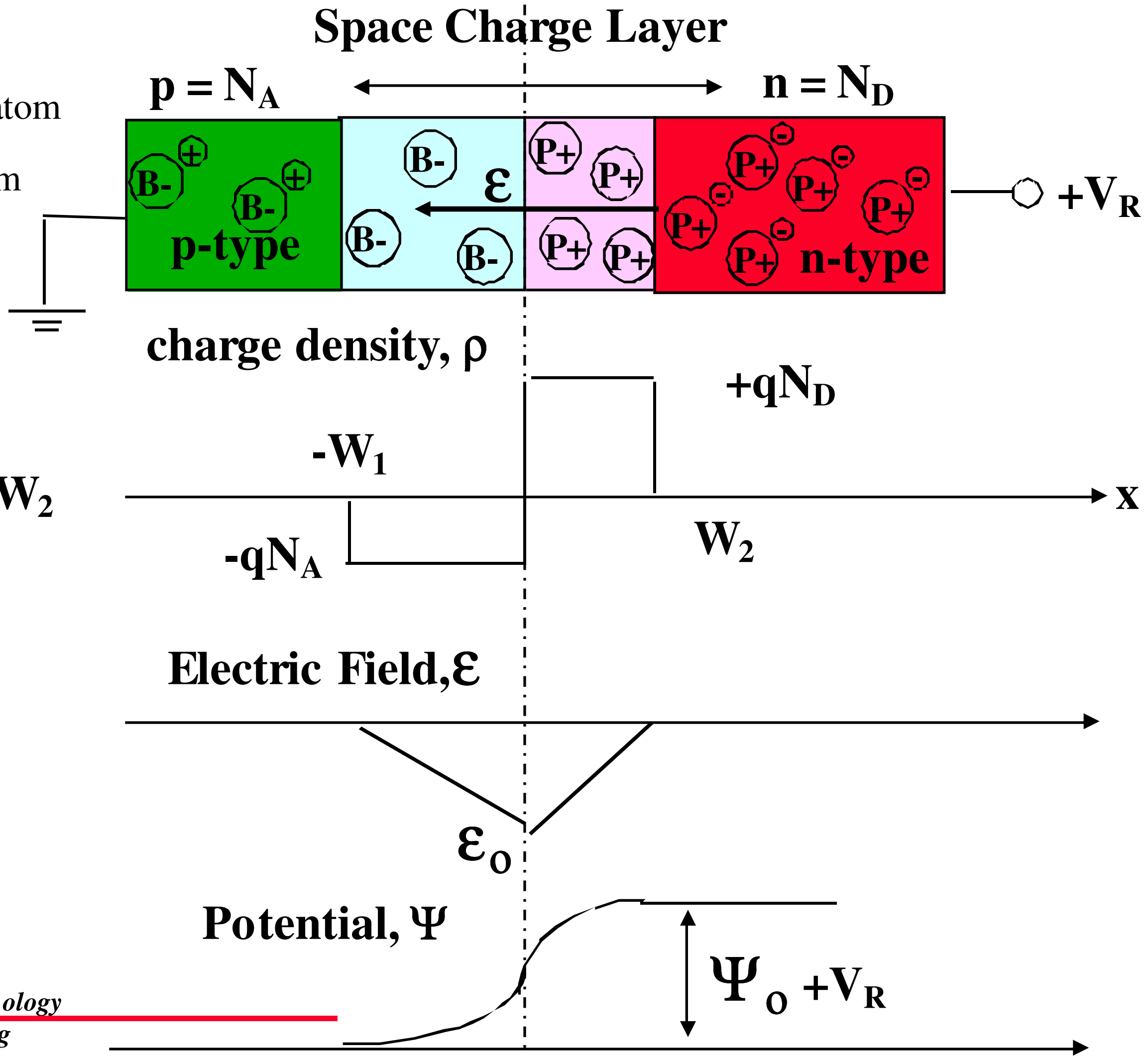
OUTLINE

Uniform Doped pn Junction
Real pn Junctions
Photodiodes
Light Sources
Diode Temperature Sensors
Solar Cells
Applications:
    Temperature
    Turbidity
    Spectral Radiometer

UNIFORMLY DOPED PN JUNCTION

- P+ ⊖ Phosphrous donor atom and electron
- P+ Ionized Immobile Phosphrous donor atom
- B- Ionized Immobile Boron acceptor atom
- B- ⊕ Boron acceptor atom and hole

$$qN_A W_1 = qN_D W_2$$



UNIFORMLY DOPED pn JUNCTION

From Physical Fundamentals:

Potential Barrier - Carrier Concentration: $\Psi_o = KT/q \ \ln (N_A N_D / ni^2)$

From Electric and Magnetic Fields :

Gauss's Law, Maxwells 1st eqn: $\rho = \nabla \cdot D$

Relationship between electric flux D and electric field E : $D = \varepsilon E$

Poisson's Equation: $\nabla^2 \Psi_o = -\rho / \varepsilon$

Definition of Electric Field: $E = -\nabla V$

Ψ_o FROM PHYSICS (FERMI STATISTICS)

$$q(Vbi) = (Ei - Ef)\text{p-side} + (Ef-Ei)\text{ n-side}$$

$$p= ni\ e^{(Ei-Ef)/KT/q} \qquad n= ni\ e^{(Ef-Ei)/KT/q}$$

$$\ln(p/ni) = \ln\ e^{(Ei-Ef)/KT/q} \qquad \ln(n/ni) = \ln\ e^{(Ef-Ei)/KT/q}$$

$$KT/q\ \ln(p/ni) = (Ei-Ef)_{\text{p-side}} \qquad KT/q\ \ln(n/ni) = (Ef-Ei)_{\text{n-side}}$$

$$\boxed{\Psi_o = KT/q\ \ \ln\ (N_A N_D /ni^2)}$$

$ni = 1.45E10\ cm^{-3}$ for silicon

Where N_A=~p in p-type silicon and N_D=~n in n-type silicon

UNIFORMLY DOPED PN JUNCTION

Built in Voltage: $\Psi_o = KT/q \ \ln (N_A N_D/ni^2)$

$ni = 1.45E10 \text{ cm}^{-3}$

Width of Space Charge Layer, W: with reverse bias of V_R volts

$$W = (W_1 + W_2) = [(2\varepsilon/q)(\Psi_o + V_R)(1/N_A + 1/N_D)]^{1/2}$$

W_1 width on p-side — W_2 width on n-side

$W_1 = W [N_D/(N_A + N_D)]$ — $W_2 = W [N_A/(N_A + N_D)]$

Maximum Electric Field:

$$E_o = - [(2q/\varepsilon)(\Psi_o + V_R)(N_A N_D/(N_A + N_D))]^{1/2}$$

Junction Capacitance per unit area:

$$C_j' = \varepsilon_o \varepsilon_r / W = \varepsilon_o \varepsilon_r / [(2\varepsilon/q)(\Psi_o + V_R)(1/N_A + 1/N_D)]^{1/2}$$

$\varepsilon = \varepsilon_o \varepsilon_r = 8.85E^{-12}\ (11.7)$ F/m

$= 8.85E^{-14}\ (11.7)$ F/cm

TEMPERATURE DEPENCENCE OF BUILT-IN VOLTAGE

Built in Voltage:

$$\Psi_o = \boxed{KT/q}\ \ln (N_A\, N_D \boxed{/ni^2)}$$

$ni = 1.45E10\ cm^{-3}$ at 300 °K

$$ni^2\,(T) = A\, T^3 \exp^{-q\, Eg/KT}$$

Where $A = 3.977E31$

$$Eg = Ego - aT^2 / (T+B)$$

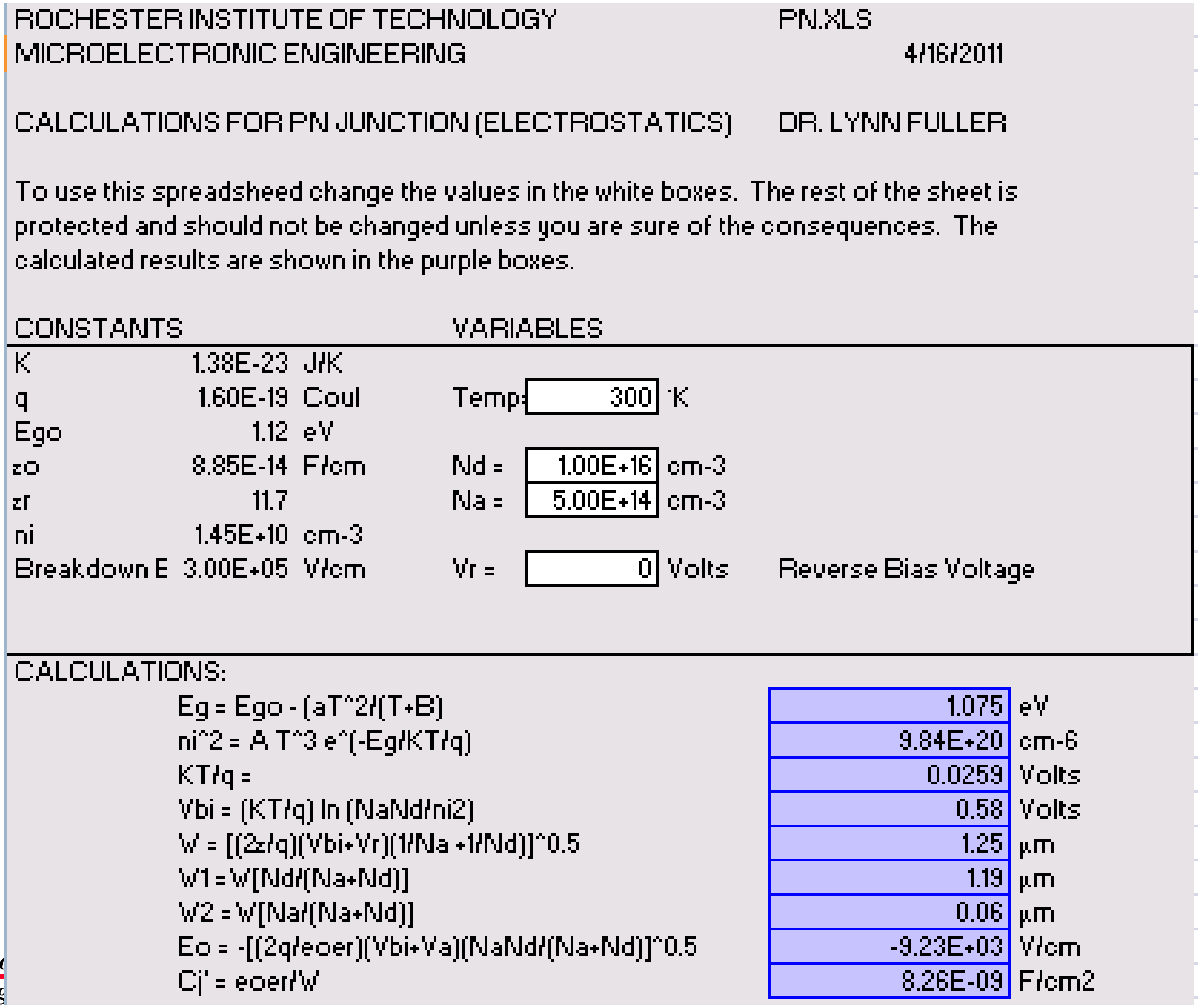
Where $a = 0.000702$

$B = 1110$

$Ego = 1.12$ eV

EXAMPLE CALCULATIONS

Width of space charge layer depends on the doping on both sides and the applied reverse bias voltage and temperature.



ROCHESTER INSTITUTE OF TECHNOLOGY PN.XLS
MICROELECTRONIC ENGINEERING 4/16/2011

CALCULATIONS FOR PN JUNCTION (ELECTROSTATICS) DR. LYNN FULLER

To use this spreadsheed change the values in the white boxes. The rest of the sheet is protected and should not be changed unless you are sure of the consequences. The calculated results are shown in the purple boxes.

CONSTANTS			VARIABLES			
K	1.38E-23	J/K				
q	1.60E-19	Coul	Temp:	300	'K	
Ego	1.12	eV				
zo	8.85E-14	F/cm	Nd =	1.00E+16	cm-3	
zr	11.7		Na =	5.00E+14	cm-3	
ni	1.45E+10	cm-3				
Breakdown E	3.00E+05	V/cm	Vr =	0	Volts	Reverse Bias Voltage

CALCULATIONS:

Eg = Ego - (aT^2/(T+B)	1.075	eV
ni^2 = A T^3 e^(-Eg/KT/q)	9.84E+20	cm-6
KT/q =	0.0259	Volts
Vbi = (KT/q) ln (NaNd/ni2)	0.58	Volts
W = [(2z/q)(Vbi+Vr)(1/Na +1/Nd)]^0.5	1.25	µm
W1 = W[Nd/(Na+Nd)]	1.19	µm
W2 = W[Na/(Na+Nd)]	0.06	µm
Eo = -[(2q/eoer)(Vbi+Va)(NaNd/(Na+Nd)]^0.5	-9.23E+03	V/cm
Cj' = eoer/W	8.26E-09	F/cm2

EXAMPLE

Example: If the doping concentrations are Na=1E15 and Nd=3E15 cm^{-3} and the reverse bias voltage is 0, then find the built in voltage, width of the space charge layer, width on the n-side, width on the p-side, electric field maximum and junction capacitance. Repeat for reverse bias of 10, 40, and 100 volts.

