Flat Optics: Controlling Wavefronts with Optical Antenna Metasurfaces

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Abstract—Conventional optical components rely on the propagation effect to control the phase and polarization of light beams. One can instead exploit abrupt phase and polarization changes associated with scattered light from optical resonators to control light propagation. In this paper, we discuss the optical responses of anisotropic plasmonic antennas and a new class of planar optical components (“metasurfaces”) based on arrays of these antennas. To demonstrate the versatility of metasurfaces, we show the design and experimental realization of a number of flat optical components: (a) metasurfaces with a constant interfacial phase gradient that deflect light into arbitrary directions; (b) metasurfaces with anisotropic optical responses that create light beams of arbitrary polarization over a wide wavelength range; (c) planar lenses and axicons that generate spherical wavefronts and non-diffracting Bessel beams, respectively; and (d) metasurfaces with spiral phase distributions that create optical vortex beams of well-defined orbital angular momentum.

Index Terms— Antenna arrays, Lenses, Metamaterials, Optical surface waves, Optical polarization, Phased arrays

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optically thin photonic devices such as optical phased arrays that can reflect and refract light into practically arbitrary directions [10,19-21], optical vortex plates [10,22], perfect propagating-wave-to-surface-wave converters [23], and flat lenses and axicons [24]. In addition, it has been shown that spatially varying polarization manipulation produces a geometrical phase modification closely related to the Pancharatnam–Berry phase [25,26], which affects the propagation of the transmitted beam [27-33]. By appropriate design of anisotropic optical antennas one can also introduce abrupt polarization changes as light is transmitted or reflected by the metasurface. Metasurfaces with giant optical birefringence have been demonstrated [34-39]. Finally by tailoring the spatial profiles of the phase discontinuities and polarization response across the metasurface novel background free and broad-band quarter-wave plates have been demonstrated [40].

This paper presents a systematic study of the properties of plasmonic antennas in changing the phase and polarization of the scattered light, and the applications of plasmonic antenna arrays in flat optical components (“metasurfaces”). Optical antennas are optical analogues of radio wave antennas. They have a wide range of potential applications [41-53]. Previous research efforts have primarily focused on the capability of optical antennas in capturing and concentrating light power into subwavelength regions [54-58]. However, their phase and polarization responses and their implications in controlling the propagation of light have not been systematically investigated.

This paper is organized as follows: Section II discusses fundamental optical properties of plasmonic antennas, including the basics of antenna resonance, eigenmodes of anisotropic antennas, and scattering properties of phased optical antenna arrays. Section III presents a few applications of metasurfaces comprising phased optical antenna arrays. These include demonstration of generalized laws of reflection and refraction, broadband background-free plasmonic wave plates, planar plasmonic lenses, and phase plates that generate optical vortices. Section IV is conclusion.

II. PHASE AND POLARIZATION RESPONSES OF OPTICAL ANTENNAS

A. Oscillator Model for Optical Antennas

The phase shift between the scattered and incident light of an optical antenna sweeps a range of $-\pi$ across a resonance. To achieve a qualitative understanding of the phase shift, one can do the following analysis. If the antenna is optically small ($l \lambda_{sp} \ll 1$, where $l_a$ is the length of the antenna and $\lambda_{sp}$ is surface plasmon wavelength [17]), its charge distribution instantaneously follows the incident field, i.e.,

$$\rho \propto E_{inc} = E_{inc} \exp\left(i\omega t\right),$$

where $\rho$ is the charge density at one end of the antenna. Therefore, the scattered electric field from the antenna, which is proportional to the acceleration of the charges (Larmor formula [59]), is

$$\vec{E}_{scat} \propto \nabla E_{inc} \propto -\omega^2 E_{inc}.$$ 

That is, the incident and scattered fields are $\pi$ out of phase. At antenna resonance ($l \lambda_{sp} \approx 1/2$), the incident field is in phase with the current at the center of the antenna, i.e., $\vec{I} \propto \vec{E}_{inc}$ and therefore drives the current most efficiently, leading to maximum charge density at the antenna ends. As a result,

$$\vec{E}_{scat} \propto \frac{\partial^2 \rho}{\partial t^2} \propto -\omega^2 E_{inc},$$

the phase difference between $\vec{E}_{scat}$ and $\vec{E}_{inc}$ is $\pi/2$. For a long antenna with length comparable to the wavelength ($l \lambda_{sp} \approx 1$), the antenna impedance (defined as the incident field divided by the current at the center of the antenna) is primarily inductive, or $I \propto -iE_{inc}$. Consequently, the scattered and incident light are almost in phase, $\vec{E}_{scat} \propto \nabla E_{inc} \propto \vec{E}_{inc}$. In summary, for a fixed excitation wavelength, the impedance of an antenna changes from capacitive, to resistive, and to inductive cross a resonance as the antenna length increases, which leads to the $0$-to-$\pi$ phase shift.

There are no analytical solutions of the phase response of antennas and the problem used to be a challenging topic for mathematical physicists [60]. The problem of antenna scattering is complex because a charge in an antenna is not only driven by the incident field but also by the retarded Coulomb forces exerted by the rest of the oscillating charges (“self-interaction”). An antenna with $l_a \approx \lambda_{sp}/2$ is resonant because electric charges at one end of the antenna rod experience the repulsive force exerted by the charges of the same sign that were at the other end of the rod a half period earlier [61]. This repulsive force leads to maximum oscillating charges. Resonant behavior can be found in any type of vibration, including mechanical, electrical, optical, and acoustic, among others, and can be utilized in the manipulation of these various kinds of waves [62,63].

In the following we summarize a simple oscillator model [64] that we recently developed for optical antennas and, in general, for any nanostructures supporting localized surface plasmon resonances (LSPRs) [65,66]. The model treats the resonant, collective oscillations of electrons in the nanostructure as a damped, driven harmonic oscillator consisting of a charge on a spring. Unlike previously proposed models in which all damping mechanisms were combined into a single loss term proportional to the charge velocity [67-69], we explicitly accounted for two decaying channels for LSPR modes: free carrier absorption (internal damping) and emission of light into free space (radiation damping).

We begin by analyzing a system in which a charge $q$ located at $x(t)$ with mass $m$ on a spring with spring constant $k$ (Fig. 1(a)) is driven by an incident electric field with frequency $\omega$, and experiences internal damping with damping coefficient $\Gamma_a$:

$$m \frac{d^2x}{dt^2} + \Gamma_a \frac{dx}{dt} + kx = qE_0e^{i\omega t} - \Gamma_s \frac{d^3x}{dt^3}$$

In addition to the internal damping force $\Gamma_s(\omega,t) = -\Gamma_s \frac{dx}{dt}$, the charge experiences an additional force $F_s(\omega,t) = -\Gamma_s \frac{dx}{dt}$ due to radiation reaction, where $\Gamma_s = q^2/6\pi\varepsilon_0$. This term
describes the recoil that the accelerating charge feels when it emits radiation that carries away momentum. The recoil is referred to as the Abraham-Lorentz force or the radiation reaction force [70], and it can also be seen as the force that the field produced by the charge exerts on the charge itself [71]. For our charge-on-a-spring model, the radiation reaction term has to be included for physical consistency, and cannot be absorbed into the internal damping coefficient $\Gamma_a$.