$\Psi_o = Vbi = KT/q \ \ln (N_A N_D / ni^2) =$

$W = (W_1 + W_2) = [(2\varepsilon/q)(\Psi_o + V_R)(1/N_A + 1/N_D)]^{1/2} =$

W1 =

W2 =

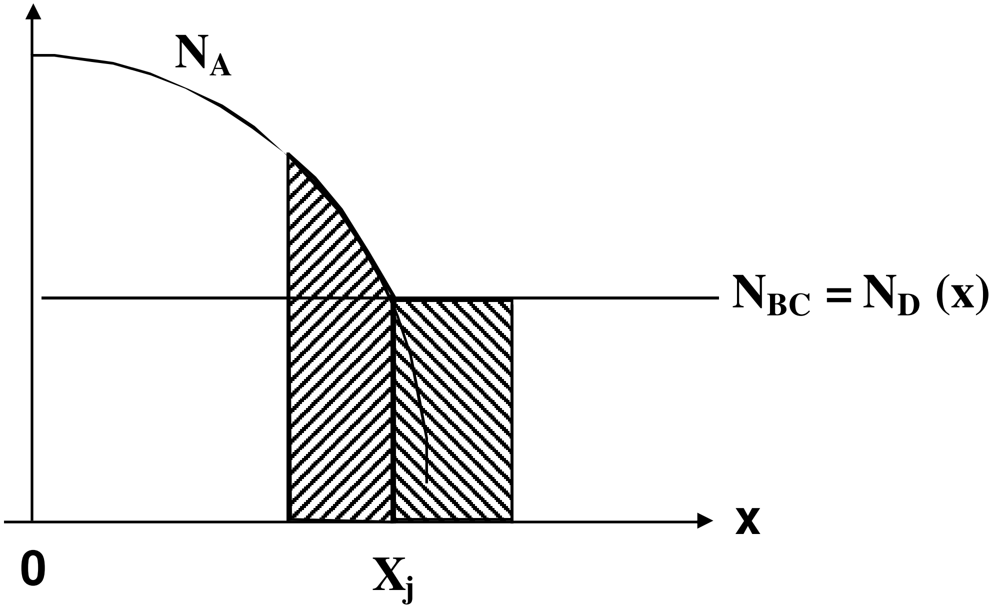
Emax =

Cj =

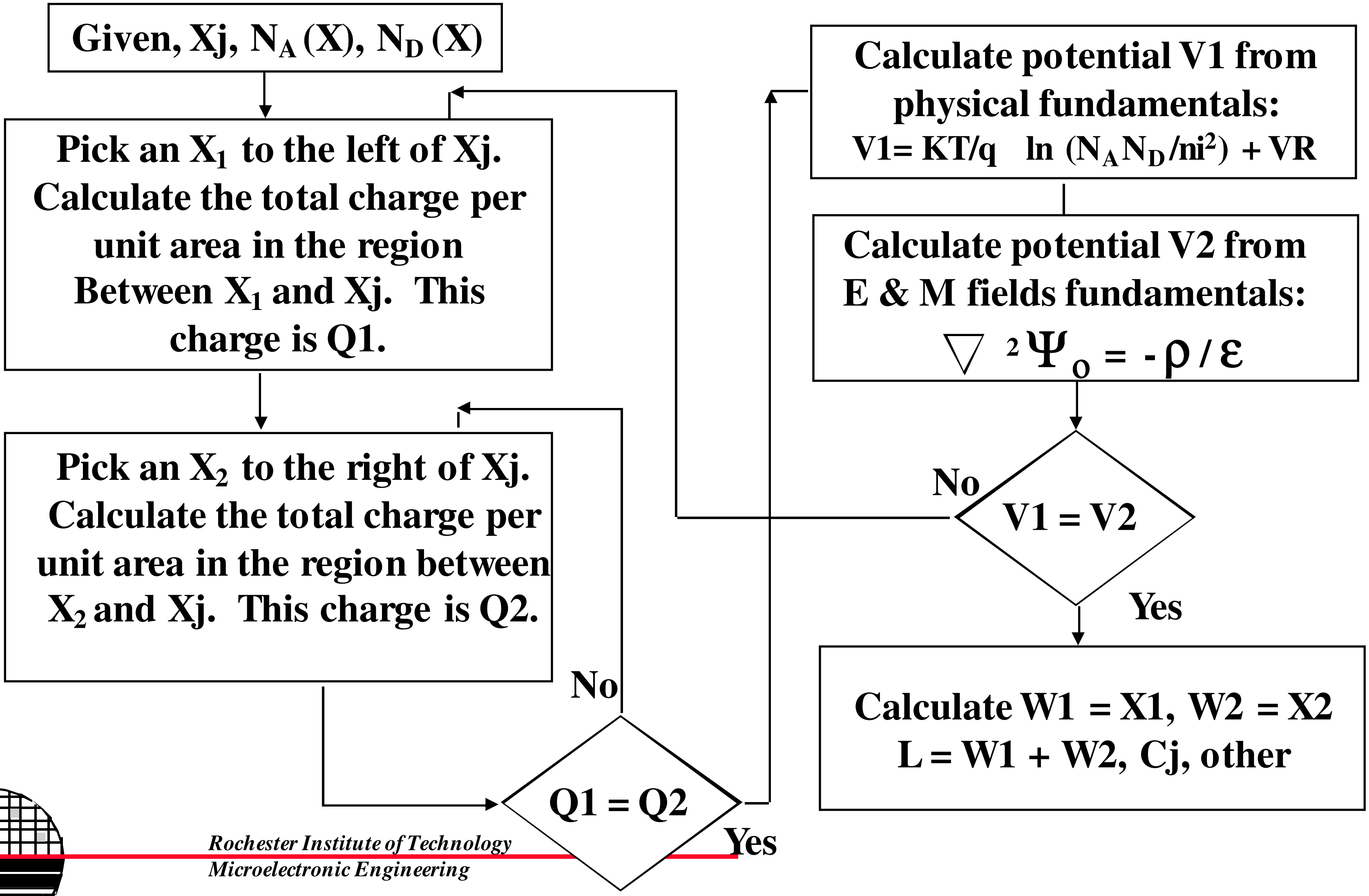
REAL JUNCTION

Real pn junctions: The uniformly doped abrupt junction is rarely obtained in integrated circuit devices. (epi layer growth is close).

Diffused pn junction:



REAL pn JUNCTION



Given, Xj, N_A (X), N_D (X)

Pick an X_1 to the left of Xj. Calculate the total charge per unit area in the region Between X_1 and Xj. This charge is Q1.

Pick an X_2 to the right of Xj. Calculate the total charge per unit area in the region between X_2 and Xj. This charge is Q2.

Q1 = Q2

Calculate potential V1 from physical fundamentals:

$V1 = KT/q \ \ln (N_A N_D / ni^2) + VR$

Calculate potential V2 from E & M fields fundamentals:

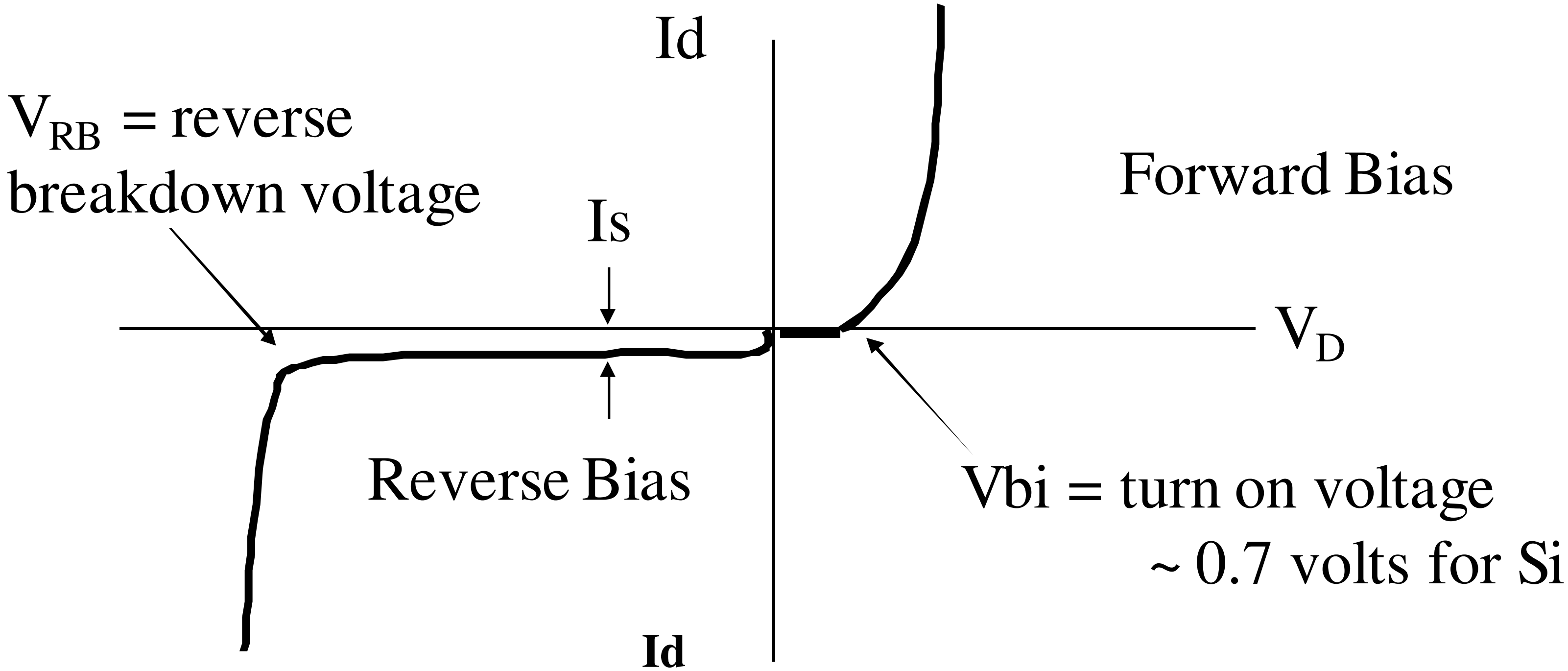
$$\nabla^2 \Psi_o = -\rho / \varepsilon$$

V1 = V2

Calculate W1 = X1, W2 = X2

L = W1 + W2, Cj, other

CURRENTS IN PN JUNCTIONS



Id

V_{RB} = reverse breakdown voltage

Is

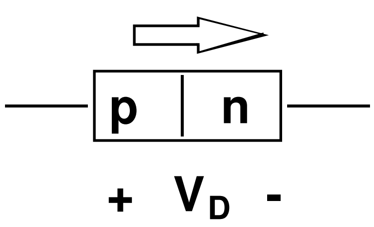
V_D

Forward Bias

Reverse Bias

Vbi = turn on voltage
~ 0.7 volts for Si

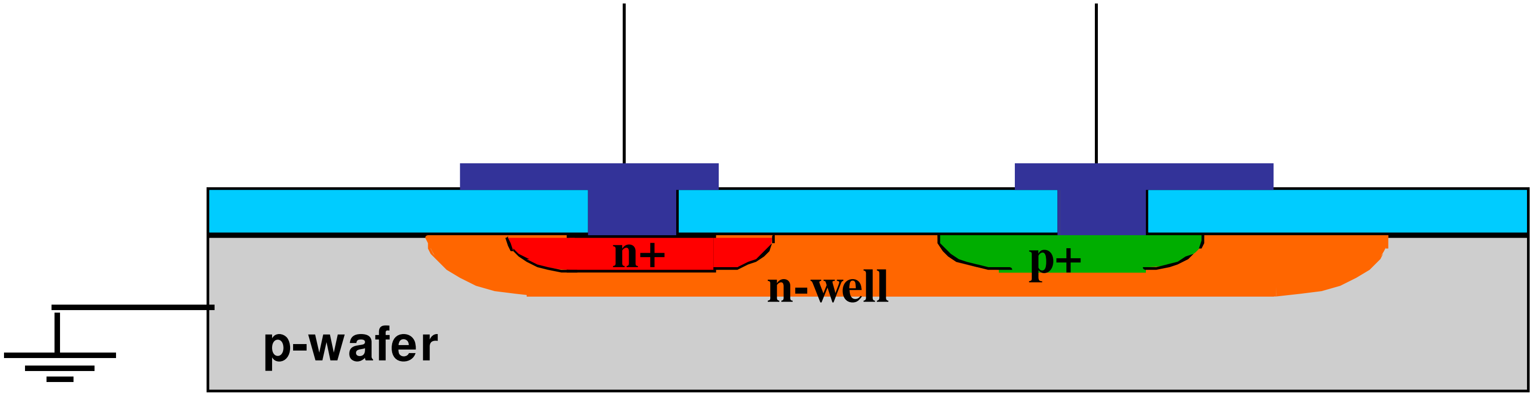
Id



$$Id = Is\,[EXP^{(q\,VD/KT)} - 1]$$

Ideal diode equation

INTEGRATED DIODES



p+ means heavily doped p-type
n+ means heavily doped n-type
n-well is an n-region at slightly higher doping than the p-wafer

Note: there are actually two pn junctions, the well-wafer pn junction should always be reverse biased

REAL DIODES

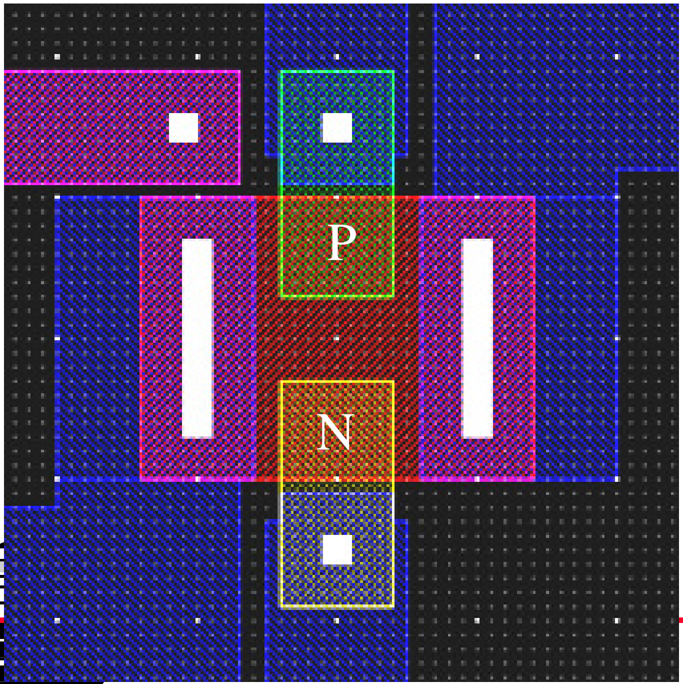
Series Resistance =1/4.82m=207

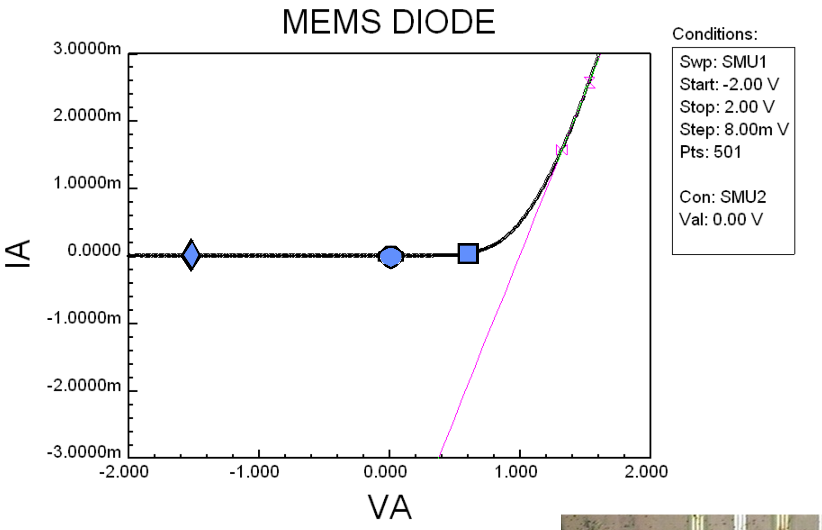
Junction Capacitance ~ 2 pF

Is = 3.02E-9 amps

BV = > 100 volts

Size 80µ x 160µ





Fit #1:	Fit #2:
Type: Cursor	None
Slp:4.82m	****
Yint:-4.79m	****
Xint:0.99	***

Cursors:	X	Y
□	0.60	0.03m
◇	-1.52	-3.02n
○	0.00	0.38n
⧗	1.53	2.57m
⋈	1.32	1.57m
△		

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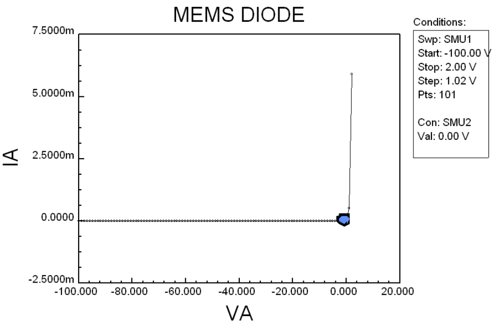
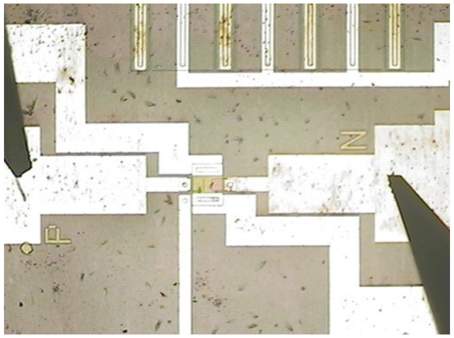
03/05/2009





DIODE SPICE MODEL

Model Parameter	Default Value	MEMS Diode Extracted Value
Is reverse saturation current	1e-14 A	3.02E-9 A
N emission coefficient	1	1
RS series resistance	0	207 ohms
VJ built-in voltage	1 V	0.6
CJ0 zero bias junction capacitance	0	2pF
M grading coefficient	0.5	0.5
BV Breakdown voltage	infinite	400
IBV Reverse current at breakdown	1E-10 A	-

```
DXXX N(anode) N(cathode) Modelname
.model Modelname  D Is=1e-14 Cjo=.1pF Rs=.1
.model RITMEMS  D IS=3.02E-9 N=1 RS=207
+VJ=0.6 CJ0=2e-12 M-0.5 BV=400
```

DIODE TEMPERATURE DEPENDENCE

$$Id = I_S\,[\mathrm{EXP}^{(q\ VD/KT)} - 1]$$

Neglect the –1 in forward bias, Solve for VD

$$VD = (KT/q)\ \ln\ (Id/I_S) = (KT/q)\ (\ln(Id) - \ln(Is)) \qquad \text{eq 1}$$

Take dVD/dT: note Id is not a function of T but Is is

$$dVD/dT = (KT/q)\ (\underbrace{d\ln(Id)/dT}_{\text{zero}} - d\ln(Is)/dT) + \underbrace{K/q\ (\ln(Id) - \ln(Is))}_{\text{VD/T from eq 1}}$$

Rewritten

$$dVD/dT = VD/T - (KT/q)\ ((1/Is)\ dIs/dT\ ) \qquad \text{eq 2}$$

Now evaluate the second term, recall

$$Is = qA\ (Dp/(LpNd) + Dn/(LnNa))ni^2$$

Note: Dn and Dp are proportional to 1/T

DIODE TEMPERATURE DEPENDENCE

and $ni^2(T) = A\,T^3\ e^{-qEg/KT}$

This gives the temperature dependence of Is

$$Is = C\,T^2\ e^{-qEg/KT} \qquad \text{eq 3}$$

Now take the natural log

$$\ln Is = \ln\left(C\,T^2\ e^{-qEg/KT}\right)$$

Take derivative with respect to T

$$(1/Is)\, d\,(Is)/dT = d\,[\ln\left(C\,T^2\ e^{-qEg/KT}\right)]/dT = (1/Is)\ d\,(CT^2e^{-qEg/KT})dT$$

$$= (1/Is)\ [CT^2\, e^{-qEg/KT}(qEg/KT^2) + (Ce^{-qEg/KT})2T]$$

$$= (1/\cancel{I}s)\ [\cancel{Is}(qEg/KT^2) + (2\cancel{Is}/T)]$$

Back to eq 2

$$dVD/dT = VD/T - (KT/q)\ [(qEg/KT^2) + (2/T)])$$

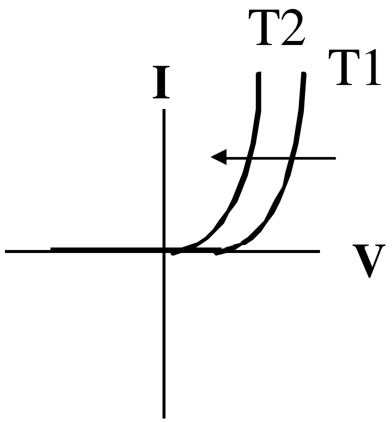
$$\boxed{dVD/dT = VD/T - Eg/T - 2K/q)}$$

EXAMPLE: DIODE TEMPERATURE DEPENDENCE

$$\boxed{dVD/dT = VD/T - Eg/T - 2K/q}$$

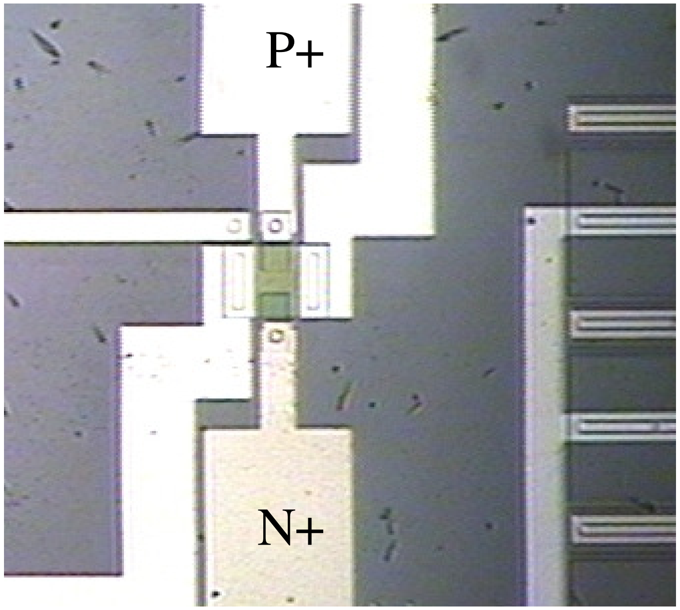
Silicon with Eg ~ 1.2 eV, VD = 0.6 volts, T=300 °K

$$dVD/dT = .6/300 - 1.2/300) - (2(1.38E-23)/1.6E-19$$
$$= -2.2 \text{ mV}/°$$

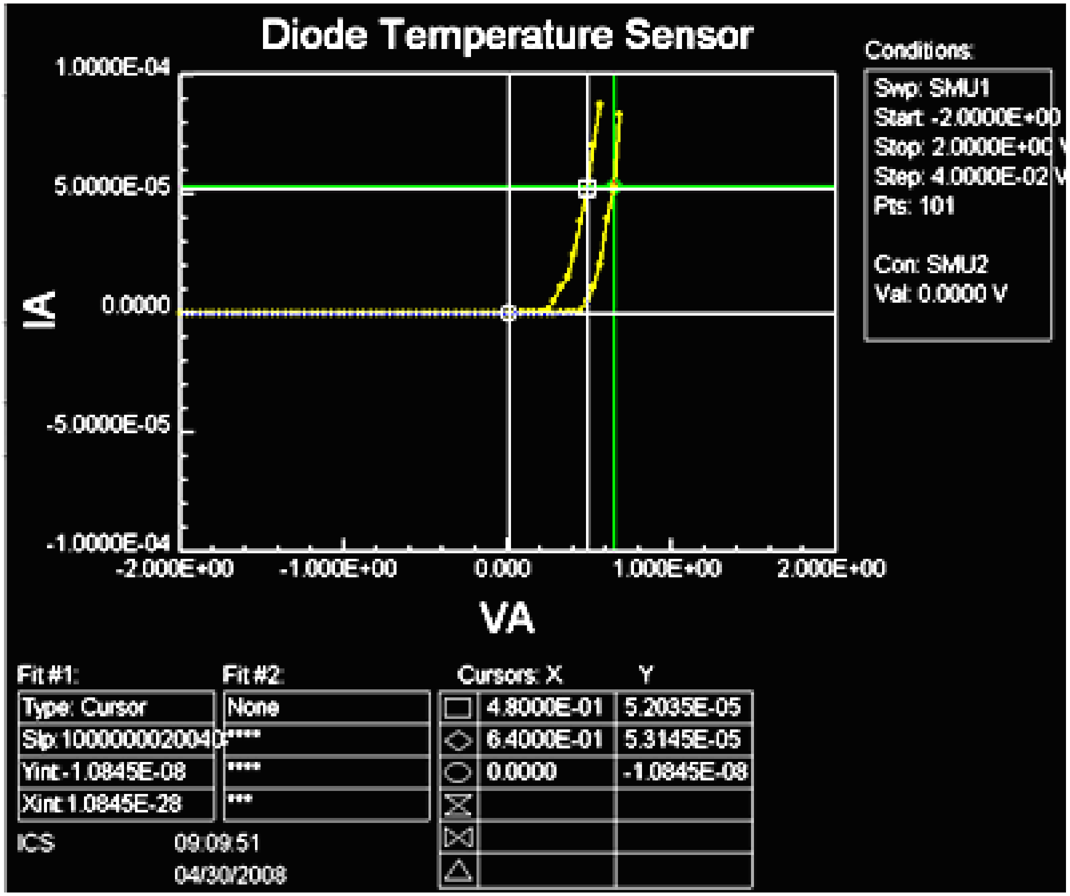


T1<T2

DIODE AS A TEMPERATURE SENSOR



Poly Heater
Buried pn Diode,
N+ Poly to Aluminum
Thermocouple



Conditions:
Swp: SMU1
Start: -2.0000E+00
Stop: 2.0000E+00
Step: 4.0000E-02
Pts: 101

Con: SMU2
Val: 0.0000 V

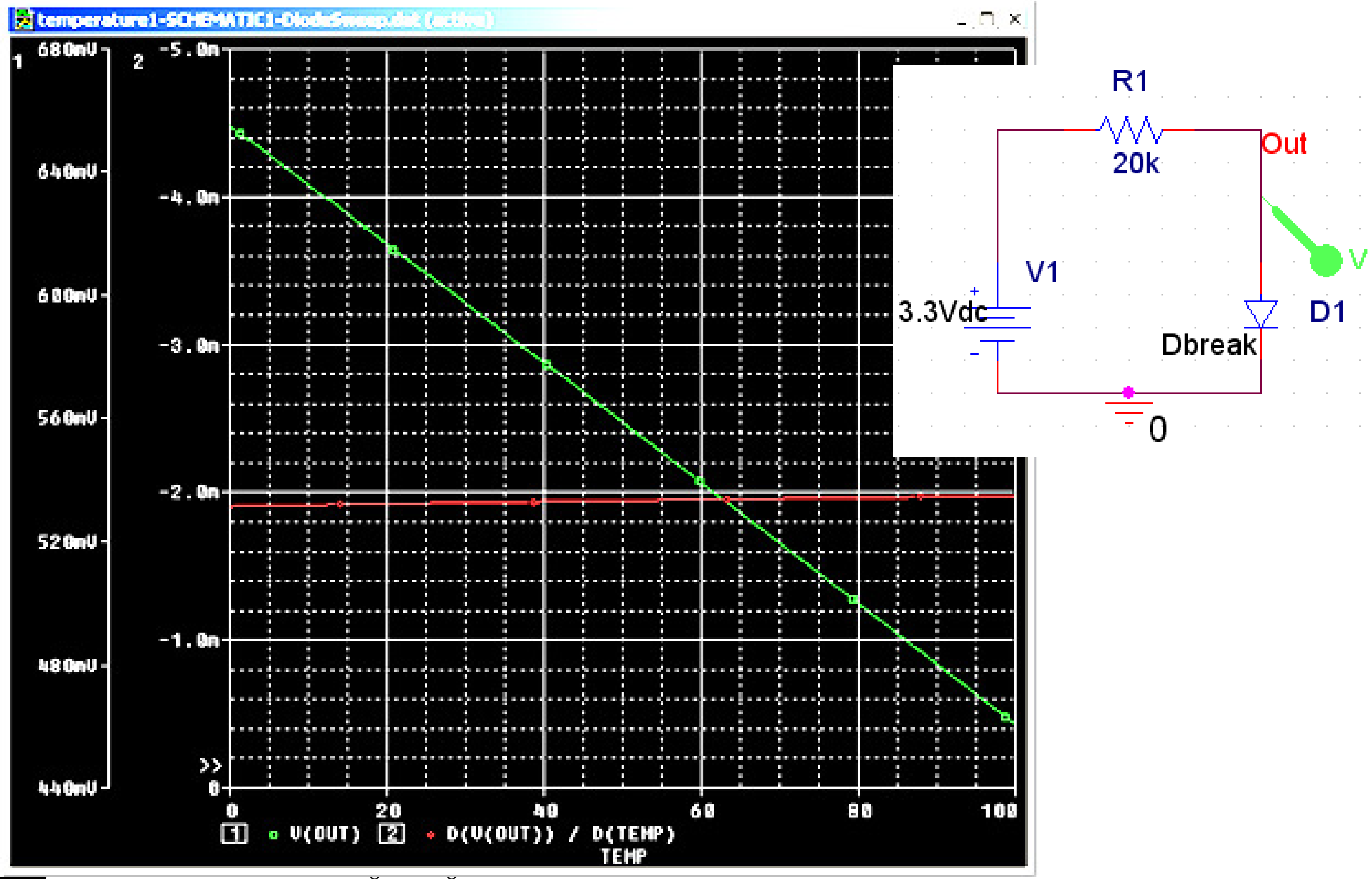
Fit #1:	Fit #2:
Type: Cursor	None
Slp: 10000000020040	****
Yint -1.0845E-08	****
Xint 1.0845E-28	***