By assuming harmonic motion $x(\omega,t) = x(\omega)e^{i\omega t}$, it follows from Eq. (1) that
\[
x(\omega,t) = \frac{(q/m)E_0}{(\omega_0^2 - \omega^2) + i\omega/\mu + (\Gamma_a + \omega^2\Gamma_s)}e^{i\omega t} = x(\omega)e^{i\omega t}
\] (2)
where $\omega_0 = \sqrt{k/m}$. The time-averaged absorbed power by the oscillator can be written as $P_{abs}(\omega) = F_a(\omega,t)^\dagger(i\omega x(\omega,t))$, where $F_a(\omega,t)^\dagger$ is the complex conjugate of the internal damping force. Similarly, the time-averaged scattered power by the oscillator is $P_{scat}(\omega) = F_i(\omega,t)^\dagger(i\omega x(\omega,t))$. Therefore we have:
\[
P_{abs}(\omega) = \omega^2\Gamma_a |x(\omega)|^2
\] (3)
\[
P_{scat}(\omega) = \omega^4\Gamma_s |x(\omega)|^2
\] (4)

Our oscillator model can shed light on the relationship between the near-field, absorption, and scattering spectra in optical antennas. If we interpret the optical antenna as an oscillator that obeys Eqs. (1)-(4), we can associate $P_{abs}$ and $P_{scat}$ in Eqs. (3) and (4) with the absorption and scattering spectra of the antenna, respectively. Furthermore, we can calculate the near-field intensity enhancement at the tip of the antenna as $|E_{near}(\omega)|^2 \propto |x(\omega)|^2$ [69].

By examining Eqs. (3) and (4) and noting that $P_{scat} \propto \omega^2 P_{abs} \propto \omega^4 |E_{near}(\omega)|^2$ we can deduce that the scattering spectrum $P_{scat}(\omega)$ will be blue-shifted relative to the absorption spectrum $P_{abs}(\omega)$, which will in turn be blue-shifted relative to the near-field intensity enhancement spectrum $|E_{near}(\omega)|^2$. This is in agreement with experimental observations that the wavelength dependence of near-field quantities such as the electric-field enhancement can be significantly red-shifted compared with far-field quantities such as scattering spectra [73-78]. These spectral differences can also be clearly seen in finite difference time domain (FDTD) simulations of gold linear antennas on a silicon substrate designed to resonate in the mid-infrared spectral range (Fig. 1(c)). We fit the simulation results presented in Fig. 1(c) with Eqs. (3) and (4) to obtain the parameters $q$, $m$, $\omega_0$, and $\Gamma_s$. The resulting model is able to explain the peak spectral position and general shape of the near-field intensity, as well as the phase response of the antenna (Fig. 1(c)). This result suggests that this model can predict the near-field amplitude and phase response from experimental far-field spectra of antennas, which are much easier to obtain than near-field measurements. Our model shows that in LSPR systems the near-field, absorption, and scattering spectra are all expected to peak at different frequencies and have distinct profiles.

It is important to note that our results remain valid only for wavelengths far enough away from any material resonances. For example, in the visible range near an interband transition for gold the absorption spectrum for nanospheres peaks at a
smaller wavelength than the scattering spectrum [78], contrary to the predictions of our model. It appears that our model can be safely applied to noble metal structures in the near- and mid-infrared spectral range and longer wavelengths. To treat the short wavelength regime our model could be augmented by introducing an additional oscillator with a coupling term to represent the resonant absorption due to an interband transition in a metal [65].

Fig. 2. (a) A V-shaped optical antenna is a simple example of a plasmonic two-oscillator element. Its two orthogonal modes, i.e., symmetric and antisymmetric modes, are shown respectively in the left and right panels. The schematic current distribution on the antenna is represented in gray scale with lighter tones indicating larger current density. The instantaneous direction of current flow is indicated by arrows with gradient. (b) SEM images of gold V-shaped antennas fabricated on a silicon substrate with opening angles Δ = 45°, 75°, 90°, and 120°. (c)-(e) Measured transmission spectra through the V-antenna arrays at normal incidence as a function of wavelength and angle Δ for fixed arm length h = 650 nm. Transmission here is the ratio of the transmitted intensity to that through the bare Si substrate to account for multiple reflections in the latter. The polarization of the incident light is indicated in the upper right corner. (f)-(h) FDTD simulations corresponding to the experimental spectra in (c)-(e), respectively. The feature at λ_o = 8-9 µm is due to the phonon resonance in the ~2 nm SiO_2 on the substrate.

B. Eigenmodes of Anisotropic Plasmonic Antennas

To gain full control over an optical wavefront, we need a subwavelength optical element able to span the phase of the scattered light relative to that of the incident light from 0 to 2π and able to control the polarization of the scattered light. A plasmonic element consisting of two independent and orthogonally oriented oscillator modes is sufficient to provide complete control of the amplitude, phase, and polarization response, and is therefore suitable for the creation of designer metasurfaces as described in detail in Refs. [39] and [79].

A large class of plasmonic elements can support two orthogonally-orientated modes. We focus on lithographically-defined nanoscale V-shaped plasmonic antennas as examples of two-oscillator systems [10,39,79,80]. The antennas consist of two arms of equal length h connected at one end at an angle
They support “symmetric” and “antisymmetric” modes (Fig. 2(a)), which are excited by electric fields parallel and perpendicular to the antenna symmetry axes, respectively. In the symmetric mode, the current and charge distributions in the two arms are mirror images of each other with respect to the antenna’s symmetry plane, and the current vanishes at the corner formed by the two arms (Fig. 2(a) left panel). This means that, in the symmetric mode, each arm behaves similarly to an isolated rod antenna of length \( h \), and therefore the first-order antenna resonance occurs at \( h \approx \lambda_{\text{eff}}/2 \). In the antisymmetric mode, antenna current flows across the joint (Fig. 2(a) right panel). The current and charge distributions in the two arms have the same amplitudes but opposite signs, and they approximate those in the two halves of a straight rod antenna of length \( 2h \). The condition for the first-order resonance of this mode is therefore \( 2h \approx \lambda_{\text{eff}}/2 \). The experiments and calculations in Figs. 2(c)-(h) indeed show that the two modes differ by about a factor of two in resonant wavelength. Systematic calculations of the current distributions of the symmetric and antisymmetric modes as a function of the antenna geometry are provided in Ref. 79. Most importantly the phase response of the scattered light from a V antenna covers the range from 0 to \( 2\pi \), as opposed to the \( \pi \) range of a linear antenna, since it results from the excitation of a linear combination of the two antenna modes for arbitrarily oriented incident polarization [10,39,79]. This full angular coverage makes it possible to shape the wavefront of the scattered light in practically arbitrary ways.

The scattered light largely preserves the polarization of the incident field when the latter is aligned parallel or perpendicular to the antenna symmetry axis, corresponding to unit vectors \( \hat{\mathbf{s}} \) and \( \hat{\mathbf{a}} \), respectively (Fig. 2(a)). For example, when the incident light is polarized along the \( \hat{\mathbf{s}} \) direction, the scattered electric field is polarized primarily along the same axis since the charge distribution is primarily dipolar. Note that a quadrupolar mode is excited in the \( \hat{\mathbf{a}} \) direction but its radiative efficiency is poor. For an arbitrary incident polarization, both the symmetric and antisymmetric antenna modes are excited, but with substantially different amplitudes and phases due to their distinctive resonance conditions. As a result, the antenna radiation can have tunable polarization states.

We characterized the spectral response of V-antennas by Fourier transform infrared (FTIR) spectroscopy and numerical simulations. In Fig. 2, we mapped the two oscillator modes of V-antennas as a function of wavelength and opening angle \( \Delta \) by showing the measured (c)-(e) and calculated (f)-(h) transmission spectra. The gold antennas fabricated on silicon wafers have arm length \( h = 650 \) nm, width \( w = 130 \) nm, thickness \( t = 60 \) nm, and opening angle ranging from 45° to 180°. Figures 2(c) and (f) correspond to excitation of only the x-oriented symmetric antenna mode, whereas (d) and (g) correspond to the y-oriented antisymmetric mode, and (e) and (h) shows both excited modes. The spectral positions of these resonances are slightly different from the first order approximation which would yield \( \lambda_x \approx 2hn_{\text{eff}} \approx 3.4 \) μm and \( \lambda_y \approx 4hn_{\text{eff}} \approx 6.8 \) μm, taking \( n_{\text{eff}} \) as 2.6 [10], with the differences attributed to the finite aspect ratio of the antennas and near-field coupling effects. The latter are especially strong for small \( \Delta \) when the arms are in closer proximity to each other, leading to a significant resonance shift (Figs. 2(d) and (g)). All of the experimental results are reproduced very well in simulations, including the feature at 8-9μm due to a phonon resonance in the 2-nm native silicon oxide layer on the silicon substrate, which is enhanced by the strong near fields formed around the metallic antennas. In Figs. 2(d), (e), (g), and (h), a higher order antenna mode is clearly visible at \( \lambda_o \approx 2.5 \) μm for large \( \Delta \).