ICS 09:09:51
04/30/2008

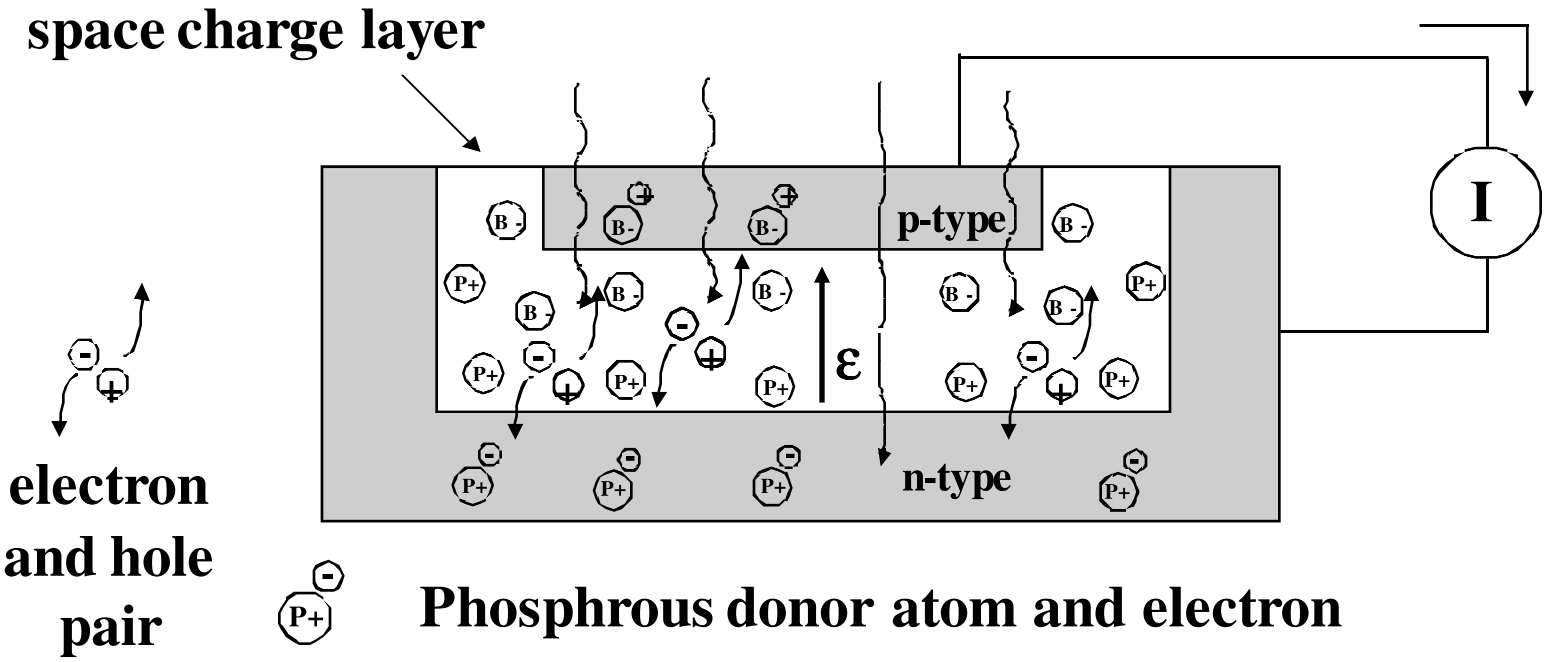
Cursors:	X	Y
□	4.8000E-01	5.2035E-05
◇	6.4000E-01	5.3145E-05
○	0.0000	-1.0845E-08
⧗		
⋈		
△		

Compare with theoretical -2.2mV/°C

SPICE FOR DIODE TEMPERATURE SENSOR



PHOTODIODE



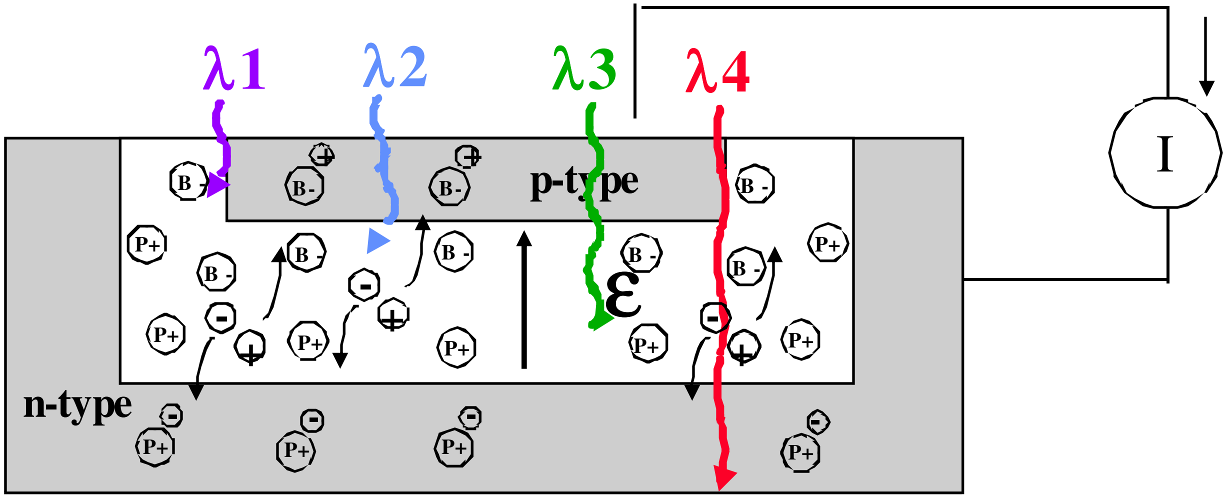
P+ ⊖	Phosphrous donor atom and electron
P+	Ionized Immobile Phosphrous donor atom
B -	Ionized Immobile Boron acceptor atom
B- ⊕	Boron acceptor atom and hole

CHARGE GENERATION vs WAVELENGTH

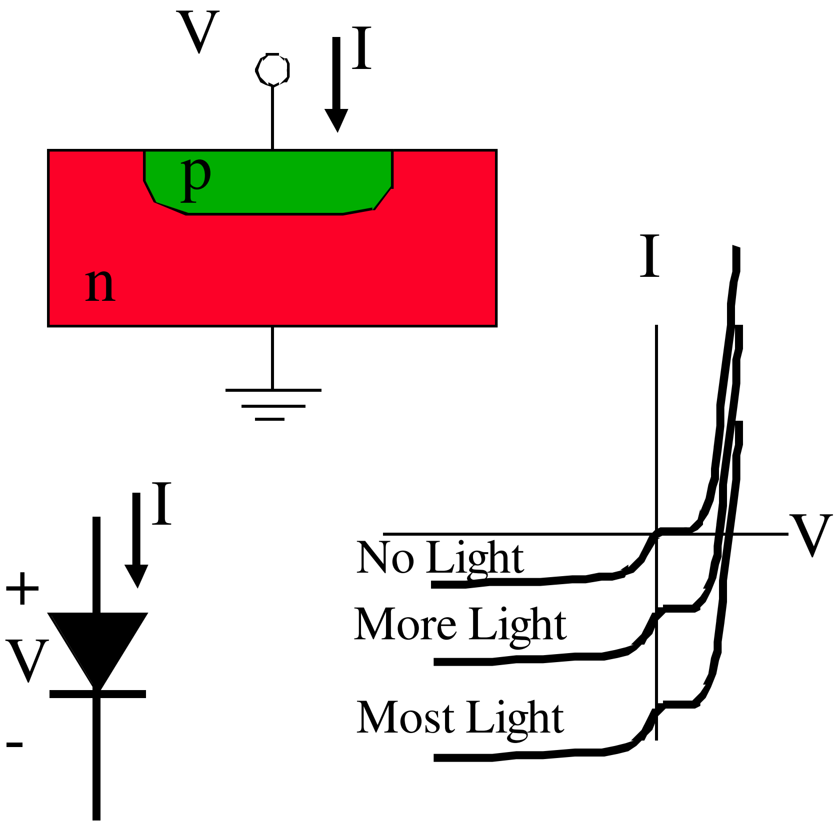
$E = h\nu = hc / \lambda$

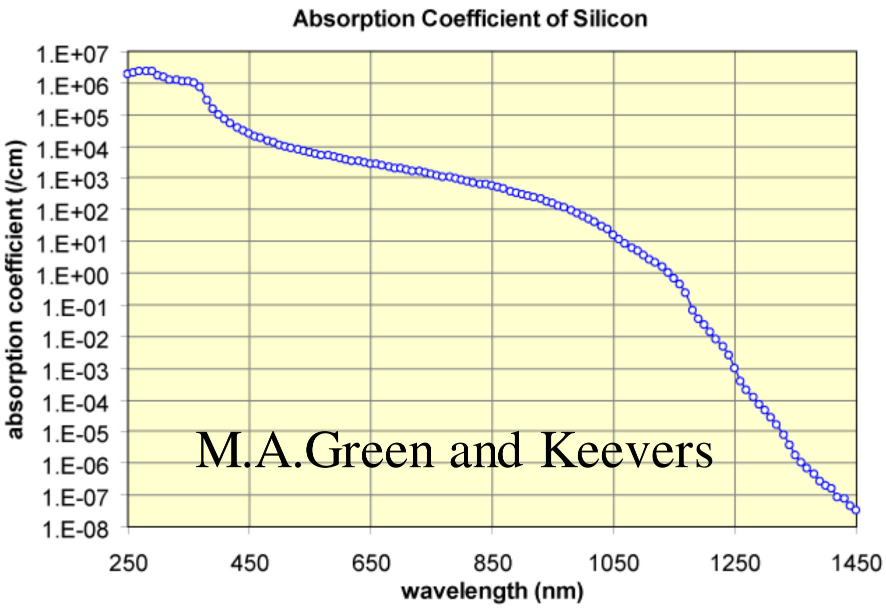
$h = 6.625\ e\text{-}34$ j/s
$= (6.625\ e\text{-}34/1.6e\text{-}19)$ eV/s

E = 1.55 eV (red)
E = 2.50 eV (green)
E = 4.14 eV (blue)



ADSORPTION VERSUS DISTANCE



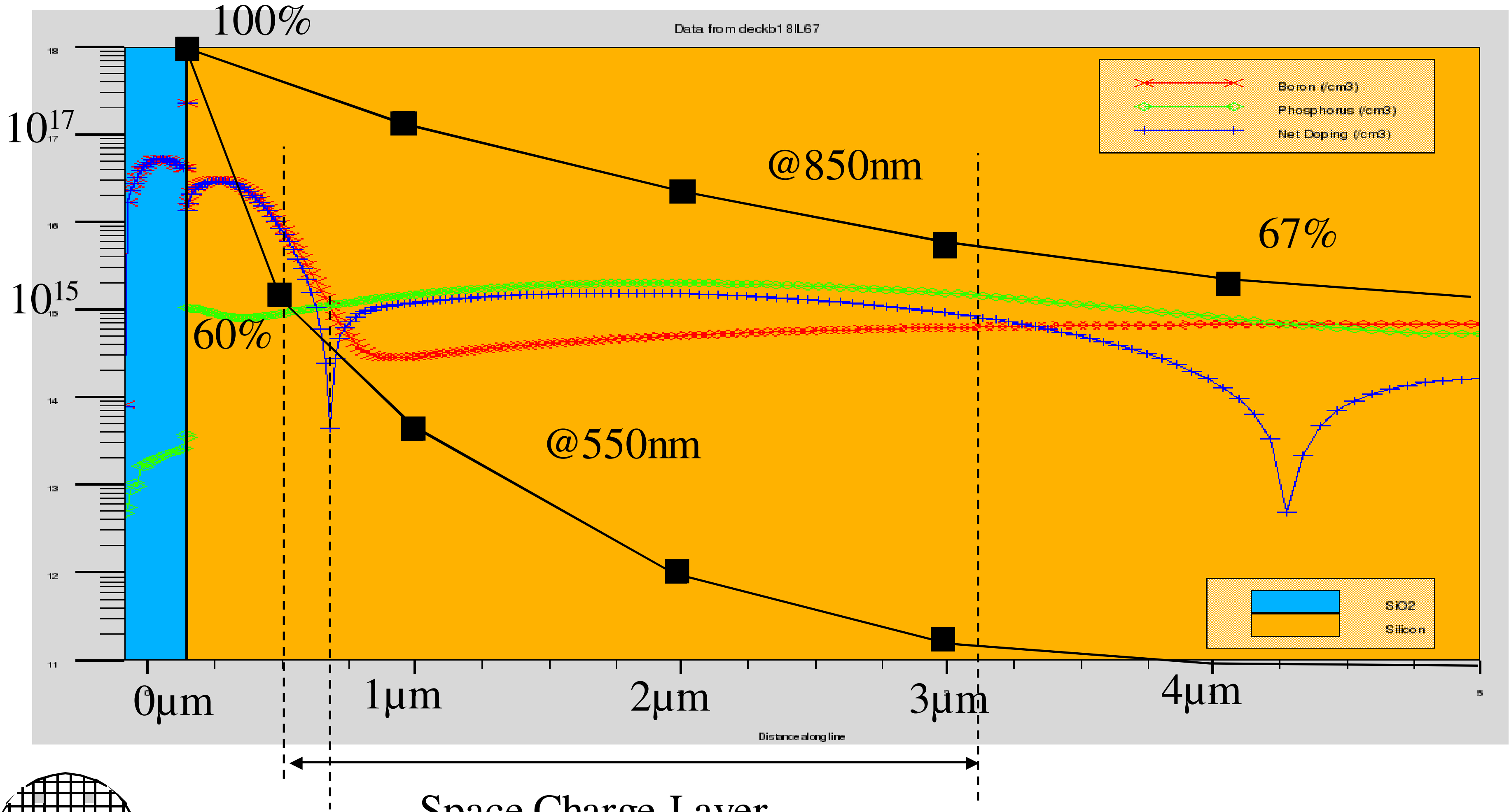


M.A.Green and Keevers

$$\phi(x) = \phi(0)\ \exp^{-\alpha\ x}$$

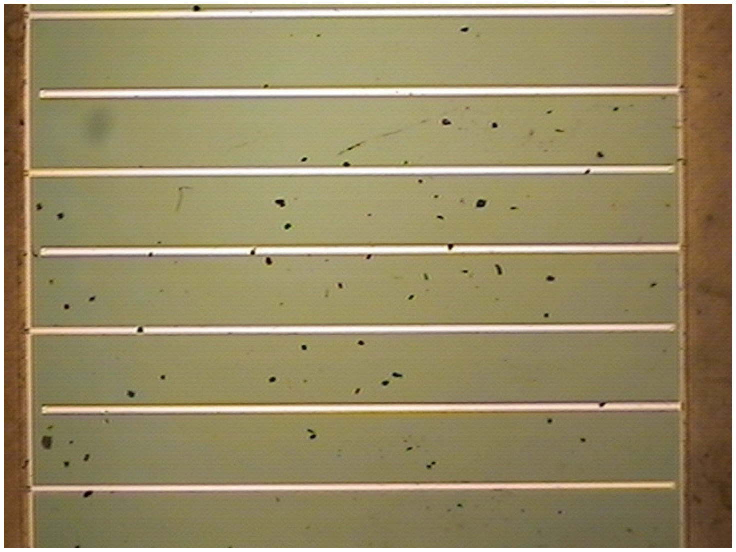
Find % adsorbed for Green light at x=5 µm and Red light at 5 µm

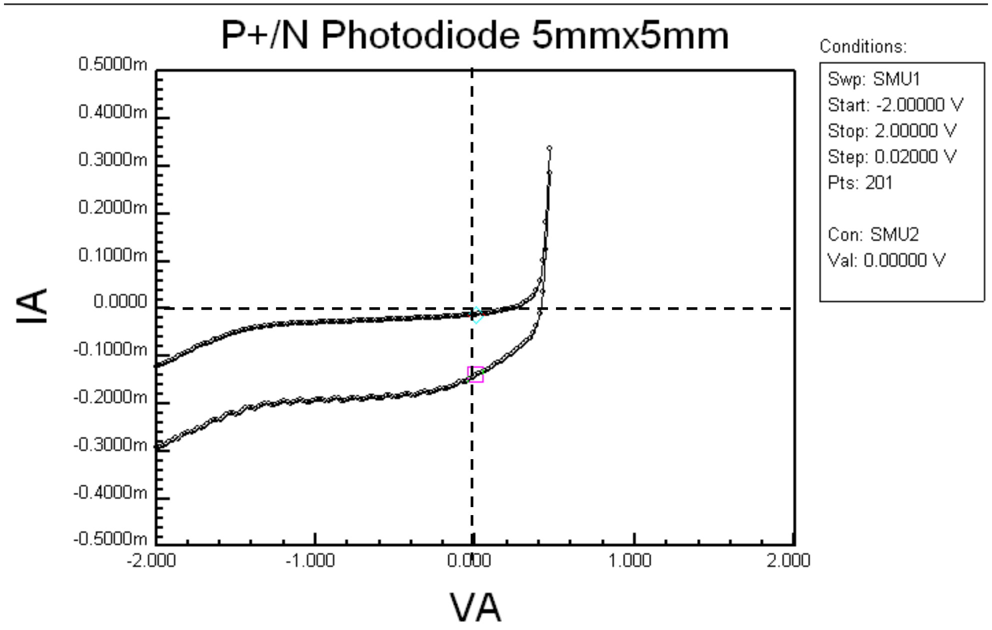
PN JUNCTION DESIGN FOR PHOTO DIODE



Space Charge Layer

LARGE 5mm X 5mm PHOTODIODE





Conditions:

Swp: SMU1
Start: -2.00000 V
Stop: 2.00000 V
Step: 0.02000 V
Pts: 201

Con: SMU2
Val: 0.00000 V

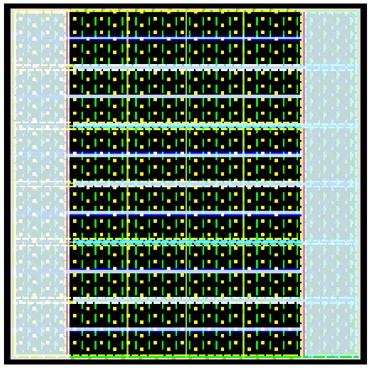
Fit #1:
None
****
****
***

Fit #2:
None
****
****
***

	Cursors: X	Y
□	0.00000	-0.14080m
◇	0.00000	-0.01383m
○		
⧖		
⋈		
△		

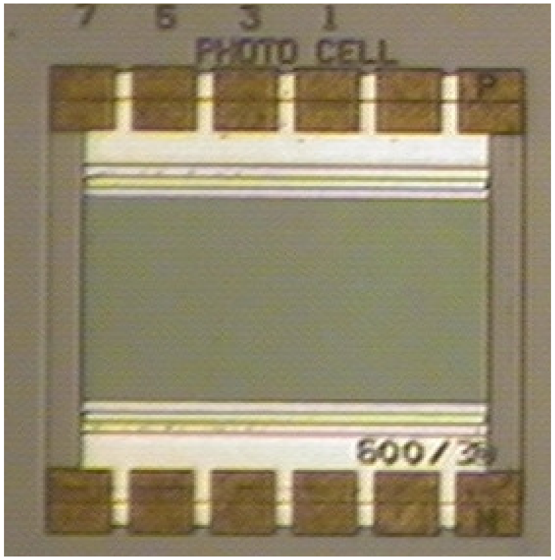
ICS 13:46:58
11/18/2009

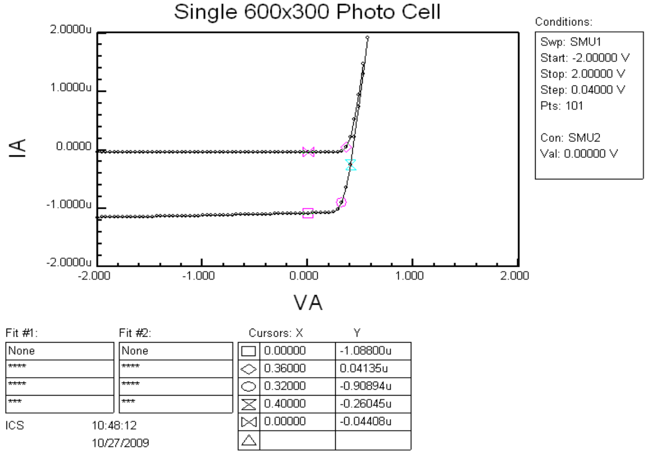
5mm
x
3.33mm



Isc = 0.15mA (short circuit current)
or 9.09 A/m2

SINGLE AND DUAL PHOTO CELL





Conditions:
Swp: SMU1
Start: -2.00000 V
Stop: 2.00000 V
Step: 0.04000 V
Pts: 101

Con: SMU2
Val: 0.00000 V

Fit #1:	Fit #2:
None	None
****	****
****	****
***	***

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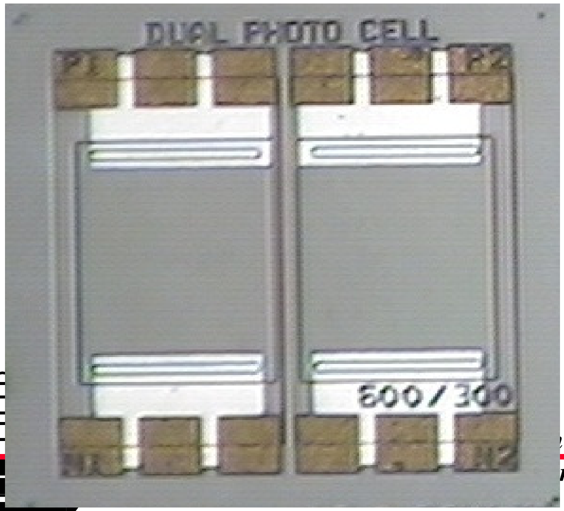
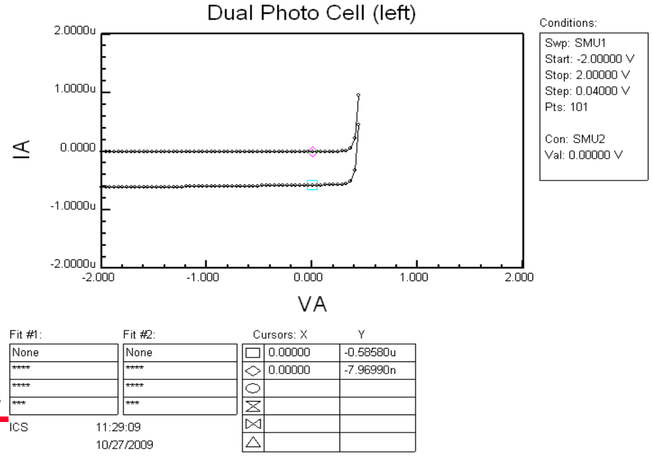
10/27/2009

Cursors:	X	Y
□	0.00000	-1.08800u
◇	0.36000	0.04135u
○	0.32000	-0.90894u
⧗	0.40000	-0.26045u
⋈	0.00000	-0.04408u
△		

Isc = 1.088 uA
or 6 A/m2





Conditions:
Swp: SMU1
Start: -2.00000 V
Stop: 2.00000 V
Step: 0.04000 V
Pts: 101

Con: SMU2
Val: 0.00000 V

Fit #1:	Fit #2:
None	None
****	****
****	****
***	***

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10/27/2009

Cursors:	X	Y
□	0.00000	-0.58580u
◇	0.00000	-7.96990n
○		
⧗		
⋈		
△		

Isc = 0.585 uA
or 3.25 A/m2

SOLAR CELL TUTORIAL

SOME TERMS AND DEFINITIONS:

Air Mass – amount of air between sun and solar cell. In space AM=0 at the equator at noon AM=1, if the sun is arriving at an angle θ , AM=1/cos θ . AM1.5 is the standard for most solar cell work in USA and gives a sum total of 1000w/m2 over the entire spectrum of wavelengths from 0.2um to 2.0um

Efficiency is the ratio of the power out of a solar cell to the power falling on the solar cell (normally 1000w/m2 with the AM1.5 spectrum) Since Si solar cells can not absorb much of the infrared spectrum from the sun, and other factors, typical efficiencies are limited to 26-29% for basic silicon solar cells.

Quantum Efficiency – normalized ratio of electrons and holes collected to photons incident on the cell at a single wavelength, given in %.

FF – Fill Factor, a figure of merit, the “squareness “ of the diode I-V characteristic in 4th quadrant with light falling on the cell.