C. Design of Phased Optical Antenna Array

The essence of metasurfaces is to use spatially inhomogeneous arrays of anisotropic optical antennas to control optical wavefronts. As an example, we designed a set of eight different V-antennas (Table I), which provide phase shifts over the entire \( 0 \)-to-\( 2\pi \) range in increments of \( \pi/4 \) and are used as basic elements for constructing metasurfaces. The four first antennas in the array have their symmetry axis fixed along the same direction. The last four antennas are obtained by rotating the first four antennas in the clock-wise direction by \( 90^\circ \). It will be shown later that this coordinate transformation introduces a \( \pi \) phase shift in the scattered light of the last four antennas.

<table>
<thead>
<tr>
<th>TABLE I ANTENNA SCATTERED FIELD</th>
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<tbody>
<tr>
<td>Projection of ( E_{\text{scat}} ) to s-axis</td>
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<tr>
<td>Projection of ( E_{\text{scat}} ) to a-axis</td>
</tr>
<tr>
<td>Scattered field of symmetric eigenmode</td>
</tr>
<tr>
<td>Scattered field of antisymmetric eigenmode</td>
</tr>
<tr>
<td>Scattered field of antennas</td>
</tr>
</tbody>
</table>

We follow three steps to calculate the scattered light from the antennas as shown in Table I. First, the incident field with an arbitrary linear polarization is decomposed into components along the \( \hat{\mathbf{s}} \) and \( \hat{\mathbf{a}} \) axes, which will excite the two eigenmodes, respectively. Second, we calculate the complex scattered fields of the eigenmodes \( (S_i\hat{\mathbf{s}} \text{ and } A_i\hat{\mathbf{a}}, i=1-4) \), which can be obtained by analytical calculations or simulations [10,39,79]. Third, the scattered field of the \( i^\text{th} \) antenna and its rotated counterpart is expressed as a linear combination of the field components radiated by the symmetric and antisymmetric modes, in the x-y reference frame, as shown in the last row of Table I. Equation (5) provides explicitly the expressions for the fields scattered by the eight antennas:
Here the angles $\alpha$ and $\beta$ represent the orientation of the incident field and of the antenna symmetry axis, respectively. Equation (5) shows that the scattered light from the antennas ($E_i$, with $i=1-8$) contains two terms, which are polarized along the $(2\beta-\alpha)$-direction and the $\alpha$-direction, respectively. Note that the minus signs of the $(2\beta-\alpha)$-polarized components of antennas #5-#8 originate from rotation of the antenna symmetry axis.

$$
\begin{align*}
\begin{pmatrix}
E_1 \\
E_2 \\
E_3 \\
E_4 \\
E_5 \\
E_6 \\
E_7 \\
E_8
\end{pmatrix} &= \begin{pmatrix}
S_1 - A_1 \\
S_2 - A_2 \\
S_3 - A_3 \\
S_4 - A_4 \\
-(S_1 - A_1) \\
-(S_2 - A_2) \\
-(S_3 - A_3) \\
-(S_4 - A_4)
\end{pmatrix} \begin{pmatrix}
\cos(2\beta - \alpha) \hat{y} \\
+ \sin(2\beta - \alpha) \hat{x}
\end{pmatrix} \begin{pmatrix}
1 \\
1/2 \\
1/2 \\
1/2 \\
1 \\
-1 \\
-1 \\
1
\end{pmatrix} \begin{pmatrix}
S_1 + A_1 \\
S_2 + A_2 \\
S_3 + A_3 \\
S_4 + A_4
\end{pmatrix} \\
&= \begin{pmatrix}
\cos \alpha \hat{y} \\
+ \sin \alpha \hat{x}
\end{pmatrix}
\end{align*}
$$

We spatially tailor the antenna geometries so that at $\lambda = 8\mu m$ the $(2\beta-\alpha)$-polarized components of all the antennas have the same amplitudes and phase increments of $\Delta \phi = \pi/4$ (Fig. 3, Figs. 4(a) and (b)). That is, $|S_i - A_i|$ is constant, with $i=1-4$, and Phase$(S_{i+1} - A_{i+1}) - \text{Phase}(S_i - A_i) = \pi/4$, with $i=1-3$. Therefore the $(2\beta-\alpha)$-polarized partial waves scattered from the antenna array form an extraordinary beam propagating away from the surface normal. On the other hand, the $\alpha$-polarized components, which have the same polarization as the incident light, have similar phase responses at $\lambda = 8\mu m$ (Fig. 4(c)). Therefore, they combine to form a wave that propagates along the surface normal and contributes to an ordinary beam. Note that one has independent control of the wavefronts and polarizations of the two beams: the wavefronts are determined by the complex scattering amplitudes $S_i$ and

Fig. 3. (a) and (b) Calculated amplitude and phase of the $x$-polarized component of the scattered light, $E_{\text{scat,x}}$, for gold V-antennas with different geometries at $\lambda = 8\mu m$. The incident field is polarized along the $y$-axis ($\alpha=0^\circ$) and the antenna symmetry axis is along $45^\circ$-direction ($\beta=45^\circ$); therefore the scattered field $E_{\text{scat,x}}$ is cross-polarized with respect to the incident light ($2\beta-\alpha=90^\circ$) as shown by the first term on the right hand side of Eq. 5). We chose four antennas indicated by circles in (a) and (b) so that they provide nearly equal scattering amplitudes and incremental phases of $\pi/4$ for the cross-polarized scattered field $E_{\text{scat,x}}$. (c) FDTD simulations of $E_{\text{scat,x}}$ for individual antennas. The antennas are excited by $y$-polarized plane waves with the same phase. The difference in the propagation distance of wavefronts emanated from neighboring antennas is $1\mu m$, corresponding to a phase difference of $\pi/4$. 

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which are in turn determined by antenna materials and geometries (i.e., arm length \(h\), opening angle \(\Delta\), arm width, thickness of metal); the polarizations are instead controlled by the orientation angles of the incident field and the antenna symmetry axis (i.e., \(\alpha\) and \(\beta\)). In most of the device applications of metasurfaces to be discussed in Section III, we choose antenna orientation \(\beta=45^\circ\) and vertical incident polarization \(\alpha=0^\circ\), so the \((2\beta-\alpha)\)-polarized component is in cross polarization.

![Graphs showing phase and amplitude of scattered light](image)

**Fig. 4.** (a) and (b) Full-wave simulations of the phase and amplitude of scattered light with cross-polarization, \(E_{\text{scat},x}\), for an array of eight gold V-antennas in the wavelength range \(\lambda = 5-16 \mu m\). The geometries of the first four antennas in the array are indicated in Fig. 3(a); the last four antennas are obtained by rotating the first four antennas 90° clock-wise. (c) and (d) Full-wave simulations of the phase and amplitude of the y-polarized component of the scattered light, \(E_{\text{scat},y}\), in the wavelength range \(\lambda = 5-16 \mu m\). All phases are referred to the phase of antenna #1, which is taken to be 0. In all the plots antenna #9 is the same as #1.

### III. APPLICATIONS OF PLASMONIC METASURFACES

#### A. Generalized Laws of Reflection and Refraction

In this section, we show one of the most dramatic demonstrations of controlling light using metasurfaces; that is, a linear phase variation along an interface introduced by an array of phased optical antennas leads to anomalously reflected and refracted beams in accordance with generalized laws of reflection and refraction. We note that antenna arrays in the microwave and millimeter-wave regime have been used for the shaping of reflected and transmitted beams in the so-called “reflectarrays” and “transmitarrays” [81-85]. There is a connection between that body of work and our work in that both use abrupt phase changes associated with antenna resonances. However the generalization of the laws of reflection and refraction we are going to present is made possible by the deep-subwavelength thickness of our optical antennas and their subwavelength spacing. It is this metasurface nature that distinguishes it from reflectarrays and transmitarrays, which typically consist of a double-layer structure separated by a dielectric spacer of finite thickness, and the spacing between the array elements is usually not subwavelength.
Equation (6) implies that the refracted beam can have an arbitrary direction in the plane of incidence, provided that a suitable constant gradient of phase discontinuity along the interface \(\frac{d\Phi}{dx}\) is introduced \([10]\). Because of the non-zero phase gradient in this modified Snell’s law, the two angles of incidence \(\pm \theta_i\) lead to different values for the angle of refraction. As a consequence, there are two possible critical angles for total internal reflection, provided that \(n_i < n_t\)

\[
\theta_c = \arcsin \left( \pm \frac{n_i}{n_t} - \frac{1}{k_n} \frac{d\Phi}{dx} \right) \quad (7)
\]

Similarly, for the reflected light we have

\[
\sin(\theta_r) - \sin(\theta_i) = \frac{1}{k_n} \frac{d\Phi}{dx} \quad (8)
\]

There is a nonlinear relation between \(\theta_i\) and \(\theta_c\), which is dramatically different from conventional specular reflection. Equation (8) predicts that there is always a critical incidence angle above which the reflected beam becomes evanescent.

In the 3D situation with arbitrary orientation of the interfacial phase gradient, one has to consider separately the conservation of wavevector parallel and perpendicular to the plane of incidence (Fig. 5(b)). For the transmitted light beam, we have

\[
\begin{align*}
&k_{x,t} = k_{x,i} + \frac{d\Phi}{dx} \\
&k_{y,t} = k_{y,i} + \frac{d\Phi}{dy}
\end{align*}
\]

where \(k_{x,t} = k_n \sin(\theta_t)\) and \(k_{y,t} = k_n \sin(\theta_t)\) are the in-plane wavevector components of the refracted and incident beams, respectively; \(k_{x,i} = k_n \cos(\theta_i) \sin(\varphi_i)\) and \(k_{y,i} = 0\) are the out-of-plane wavevector components of the refracted and incident beams, respectively. Therefore Eq. 10 can be rewritten as

\[
\begin{align*}
&n_i \sin(\theta_i) - n_t \sin(\theta_t) = \frac{1}{k_o} \frac{d\Phi}{dx} \quad (11) \\
&\cos(\theta_t) \sin(\varphi_t) = \frac{1}{n_k} \frac{d\Phi}{dy}
\end{align*}
\]

which is the generalized law of refraction in 3D \([20]\). Similarly, the generalized law of reflection in 3D reads

\[
\begin{align*}
&\sin(\theta_r) - \sin(\theta_i) = \frac{1}{n_k} \frac{d\Phi}{dx} \\
&\cos(\theta_r) \sin(\varphi_r) = \frac{1}{n_k} \frac{d\Phi}{dy}
\end{align*}
\]

The first equations in (11) and (12) are the same as the 2D situation (Eqs. (6) and (8)). The out-of-plane deflection angles \(\varphi_i\) and \(\varphi_r\) are determined by the out-of-plane phase gradient \(\frac{d\Phi}{dy}\).

The interfacial phase gradient originates from inhomogeneous plasmonic structures on the interface and it
provides an extra momentum to the reflected and transmitted photons. In return, the photons should exert a recoil force to the interface. Note that the generalized laws (Eqs. (6), (8), (11), and (12)) can also be derived following Fermat’s principle (or the principle of stationary phase) \[10,20\]. The latter states that the total phase shift \( \Phi(\hat{r}) + \int_A^B k(n(\hat{r})) \, d\hat{r} \) accumulated must be stationary for the actual path a light beam takes. Here the total phase shift includes the contribution due to propagation \( \int_A^B k(n(\hat{r})) \, d\hat{r} \) and abrupt phase changes \( \Phi(\hat{r}) \) acquired when the light beam transverses an interface; \( \hat{r} \) is the position along the interface.

\[
\Phi(\hat{r}) = \text{constant} \quad \text{for} \quad \text{linear network}.
\]

By using the eight V-antennas described in Table I as building blocks, we created metasurfaces that imprint a linear distribution of phase shifts to the optical wavefronts. A representative fabricated sample with the highest packing density of antennas is shown in Fig. 6(a). A periodic antenna arrangement with a constant incremental phase is used here for convenience, but it is not necessary to satisfy the generalized laws. It is only necessary that the phase gradient is constant along the interface. The phase increments between nearest neighbors do not need to be constant, if one relaxes the unnecessary constraint of equal spacing between neighboring antennas.

Figure 6(b) shows the schematic setup used to demonstrate the generalized laws. Large arrays (~230 \( \mu \)m \times 230 \( \mu \)m) were fabricated to accommodate the size of the plane-wave-like excitation with a beam radius of ~100 \( \mu \)m. A buried-heterostructure quantum cascade laser (QCL) with central wavelength 8 \( \mu \)m and spectral width ~0.2 \( \mu \)m was used as the light source. The laser beam was incident from the back side of the silicon wafer, which was not decorated with antennas. The sample was mounted at the center of a motorized rotation stage, and a liquid-nitrogen-cooled mercury-cadmium-telluride (MCT) detector positioned ~15 cm away from the sample was scanned to determine the angular positions of the two ordinary beams and the two extraordinary beams. Our measurements were performed with an angular resolution of 0.2 degrees.

Figure 6(c) shows experimental far-field scans at excitation wavelengths from 5.2 to 9.9 \( \mu \)m. Three samples with \( \Gamma = 13 \), 15, and 17 \( \mu \)m were tested. For all samples and excitation wavelengths, we observed the ordinary and extraordinary

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refraction and negligible optical background. Samples with smaller \( \Gamma \) create larger phase gradients and therefore deflect extraordinary refraction into larger angles from the surface normal; smaller \( \Gamma \) also means a higher antenna packing density and therefore more efficient scattering of light into the extraordinary beams (Fig. 6(c)). The observed angular positions of the extraordinary refraction agree very well with the generalized law of refraction, \( \theta_i = \arcsin(\lambda/\Gamma) \) (Eq. 6).

The antenna arrays can provide phase coverage from 0 to \( 2\pi \) with an increment of \( -\pi/4 \) over a wide range of wavelengths (Fig. 4(a)). Therefore, the metasurface can generate well-defined extraordinary refraction over a broad spectral range, which is confirmed in experiments (Fig. 6(c)). This broadband performance can be understood by referring to the phase plot in Fig. 3(b). The latter is calculated for a fixed wavelength of 8 \( \mu \text{m} \). Increasing (decreasing) the operating wavelength is equivalent to scaling down (up) the size of the antennas, or moving the circles representing antenna geometries in the diagram downward (upward). Within a certain range of the movement, the circles sample the linear distribution of the phase plot (Fig. 3(b)); therefore the relative phase between the antennas is maintained. This is why the antenna designs are quite robust against wavelength change.