SOLAR CELL TUTORIAL

From: Solar Cells, Martin A. Green, Prentice Hall

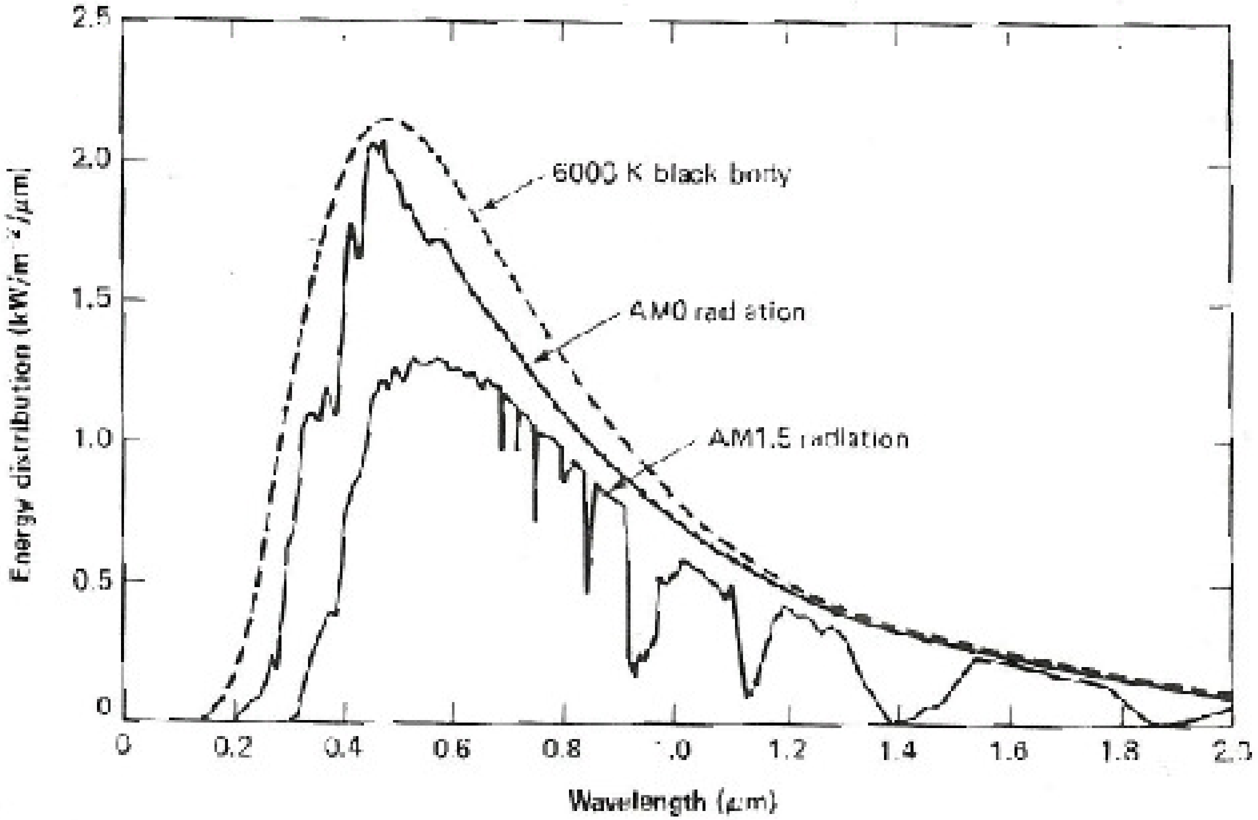
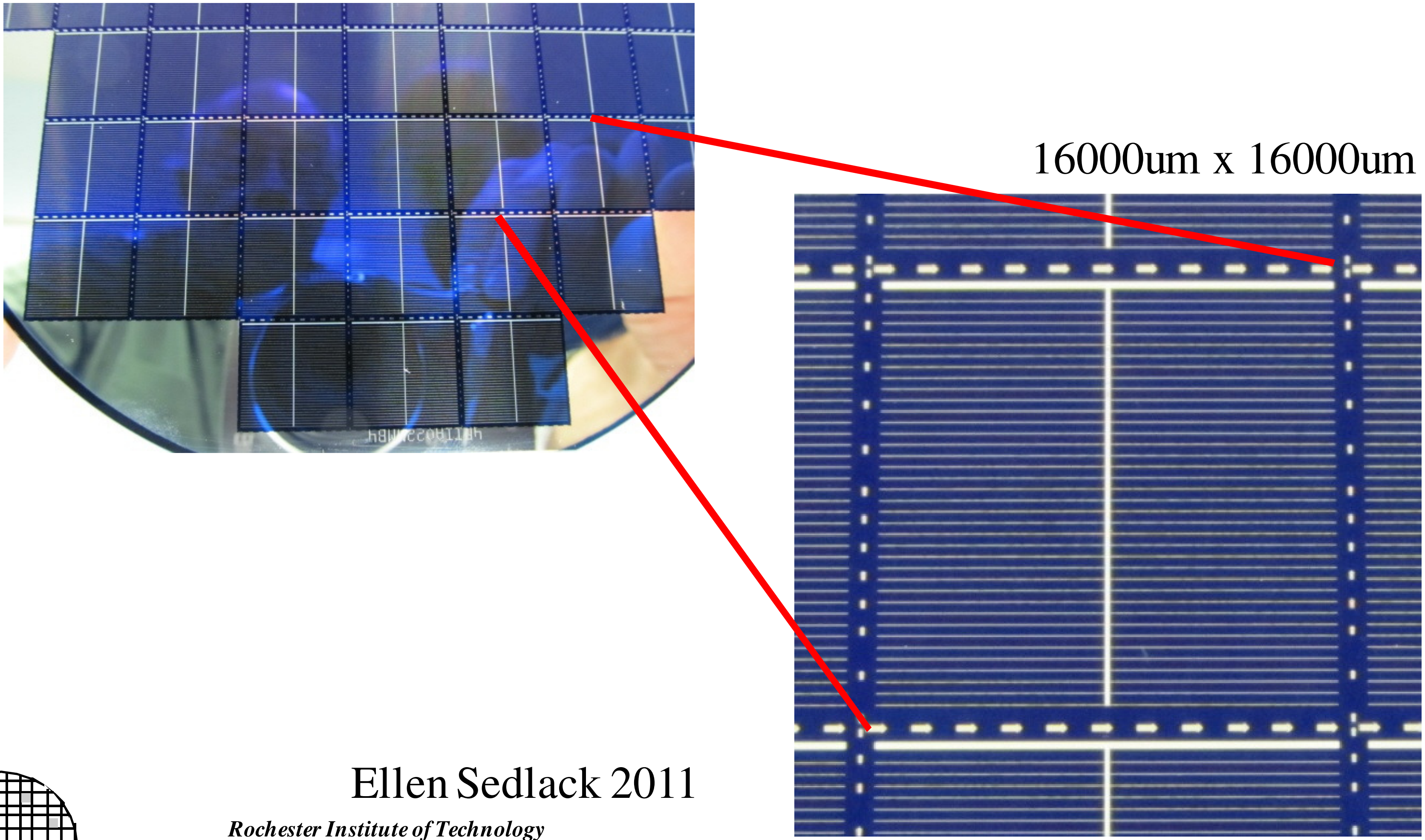


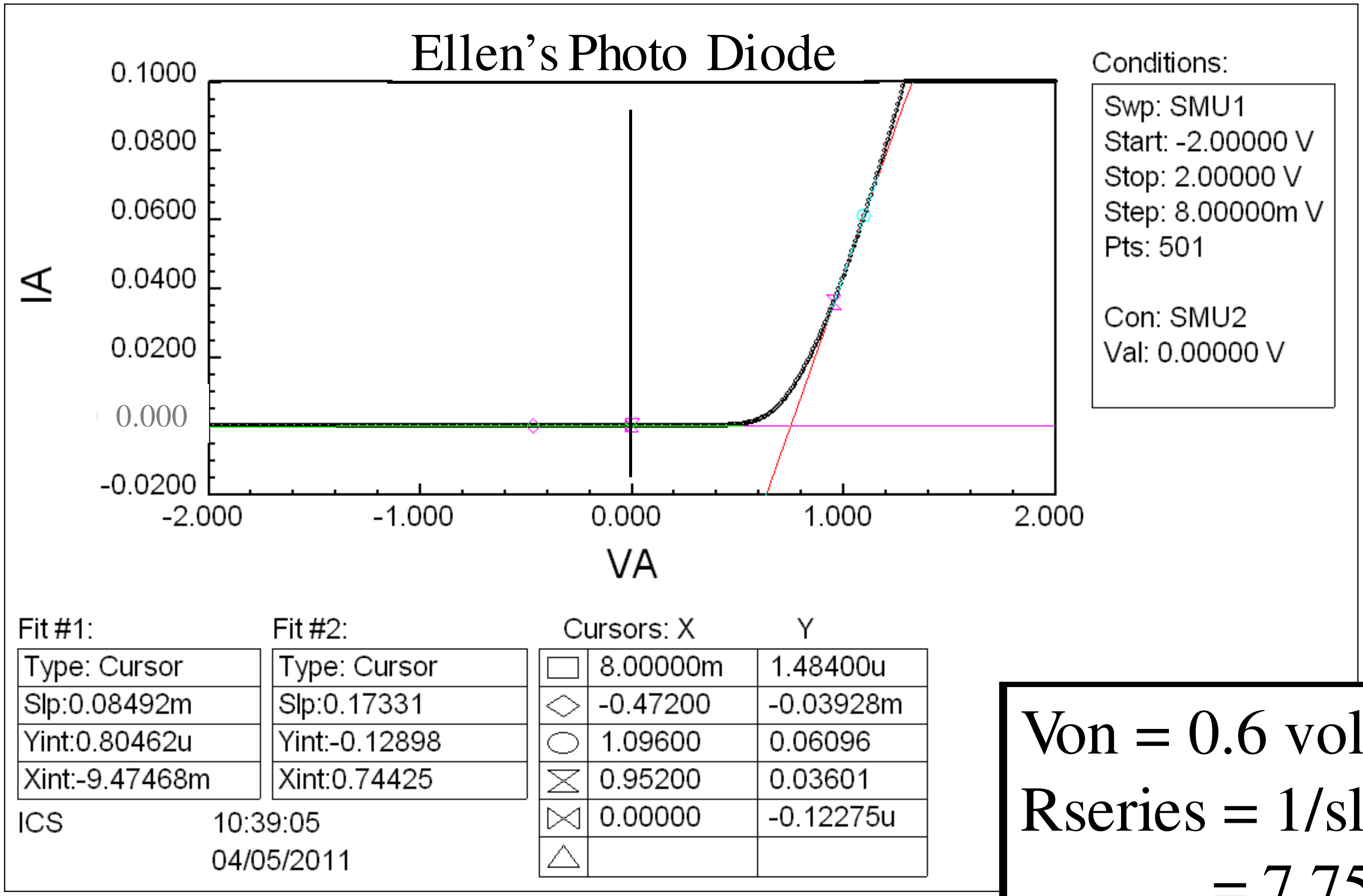
Figure 1.3. Spectral distribution of sunlight. Shown are the cases of AM0 and AM1.5 radiation together with the radiation distribution expected from the sun if it were a black body at 6000 K.

PHOTOCELL–ELLEN SEDLACK SENIOR PROJECT



Ellen Sedlack 2011

I-V CHARACTERISTICS OF PHOTO CELL



Conditions:

Swp: SMU1
Start: -2.00000 V
Stop: 2.00000 V
Step: 8.00000m V
Pts: 501

Con: SMU2
Val: 0.00000 V

Fit #1:

Type: Cursor
Slp:0.08492m
Yint:0.80462u
Xint:-9.47468m

Fit #2:

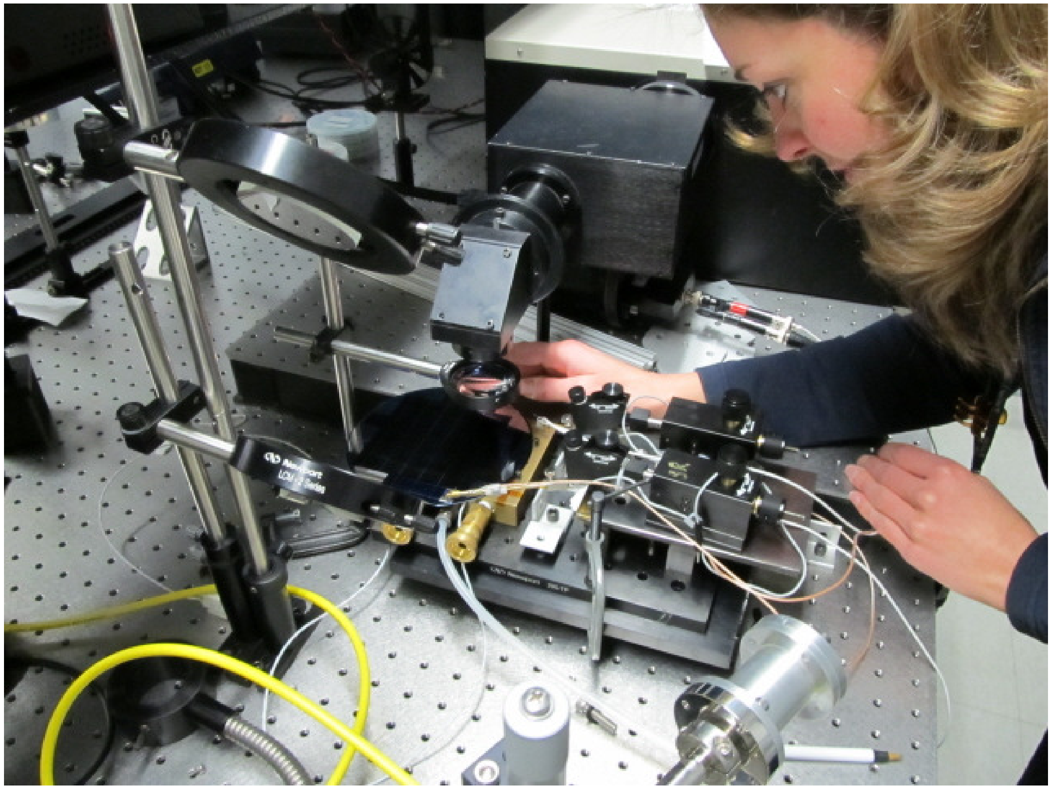
Type: Cursor
Slp:0.17331
Yint:-0.12898
Xint:0.74425

Cursors:	X	Y
□	8.00000m	1.48400u
◇	-0.47200	-0.03928m
○	1.09600	0.06096
⌧	0.95200	0.03601
⋈	0.00000	-0.12275u
△		

ICS 10:39:05
04/05/2011

Von = 0.6 volts
Rseries = 1/slope = 1/0.129
= 7.75ohms
Is = 1.48uA (in room light)

PHOTOCELL – QUANTUM EFFICIENCY



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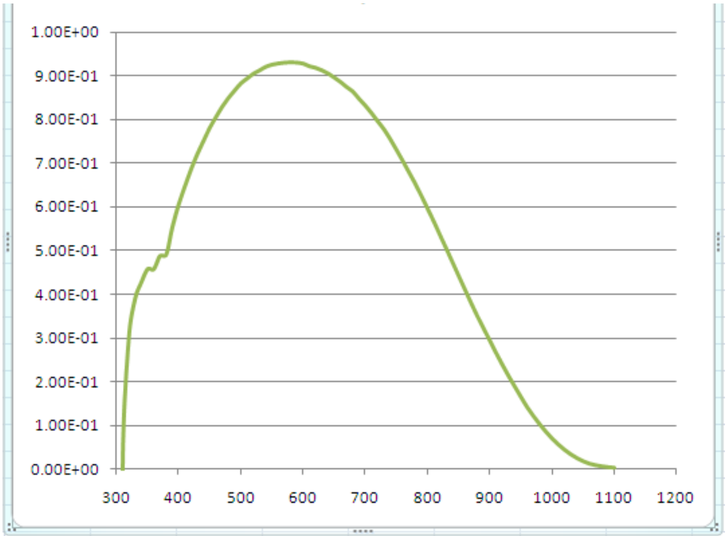
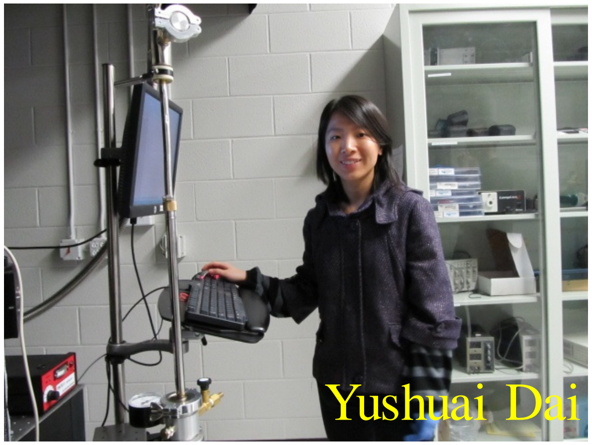
93% between 550nm and 650nm