The broadband performance of the metasurface is further demonstrated in Figs. 6(d) and (e). The ratio \( R \) between the intensity of extraordinary refraction at \( -\arcsin(\lambda/\Gamma) \) and that of the parasitic diffraction at \( \arcsin(\lambda/\Gamma) \) is in excess of a few hundred between \( \lambda=8\mu\text{m} \) and \( 10\mu\text{m} \) (Fig. 6(e)). In this optimal wavelength range, the cross-polarized component of the scattered field across the antenna array maintains a linear phase response and a relatively uniform amplitude response (Figs. 4(a) and (b)). Away from the optimal range the scattering amplitude into the cross-polarization decreases and its variation increases (Fig. 4(b)), but the linear phase response is still well maintained (Fig. 4(a)). This leads to reduced intensity in the extraordinary refraction and reduced suppression ratio \( R \) (Figs. 6(c) and (e)). The value of \( R \) is still above 10 at \( \lambda=5\mu\text{m} \) and 14\( \mu\text{m} \).

The strong suppression of diffraction at the angle \( -\arcsin(\lambda/\Gamma) \) compared to \( \arcsin(\lambda/\Gamma) \) illustrates the difference between our metasurface and a conventional diffractive grating with period \( \Gamma \) where light is equally diffracted at positive and negative orders. In fact the high suppression ratio \( R \) means that the structure operates functionally as a blazed grating where light is preferentially diffracted into a single order [5]. We have verified both experimentally and numerically that this also holds true for non-coplanar refraction [20], Larouche and Smith have in fact established the formal equivalence between the generalized law of refraction [10] and diffraction from a blazed grating [86]. However there is a fundamental difference in the way the phase of the scattered light is controlled in metasurfaces with a constant phase gradient compared to a blazed grating, due to major structural differences. In the latter it is governed by optical path differences between light scattered by grating grooves with a sawtooth profile, that is by a propagation effect, whereas in our optically thin plasmonic interface, it is controlled by abrupt phase shifts (phase discontinuities) since light scatters off deep subwavelength thick optical antennas. An important consequence of this is that our metasurface is broadband in contrast to conventional blazed gratings. Finally we wish to point out that by symmetry considerations there is no intensity at the zero diffraction order \( (\theta_i = 0') \) in cross-polarization in our metasurfaces because the complex amplitude of the cross-polarized light scattered by the four antennas \((i = 1, 2, 3, \text{and} 4 \text{ in Table I)}\) is exactly canceled by the amplitude scattered by the other four antennas, since they are the mirror images of the first four when antenna orientation \( \beta = 45' \). The light measured at \( \theta_i = 0' \) is the ordinary refraction polarized as the incident radiation.

For a complete characterization of the metasurfaces, we changed the incident angle \( \theta_i \) and scanned the detector over a larger angular range to capture both the extraordinary refraction and reflection. The experimental far-field scans for a sample with \( \Gamma=15\mu\text{m} \) at \( \lambda=8\mu\text{m} \) are shown in Fig. 7(a). A polarizer was added between the sample and the detector to isolate cross-polarized extraordinary beams and the results are shown in Fig. 7(b). The intensity of the extraordinary refraction (beams “1”) is about one third of that of the ordinary refraction (beams “0”). At \( \lambda=8\mu\text{m} \) the metasurface scatters approximately 10\% of the incident light into the extraordinary refraction and approximately 15\% into the extraordinary reflection. The efficiency can be increased by using denser antenna arrays or by exploiting antenna designs with higher scattering amplitudes (e.g., antennas with a metallic back plane operating in reflection mode).

Figures 7(c) and (d) show the angles of refraction and reflection, respectively, as a function of incident angle \( \theta_i \) for both the silicon-air interface and the metasurface with linear interfacial phase gradient. In the range of \( \theta_i = 0-9' \), the cross-polarized extraordinary beams exhibit negative angle of refraction and reflection (red curves in Figs. 7(a) and (b), red circles in Figs. 7(c) and (d)). The critical angle for total internal reflection is modified to about -8' and +27' for the metasurface in accordance with Eq. (7) compared to ±17' for the silicon-air interface (Fig. 7(c)); the anomalous reflection does not exist beyond \( \theta_i = -57' \) (Fig. 7(d)).
Fig. 7. (a) Measured far-field intensity as a function of the angular position $\gamma$ of the detector (defined in Fig. 6(b)) at different angles of incidence $\theta_i$. The unit cell of the metasurface has a lateral periodicity of $\Gamma = 15$ µm. Beams "0", "1", "2", and "3" correspond to those labeled in Fig. 6(b). The very weak beams "4" and "5" originate from the second-order diffraction of the antenna array of periodicity $\Gamma$. Their non-zero intensity is due to imperfections in the antenna array (i.e., slight mismatch in scattering amplitudes and deviation from a linear phase distribution). At $\theta_i = 4.3^\circ$ beam "2" cannot be measured in our setup because it is counter-propagating with respect to the incident beam. (b) Measured far-field intensity profiles with a polarizer in front of the detector. The polarizer filters the scattered light that is cross-polarized with respect to the incident light. (c) Angle of refraction versus angle of incidence for the ordinary and extraordinary refraction for the sample with $\Gamma = 15$ µm. The curves are theoretical calculations using the generalized Snell’s law (Eq. (6)) and the symbols are experimental data. The two arrows indicate the modified critical angles for total internal reflection. The red symbols correspond to negative angle of refraction. (d) Angle of reflection versus angle of incidence for the ordinary and extraordinary reflection for the sample with $\Gamma = 15$ µm. The inset is the zoom-in view. The curves are theoretical calculations using Eq. (8) and the symbols are experimental data. The arrow indicates the critical incidence angle above which the anomalously reflected beam becomes evanescent. The red symbols correspond to negative angle of reflection.

We demonstrated out-of-plane refraction (Fig. 8) in accordance with the 3D generalized law (Eq. (11)) using the same metasurface patterned with phased optical antenna arrays shown in Fig. 6(a) oriented in such a way that the interfacial phase gradient forms a nonzero angle with respect to the plane of incidence (Fig. 5(b)). Because of the tangential wavevector provided by the metasurface, the incident beam and the extraordinary reflection and refraction are in general non-coplanar. The extraordinary beams’ direction can be controlled over a wide range by varying the angle between the plane of incidence and the phase gradient, as well as the magnitude of the phase gradient.

Fig. 8. Experimental observation of out-of-plane refraction. Angles of refraction $\theta_t$ and $\phi_t$ (defined in Fig. 5(b)) versus angles of incidence for a phase gradient perpendicular to the plane of incidence. Angle $\phi_t$ represents the
angle between the extraordinary refraction and the plane of incidence and it is proportional to the out-of-plane interfacial phase gradient \( \frac{d\phi}{dy} \). The experimental data, 3D generalized Snell’s law, and FDTD simulations are in good agreement.

**B. Broadband Metasurface Wave Plate**

Considerable attention has been drawn to the optical properties of assemblies of anisotropic metallic and dielectric structures, which can mimic the polarization-altering characteristics of naturally-occurring birefringent and chiral media. Planar chiral metasurfaces change the polarization state of transmitted light to a limited degree [87-92]. Circular polarizers based on three-dimensional chiral metamaterials primarily pass light of circular polarization of one handedness while the transmission of light of the other handedness is suppressed (circular dichroism) [93,94]. Because of the difficulty of fabricating thick chiral metamaterials, the demonstrated suppression ratio between circular polarizations of different handedness is quite small (<10). One way to overcome this difficulty is to use planar structures comprising strongly-scattering anisotropic particles that are able to abruptly change the polarization of light. V-antennas are one example [10,20-22,24,39,79,80]; other examples include arrays of identical rod or aperture metallic antennas [34-38,95,96] or meander-line structures [97-99]. Light scattered from such particles changes polarization because they have different spectral responses (in amplitude and phase) along the two principle axes [100-109].

We measured cross-polarized scattering from arrays of identical V-antennas (Fig. 2(b)) using an FTIR setup in transmission mode. The spectra for 45° incident polarization, corresponding to equal excitation of the symmetric and antisymmetric antenna modes, normalized to the light directly transmitted through the bare silicon substrate, are shown in Fig. 9(a). The corresponding FDTD simulations are shown in Fig. 9(b), which retains the same features as the experiments, though the simulated polarization conversion spectra are more clearly broken up into two resonances. In simulations we modeled the silicon substrate as infinitely thick for convenience. More elaborate simulations including Fabry-Pérot resonances in the silicon wafer produce spectra with smeared-out features.

Metasurface wave plates consisting of identical plasmonic scatterers have a number of shortcomings. First, their performance is usually degraded by the optical background originating from direct transmission due to the finite metasurface filling factor (i.e., transmitted light not scattered by plasmonic structures). Second, their spectral response is limited because of the relatively narrow plasmonic resonance. For example, once a plasmonic quarter-wave plate operates away from the optimal wavelength, the ratio of scattering amplitude \( R \) between the two eigenmodes deviates from unity and their differential phase \( \Psi \) is no longer \( \pi/2 \) (Fig. 10(a)).

![Fig. 9. Measurements (a) and FDTD simulations (b) of the cross-polarized scattering for the V-antenna arrays in Fig. 2(b). The arrows indicated the polarizations of the incident and output light. As expected, the polarization conversion peaks in the \( \lambda=3-8 \mu m \) range, in the vicinity of the two antenna eigenmodes shown in Fig. 2. The peak polarization conversion efficiency is about 15%.](image)
Fig. 10. (a) Conventional plasmonic quarter-wave plates are based on arrays of identical anisotropic plasmonic structures that support two orthogonal plasmonic eigenmodes V and H, with spectral response shown by solid and dashed curves. The devices operate as quarter-wave plates only within a narrow wavelength range (gray area) in which the two eigenmodes have approximately equal scattering amplitudes and a phase difference of $\phi = \pi/2$. (b) Amplitude and phase responses of S, A, and S – A for a representative V-antenna obtained by full-wave simulations; here S and A represent the complex scattering amplitudes of the symmetric and antisymmetric eigenmodes, respectively. The arm length of the V-antenna is 1.2 $\mu$m and the angle between the two arms is 90° (i.e., the second antenna from the left in the unit cell in Fig. 6(a)). The scattered light from the antenna can be decomposed into two components ($S + A$) and ($S – A$) according to Table I and Eq. 5. By properly designing the phase and amplitude responses of these components in the antenna arrays, we can spatially separate them so that ($S + A$) and ($S – A$) lead to, respectively, the ordinary and extraordinary beams propagating in different directions; therefore the extraordinary beam with controlled polarization is free of optical background. Because of the much broader plasmonic resonance as a result of the combined responses (i.e., $S – A$ as compared to $S$ or $A$), as shown by the solid curve in the upper panel of (b), our metasurface wave plates can provide significant scattering efficiency over a broader wavelength range. The combined plasmonic resonances can also provide a larger coverage in the phase response (i.e., ~1.5$\pi$ for $S – A$ as compared to ~0.75$\pi$ for $S$ or $A$), as is shown in the lower panel of (b).

Fig. 11. (a) Schematics showing the polarizations of the ordinary and extraordinary beams generated from the metasurface in Fig. 6(a). Incident linear polarization is along $\alpha$-direction from the y-axis. (b)-(f) Measured intensity of the ordinary and extraordinary beams, represented by black and red circles, respectively, as a function of the rotation angle of a linear polarizer in front of the detector for different incident polarizations: $\alpha = 0, 30, 45, 60,$ and 90° from (b) to (f). The free-space wavelength is 8 $\mu$m. We obtained very similar experimental results at $\lambda = 5.2$ and 9.9 $\mu$m. (g) Far-field scans of transmitted light through the metasurface in Fig. 6(a). Plane-wave
excitations at \( \lambda = 8 \) \( \mu m \) with different polarizations were used and four samples with different period \( \Gamma \) were tested.

We studied the birefringent properties of the metainterface shown in Fig. 6(a), which works as a half-wave plate. If we write the incident field as a Jones vector \( \mathbf{E}_{\text{inc}} = \begin{pmatrix} \sin \alpha \\ \cos \alpha \end{pmatrix} \), where \( \alpha \) is the orientation of the incident linear polarization (Table I), then according to Eq. (1), the extraordinary beam can be written as

\[
\mathbf{E}_{\text{ex}} = \frac{1}{2} \mathbf{S} \mathbf{A} \begin{pmatrix} -\cos(2\beta) & \sin(2\beta) \\ \sin(2\beta) & \cos(2\beta) \end{pmatrix} \mathbf{E}_{\text{inc}}
\]

(13)

The matrix on the right hand side of the equation is the Jones matrix of a half-wave plate \([110]\) and \( \beta \), which represents the orientation of the antenna symmetry axis (Table I), is the axis of the wave plate. In our design \( \beta = 45^\circ \), then according to Table I and Eq. (5), the extraordinary beam will be polarized along \( 2\beta - \alpha = 90^\circ - \alpha \) direction from the y-axis (Fig. 11(a)). This property is confirmed by state-of-polarization analyses. Specifically, we measured the intensity of the ordinary and extraordinary beams as a function of the rotation angle of a linear polarizer in front of the detector. The results are plotted in Figs. 11(b)-(f) and show the typical responses of a half-wave plate with its optical axis oriented along 45° direction. Figure 11(g) shows measured far-field scans for normal incidence at \( \lambda = 8 \) \( \mu m \) and for four samples with different \( \Gamma \) ranging from 11 to 17 \( \mu m \). We changed incident polarization directions as indicated by the arrows in the figure. In all the cases, we observe two beams in transmission and negligible optical background. The ordinary beam is located at zero degrees. The extraordinary beam bends away from the surface normal and its intensity does not change for different incident polarizations, which agrees with our theoretical analyses (Eq. (5)).

In addition to the half-wave plate, we also demonstrated a quarter-wave plate that features ultra-broadband and background free performance, and works for any orientation of the incident linear polarization. The schematic of our metasurface quarter-wave plate is shown in Fig. 12(a) and the SEM image of one sample is shown in Fig. 13(a). The unit cell consists of two sub-units, which generate two co-propagating waves with equal amplitudes, orthogonal polarizations, and a \( \pi/2 \) phase difference. The waves coherently interfere, producing a circularly polarized extraordinary beam that bends away from the propagation direction of the ordinary beam (Fig. 12(a)). The waves have equal amplitudes because the corresponding antennas in the two sub-units have the same geometries (i.e., arm length and opening angle of the \( V \)-structures). Cross polarization between the copropagating waves is achieved by choosing antenna orientations \( \beta_1 = 67.5^\circ \) and \( \beta_2 = 112.5^\circ \) so that \( (2\beta_2 - \alpha) - (2\beta_1 - \alpha) = 90^\circ \) (Fig. 13(a)). The \( \pi/2 \) phase difference between the waves is introduced by choosing the offset \( d = \Gamma/4 \), so that \( \Psi = k_0 d \sin(\theta_0) = 2nd\Gamma = \pi/2 \) (Fig. 12(a)). Note that once \( \beta_2 - \beta_1 = 45^\circ \), the two waves will always be cross-polarized, which is independent of the orientation angle \( \alpha \) of the linearly polarized incident light (Fig. 13(b)).

The above design has the major advantage of ultra-broadband performance. Away from the optimal range of operation \( \lambda = 8-10\mu m \), the phase and amplitude responses of the antenna arrays will deviate from their designed values (Fig. 4); nevertheless, the two waves scattered from the two sub-units always have the same wavefronts (Fig. 12(c)) so they always contribute equally to the extraordinary beam, resulting in a pure circular polarization state.