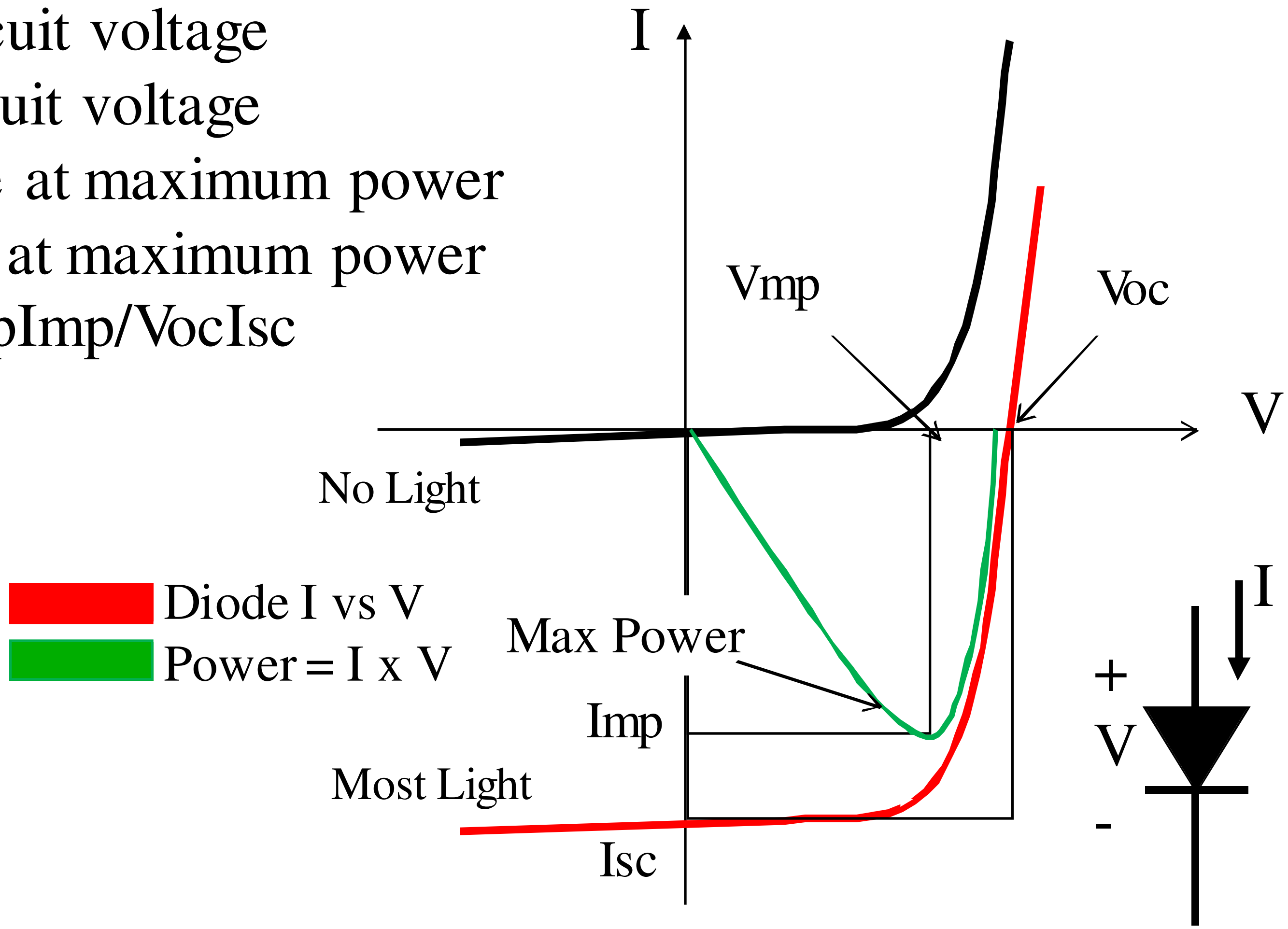




Yushuai Dai

SOLAR CELL TUTORIAL

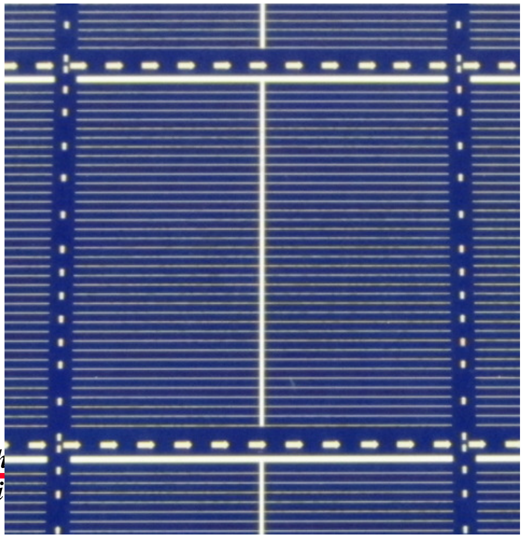
Voc - open circuit voltage
Isc – short circuit voltage
Vmp – Voltage at maximum power
Imp – Current at maximum power
FF – FF = VmpImp/VocIsc

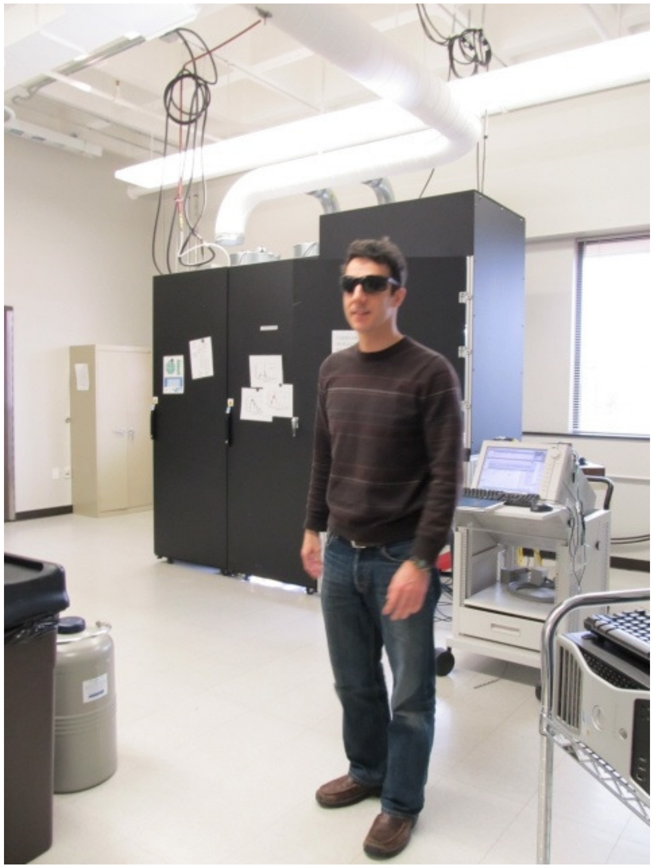


PHOTOCELL – POWER EFFICIENCY

AM 1.5 Light Source

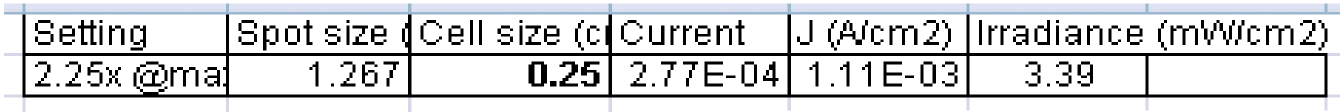
Zachary Bittner



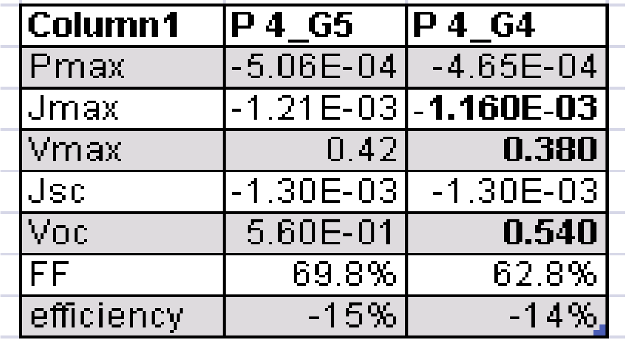


Ivan Puchades

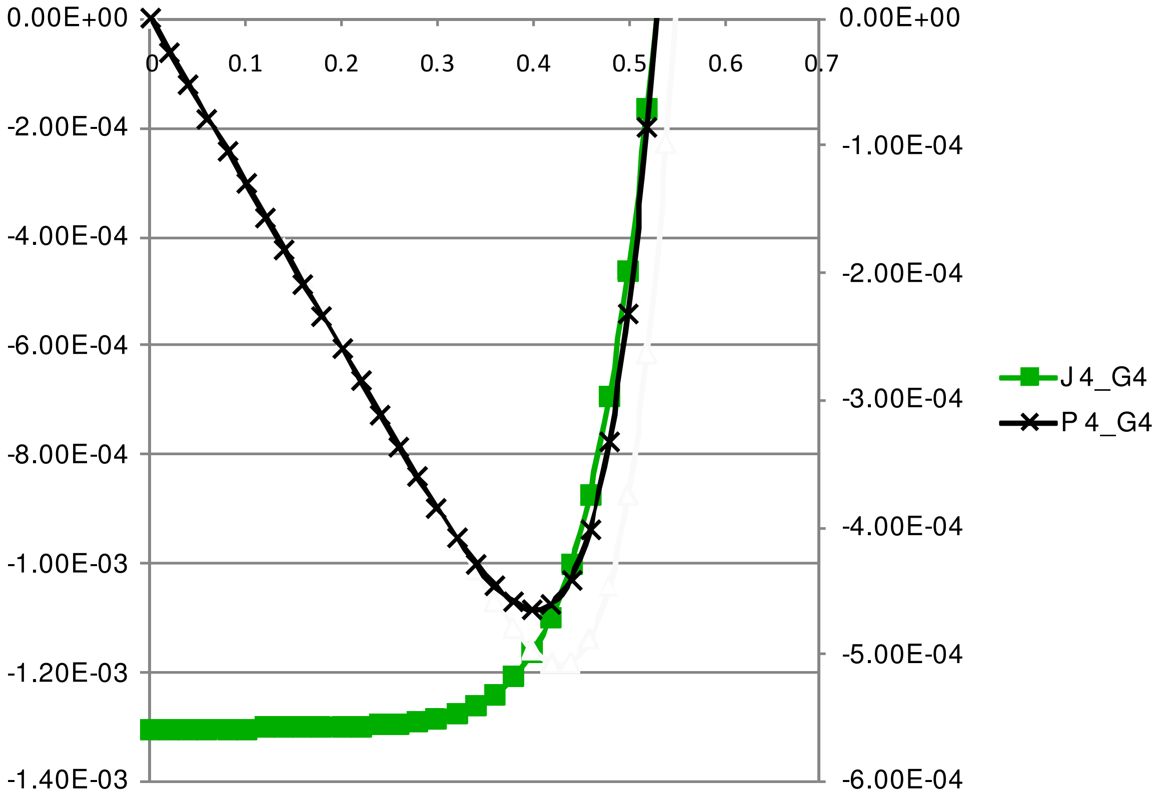
POWER, EFFICIENCY, Isc, Voc



Setting	Spot size (	Cell size (c	Current	J (A/cm2)	Irradiance (mW/cm2)	
2.25x @ma	1.267	**0.25**	2.77E-04	1.11E-03	3.39	