![Fig. 12.](image-url)
Figure 13(c) shows the phase difference $\Psi$ and amplitude ratio $R$ between the two waves scattered from the sub-units, as calculated via FDTD simulations. It is observed that $\Psi$ and $R$ are in the close vicinity of 90° and 1, respectively, over a wide wavelength range from $\lambda=5$ to 12 $\mu$m; correspondingly, a high degree of circular polarization (DOCP) close to unity is maintained over the wavelength range (Fig. 13(d)). Here DOCP is defined as $|I_{\text{RCP}}-I_{\text{LCP}}|/|I_{\text{RCP}}+I_{\text{LCP}}|$, where $I_{\text{RCP}}$ and $I_{\text{LCP}}$ stand for the intensities of the right and left circularly polarized components in the extraordinary beam, respectively [99]. Our experimental findings confirm that high-purity circular polarization can be achieved over the wavelength range from −5 to −10 $\mu$m (Fig. 13(e)). We have verified that the circular polarization of the extraordinary beam is independent of the orientation of the incident linear polarization. The extraordinary beam reaches its peak intensity at $\lambda \approx 7$ $\mu$m (Fig. 13(d)). The intensity decreases towards longer and shorter wavelengths because the S − A components of the scattered light from the antenna array have mismatched amplitudes and a nonlinear phase distribution.

We define the bandwidth $\Delta \lambda_{\text{qw}}$ of a quarter-wave plate as the wavelength range over which the DOCP is sufficiently close to 1 (e.g., >0.95) and an output with high intensity can be maintained (e.g., intensity larger than half of the peak value). According to this definition, the bandwidth of our metasurface quarter-wave plates is approximately 4 $\mu$m (DOCP > 0.97 over $\lambda=5$ to 12 $\mu$m, and intensity larger than half-maximum over $\lambda=6$ to 10 $\mu$m; see Fig. 13(d)), which is about 50% of the central operating wavelength $\lambda_{\text{central}}$. For comparison, the bandwidth of quarter-wave plates based on arrays of identical anisotropic rod or aperture antennas is typically $\Delta \lambda_{\text{qw}} = 0.05-0.1 \lambda_{\text{central}}$ [34-38,95,96].

The offset between the sub-units, $d$, controls the phase difference between the two scattered waves and therefore the polarization of the extraordinary beam. The phase difference is $\Psi = k_d \sin(\theta) = 2\pi d/d'$. Therefore, $d = 0$ or $d'/2$ leads to linear polarization, shown in Fig. 14(a); $d = d'/4$ leads to circular polarization, shown in Fig. 13; other choices lead to elliptical polarization states and those with $\Psi = \pi/4$ and $3\pi/4$ are shown in Fig. 14(b).

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In (a) and (b), the left panels are SEM images of the unit cells and the right panels are the results of the state-of-polarization analysis. The symbols are measurements and the curves are analytical calculations assuming that the two waves scattered from the sub-units have equal amplitudes and a phase difference equal to the value of $\Psi$ indicated in the figure.

C. Planar Lenses and Axicons

The fabrication of refractive lenses with aberration correction is difficult and low-weight small-volume lenses based on diffraction are highly desirable. At optical frequencies, planar focusing devices have been demonstrated using arrays of nanoholes [111], optical masks [112-114], nanoslits [115], and loop antennas [116]. In addition flat metamaterial-based lenses such as hyperlenses and superlenses have been used to demonstrate optical imaging with resolution finer than the diffraction limit [117-121]. We designed and demonstrated planar lenses and axicons based on metasurfaces at telecom wavelength $\lambda=1.55 \mu m$. Planar lenses can mold incident planar wavefronts into spherical ones and therefore eliminate spherical aberration; axicons are conical shaped lenses that can convert Gaussian beams into non-diffracting Bessel beams and can create hollow beams [2,122].

Fig. 15. Schematics showing the design of flat lenses and axicons. To focus a plane wave to a single point at distance $f$ from the metasurface, a hyperboloidal phase profile must be imparted onto the incident wavefront. (a) The phase shift at a point $P_L$ on the flat lens is designed to be proportional to the distance between $P_L$ and its corresponding point $S_L$ on the spherical surface of radius $f$ and is given by Eq. (14). (b) The axicon images a point source onto a line segment along the optical axis; the length of the segment is the depth of focus (DOF). The phase (Eq. (15)) in point $P_A$ on the flat axicon is proportional to the distance between $P_A$ and its corresponding point $S_A$ on the surface of a cone with base angle $\xi=\arctan(r/DOF)$, where $r$ is the radius of the flat axicon.

Fig. 16. (a) FDTD simulations of the phase shifts and scattering amplitudes in cross-polarization for the eight V-antennas designed to operate at $\lambda=1.55\mu m$. The parameters characterizing the elements from 1 to 4 are: $h=180nm$, 140nm, 130nm, 85nm and $\Delta=79^\circ$, 68°, 104°, 175°. Elements #5-#8 are obtained by rotating #1-#4 by an angle of 90° counter-clockwise. The antenna width is fixed at 50 nm. (b) Experimental setup: a diode laser beam at $\lambda=1.55\mu m$ is incident onto the sample with y-polarization. The light scattered...
by the metasurface in x-polarization is isolated with a polarizer. A detector mounted on a 3-axis motorized translational stage detects the light passing through a pinhole, attached to the detector, with an aperture of 50μm. The lenses and axicon also work for x-polarized illumination because of symmetry in our design: the antennas have their symmetry axis along the 45°-direction; therefore x-polarized illumination will lead to y-polarized focused light. In general, our flat optical components can focus light with any arbitrary polarization because the latter can always be decomposed into two independent components polarized in the x- and y-directions. (c) Left panel: SEM image of the fabricated lenses with 3cm focal length. Right panel: Phase profile calculated from Eq. 14 and discretized according to the phase responses of the eight antennas. Insets: zoom-ins of fabricated antennas. The antenna array has a square lattice with a lattice constant of 750nm.

The design of flat lenses is obtained by imposing a hyperboloidal phase profile on the metasurface. In this way, secondary waves emerging from the latter constructively interfere at the focal point similar to the waves that emerge from conventional lenses [4]. For a given focal length f, the phase shift \( \phi \) imposed on every point \( P(x,y) \) on the flat lens must satisfy the following equation (Fig. 15(a))

\[
\phi(x,y) = \frac{2\pi}{\lambda} P S e^{-\frac{2\pi}{\lambda} \left( \sqrt{x^2 + y^2 + f^2} - f \right)} \quad (14)
\]

For an axicon with angle \( \xi \), the phase delay has to increase linearly with the distance from the center. The phase shift \( \phi_\xi \) at every point \( P(x,y) \) has to satisfy the following equation

\[
\phi_\xi(x,y) = \frac{2\pi}{\lambda} P S A e^{-\frac{2\pi}{\lambda} \sqrt{x^2 + y^2} \sin \xi} \quad (15)
\]

The design of this new class of focusing devices is free from spherical aberration. A spherical lens introduces a variety of aberrations under the non-paraxial conduction [4]. To circumvent this problem, aspheric lenses or multi-lens designs are implemented [4, 123]. In our case, the hyperboloidal phase distribution imposed at the metasurfaces produces a wavefront that remains spherical for a large plane wave normally impinging on the metasurfaces, which leads to high numerical-aperture (NA) focusing without spherical aberration. Other monochromatic aberrations such as coma are reduced compared to spherical lenses but are not completely eliminated.