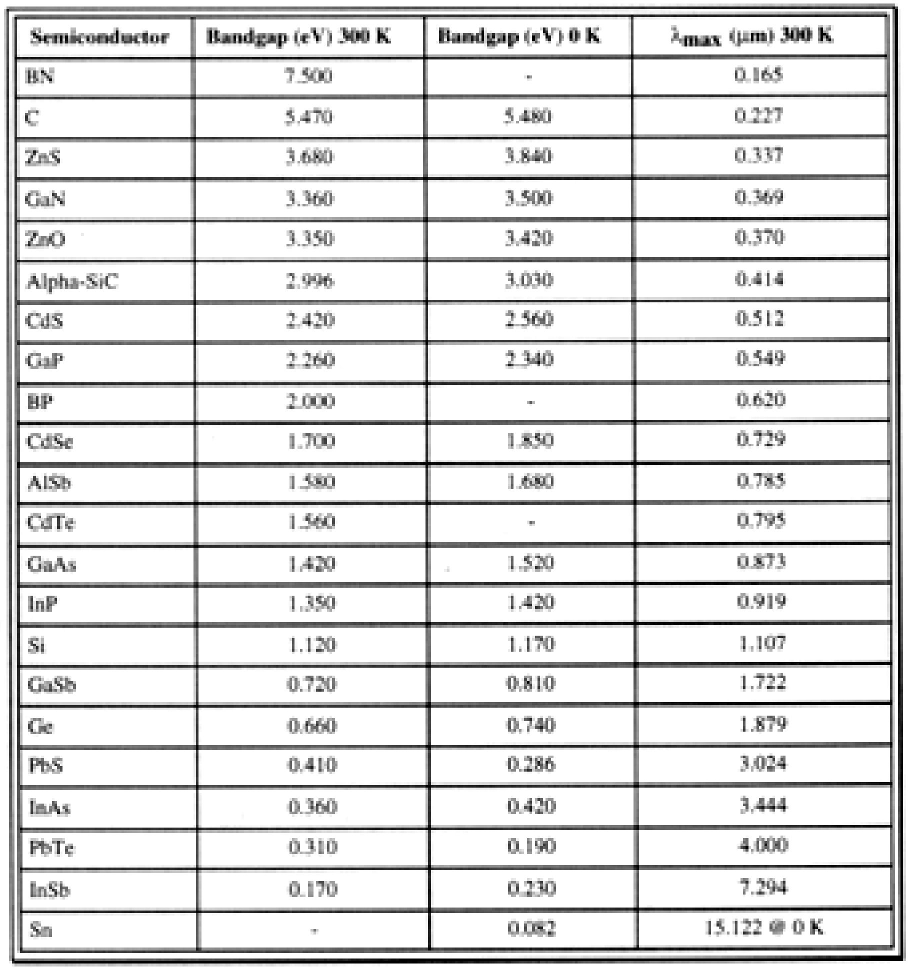
Column1	**P 4_G5**	**P 4_G4**
Pmax	-5.06E-04	-4.65E-04
Jmax	-1.21E-03	**-1.160E-03**
Vmax	0.42	**0.380**
Jsc	-1.30E-03	-1.30E-03
Voc	5.60E-01	**0.540**
FF	69.8%	62.8%
efficiency	-15%	-14%



BANDGAP OF VARIOUS SEMICONDUCTORS

$$E = h\nu = hc / \lambda$$

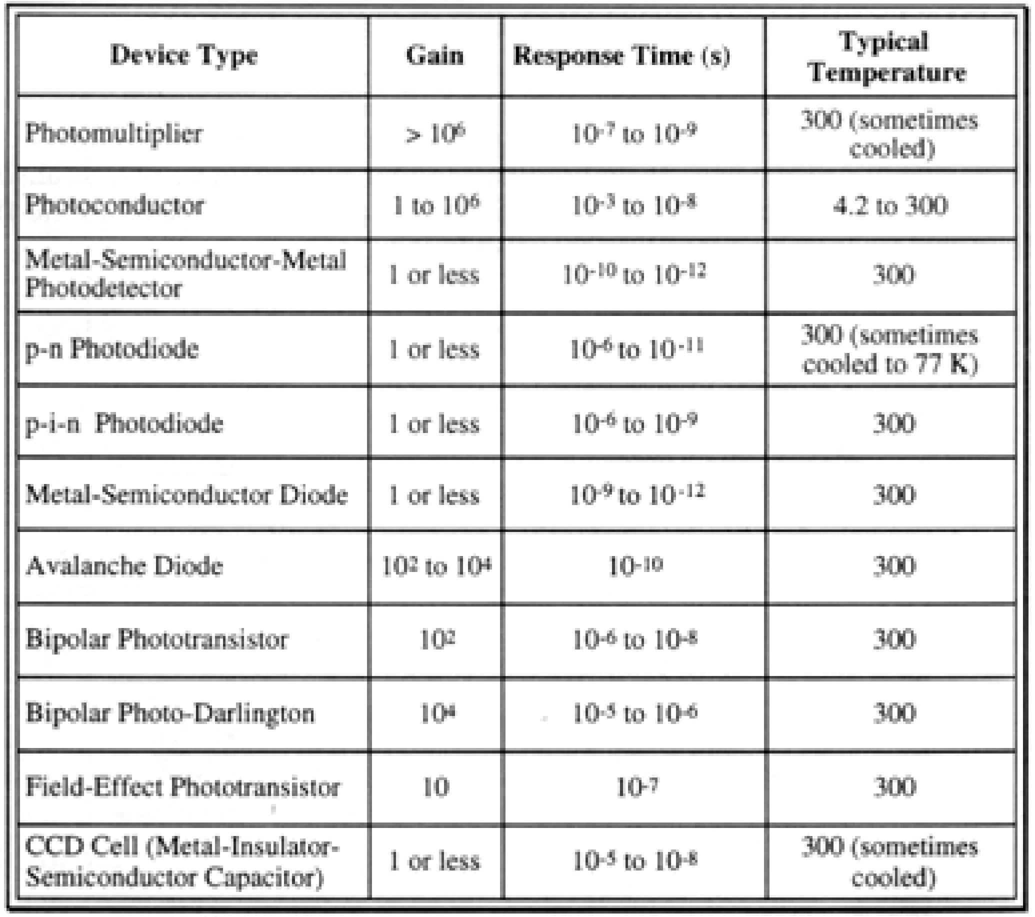
What wavelengths will not generate e-h pairs in silicon. Thus silicon is transparent or light of this wavelength or longer is not adsorbed?



Semiconductor	Bandgap (eV) 300 K	Bandgap (eV) 0 K	λ_{max} (μm) 300 K
BN	7.500	-	0.165
C	5.470	5.480	0.227
ZnS	3.680	3.840	0.337
GaN	3.360	3.500	0.369
ZnO	3.350	3.420	0.370
Alpha-SiC	2.996	3.030	0.414
CdS	2.420	2.560	0.512
GaP	2.260	2.340	0.549
BP	2.000	-	0.620
CdSe	1.700	1.850	0.729
AlSb	1.580	1.680	0.785
CdTe	1.560	-	0.795
GaAs	1.420	1.520	0.873
InP	1.350	1.420	0.919
Si	1.120	1.170	1.107
GaSb	0.720	0.810	1.722
Ge	0.660	0.740	1.879
PbS	0.410	0.286	3.024
InAs	0.360	0.420	3.444
PbTe	0.310	0.190	4.000
InSb	0.170	0.230	7.294
Sn	-	0.082	15.122 @ 0 K

Table of various semiconductors in order of increasing λ_{max}. From Sze (1981).

TYPES OF PHOTODETECTORS

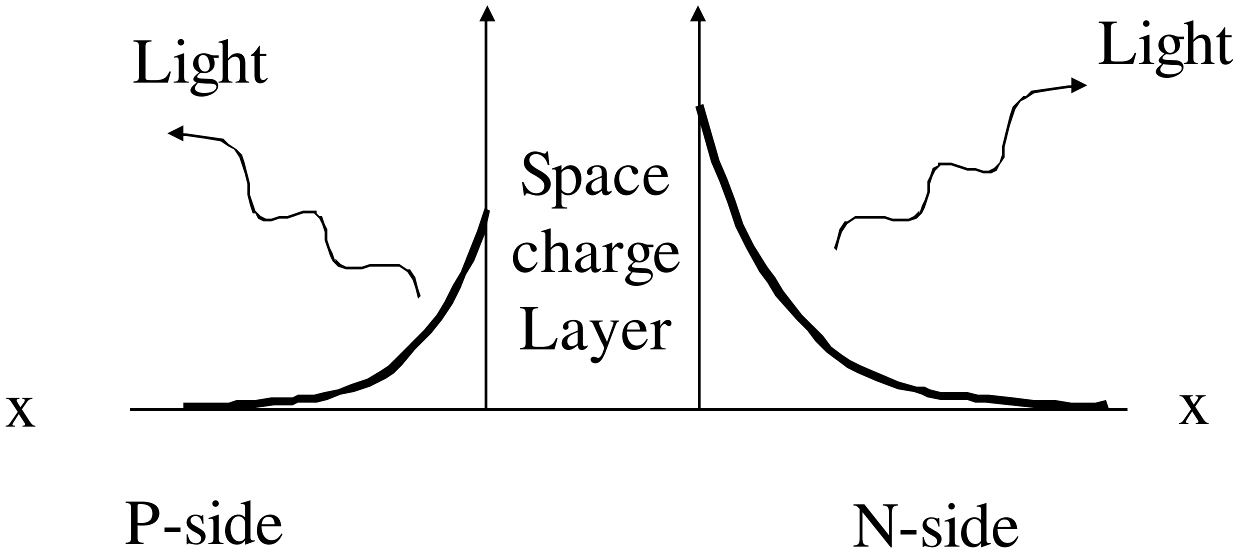


Device Type	Gain	Response Time (s)	Typical Temperature
Photomultiplier	> 10^6	10^{-7} to 10^{-9}	300 (sometimes cooled)
Photoconductor	1 to 10^6	10^{-3} to 10^{-8}	4.2 to 300
Metal-Semiconductor-Metal Photodetector	1 or less	10^{-10} to 10^{-12}	300
p-n Photodiode	1 or less	10^{-6} to 10^{-11}	300 (sometimes cooled to 77 K)
p-i-n Photodiode	1 or less	10^{-6} to 10^{-9}	300
Metal-Semiconductor Diode	1 or less	10^{-9} to 10^{-12}	300
Avalanche Diode	10^2 to 10^4	10^{-10}	300
Bipolar Phototransistor	10^2	10^{-6} to 10^{-8}	300
Bipolar Photo-Darlington	10^4	10^{-5} to 10^{-6}	300
Field-Effect Phototransistor	10	10^{-7}	300
CCD Cell (Metal-Insulator-Semiconductor Capacitor)	1 or less	10^{-5} to 10^{-8}	300 (sometimes cooled)

Gains and response times of some typical photodetectors (some are optimistic!). After Sze (1981). Note that the CCD cell, and some extrinsic photoconductors, are integrating detectors, and thus the response time figures can be somewhat misleading.

LIGHT EMITTING DIODES (LEDs)

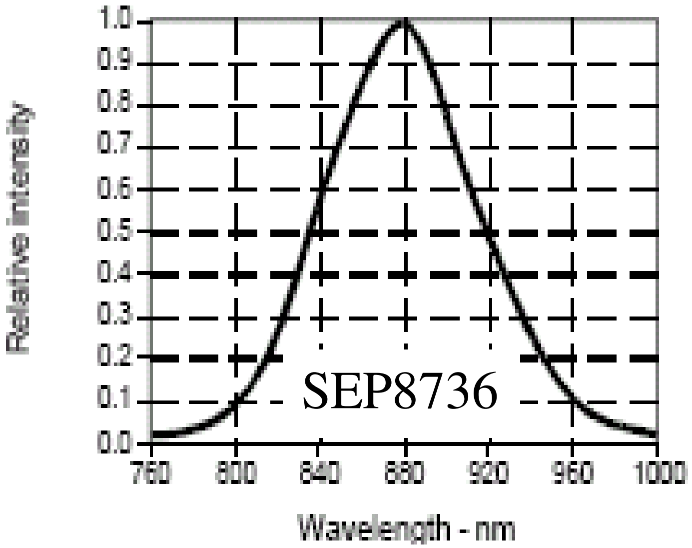
Electron concentration vs distance Hole concentration vs distnace

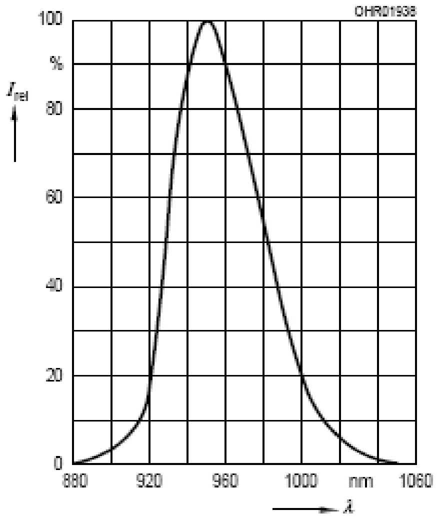


In the forward biased diode current flows and as holes recombine on the n-side or electrons recombine on the p-side, energy is given off as light, with wavelength appropriate for the energy gap for that material. $\lambda = h\ c\ /\ E$

h = Plank's constant
c = speed of light

LEDs





SFH4110

SEP8736

IMRA-807 F

SEP8736

AlGaAs Infrared Emitting Diode

ELECTRICAL CHARACTERISTICS (25°C unless otherwise noted)

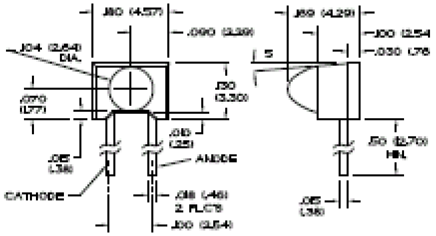
PARAMETER	SYMBOL	MIN	TYP	MAX	UNITS	TEST CONDITIONS
Irradiance (1)	H				mW/cm²	I_F=20 mA
SEP8736-001		0.5				
SEP8736-002		1.2		3.0		
SEP8736-003		1.7				
Forward Voltage	V_F			1.7	V	I_F=20 mA
Reverse Breakdown Voltage	V_{BR}	3.0			V	I_R=10 μA
Peak Output Wavelength	λ_P		880		nm	
Spectral Bandwidth	Δλ		80		nm	
Spectral Shift With Temperature	$\Delta\lambda_P/\Delta_T$		0.2		nm/°C	
Beam Angle (2)	Ø		10		degr.	I_F=Constant
Radiation Rise And Fall Time	t_r, t_f		0.7		μs	

Notes

1. Measured in mW/cm² into a 0.104 (2.64) diameter aperture placed 0.500(12.7) from the lens tip.
2. Beam angle is defined as the total included angle between the half intensity points.

OUTLINE DIMENSIONS in inches (mm)

Tolerance	3 plc decimals	±0.005(0.12)
	2 plc decimals	±0.020(0.51)



REFERENCES

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2. Microsystem Design, Stephen D. Senturia, Kluwer Academic Press, 2001.
3. IEEE Journal of Microelectromechanical Systems.
4. Solar Cells, Martin A. Green, Prentice-Hall

HOMEWORK – DIODE SENSORS THEORY

1. Calculate the temperature change if a diodes forward voltage increases from 0.65 volts to 0.69 volts. Repeat for a change from 0.65 volts to 0.55 volts.

2. Calculate the change in capacitance expected for a diode heated from room temperature 300°K to 400°K.