To demonstrate this new flat lens we designed eight different plasmonic V-antennas that scatter light in cross-polarization with relatively constant amplitudes and incremental phases of \( \pi/4 \) in the near-infrared (Fig. 16(a)). These antennas are used to form arrays according to the phase distributions specified in Eqs. (14) and (15) to create two flat lenses (\( r=0.45mm, f=3cm, \) corresponding to \( NA=0.015 \); \( r=0.45mm, f=6cm, \) corresponding to \( NA=0.075 \)), and an axicon (\( r=0.45mm, \xi=0.5° \)). These flat optical components are fabricated by patterning double-side-polished undoped silicon wafers with gold nano-antennas using electron beam lithography (EBL). To avoid multi-reflections in the silicon wafer, a \( \lambda/4 \) anti-reflective coating consisting of 240nm of SiO with refractive index \( \approx 1.6 \) was evaporated on the backside of the wafer that is not decorated with antennas. A schematic experimental setup is shown in Fig. 16(b).

The measured far-field for the metasurface lens with 3 cm focal distance and the corresponding analytical calculations are presented in Figs. 17(a)-(c). The results for the metasurface axicon and for an ideal axicon are presented in Figs. 17(d)-(f). We found good agreement between experiments and calculations. In the calculations, the metasurfaces are modeled as an ensemble of dipolar emitters with identical scattering amplitudes and phase distributions given by Eqs. 14 and 15. Note that the actual non-diffracting distance of the metasurface axicon is slightly shorter than the ideal DOF because the device is illuminated with a collimated Gaussian beam instead of a plane wave [124]. The efficiency in focusing light of the flat lens in Fig. 16(c) is about \( \sim 1\% \) because of the relatively large antenna spacing of 750 nm, which is limited by the fabrication time of EBL; it can be improved to \( \sim 10\% \) by using a higher antenna packing density with an antenna spacing of 220 nm according to our calculations.

The ability to design phase shifts on flat surface over a 0-to-2\( \pi \) range with a subwavelength spatial resolution is significant. For example, it is possible to produce large phase gradients, which are necessary to create high NA planar lenses. FDTD
Optical Vortex Beams Created by Metasurfaces

To demonstrate the ability of metasurfaces in molding complex optical wavefronts, we fabricated phased antenna arrays able to create optical vortex beams [10,22]. The latter are peculiar beams that have doughnut-shaped intensity profiles in the cross-section and helicoïdal wavefronts [125,126]. Unlike plane waves the Poynting vector (or the energy flow) of which is always parallel to the propagation direction of the beam, the Poynting vector of a vortex beam follows a spiral trajectory around the beam axis (Fig. 19(a)). This circulating flow of energy gives rise to an orbital angular momentum [126].

The wavefront of an optical vortex has an azimuthal phase dependence, \( \exp(i\ell \theta) \), with respect to the beam axis. The number of twists, \( \ell \), of the wavefront within a wavelength is called the topological charge of the beam and is related to the orbital angular momentum \( L \) of photons by the relationship \( L = \ell h \) [126,127], where \( h \) is Planck’s constant. Note that the polarization state of an optical vortex is independent of its topological charge. For example, a vortex beam with \( \ell = 1 \) can be linearly or circularly polarized. The wavefront of the vortex beam can be revealed by a spiral interference pattern produced by the interference between the beam and the spherical wavefront of a Gaussian beam (Fig. 19(d)). The topological charge can be identified by the number of dislocated interference fringes when the optical vortex and a plane wave intersect with a small angle (Fig. 19(e)).

Optical vortices are conventionally created using spiral phase plates [128], spatial light modulators [129], or holograms with fork-shaped patterns [130]. They can also be directly generated in lasers as intrinsic transverse modes [131]. Optical vortices are of great fundamental interest since they carry optical singularities [125,132], and can attract and annihilate each other in pairs, making them the optical analogue of superfluid vortices [133,134]. Vortex beams are also important for a number of applications, such as stimulated emission depletion microscopy [135], optical trapping and manipulation [136,137], and optical communication systems, where the quantized orbital angular momentum can increase the spectral efficiency of a communication channel [138, 139].

Figure 20 shows the experimental setup used to generate and characterize the optical vortices. It consists of a Mach-Zehnder interferometer where the optical vortices are generated in one arm and their optical wavefronts are revealed by interference with a reference beam prepared in the other arm. Continuous-wave monochromatic light at \( \lambda_0 = 7.75 \) \( \mu \)m with \(-10\) mW power emitted from a distributed feedback QCL was used as the light source and subsequently collimated and splitted into the two arms of the spectrometer by a beam.
splitter. The polarization of the beam in one arm is rotated by 90° using a set of mirrors, forming the reference beam. The beam in the other arm is focused onto a metasurface phase mask using a ZnSe lens (20-inch focal length, 1-inch diameter). The phase mask comprises a silicon-air interface decorated with a 2D array of V-shaped gold plasmonic antennas designed and arranged so that it introduces a spiral phase distribution to the scattered light cross-polarized with respect to the incident polarization (bottom inset of Fig. 20). We chose a packing density of about 1 antenna per 1.5 μm² (−λ²/40), to achieve a high scattering efficiency while avoiding strong near-field interactions. About 30% of the light power impinging on the metasurfaces is transferred to the vortex beams.

Figure 20. Experimental setup based on a Mach-Zehnder interferometer used to generate and characterize optical vortices. The bottom inset is an SEM image showing a metasurface phase plate corresponding to topological charge one. The plate comprises eight regions, each occupied by one of the eight elements in Table I. The antennas are arranged to generate a phase shift that varies azimuthally from 0 to 2π, thus producing a helicoidal scattered wavefront.

Figure 21(a) shows interferograms created by the interference between plane-wave-like reference beams and vortex beams. The dislocation at the center of the interferograms indicates a phase defect at the core of the vortex beam. The orientation and the number of the dislocated fringes of the interferograms give the sign and the topological charge l of the vortex beams.

Figure 22 shows FDTD simulations of the evolution of the cross-polarized scattered field after a Gaussian beam at λ = 7.7 μm impinges normally onto a metasurface plate with spiral phase distribution. The features of an optical vortex beam include phase singularity (i.e., a spatial location where optical phase is undefined) and zero optical intensity at the beam axis. These features are observed at a sub-wavelength distance of one micrometer (~λ/8) from the interface. The fact that a metasurface “instantaneously” molds the incident wavefront into arbitrary shapes presents an advantage over conventional optical components, such as liquid-crystal spatial light modulators, which are optically thick, and diffractive optical components, which require observers to be in the far-field zone characterized by Fraunhofer distance 2D²/λ, where D is the size of the component [4].

We conducted a quantitative analysis of the quality of the generated optical vortices in terms of the purity of their topological charge. The amplitude distribution of the optical vortex is obtained from the measured doughnut intensity distribution, and its phase profile is retrieved from the interferogram by conducting Fourier analysis [22]. The complex wavefront E_{vortex} of the vortex beam is then decomposed on a complete basis set of optical modes with angular momentum of different values, i.e. the Laguerre-Gaussian (LG) modes (E_{LG}^{l,p}) [140]. The weight of a particular LG mode in the vortex beam is given by C_{l,p}^{LG} = \int E_{vortex}^{*} E_{l,p}^{LG} dx dy, where the star denotes complex conjugate, and the integers l and p are the azimuthal and radial LG mode indices, respectively. The weight of a particular component with topological charge l, is finally obtained by
summing all the LG modes with the same azimuthal index \( l = \sum p C_{p}^{LG} \). The purity of the vortex beam created with our discretized metasurface phase mask shown in Fig. 20 is above 90% (i.e., \( C_{l} > 0.9 \)), similar to the purity of vortex beams generated with conventional spatial light modulators.

![Intensity and Phase Simulation](image)

**Fig. 22.** FDTD simulations of the cross-polarized scattered field as a function of distance from a metasurface phase plate designed to create a singly-charged optical vortex. The phase plate has a foot print of 50 x 50 \( \mu \text{m} \). The characteristic zero intensity at the center of the beam and the phase singularity develop as soon as the evanescent near-field components vanish, i.e., about 1 \( \mu \text{m} \) or one eighth of wavelength from the metasurface.

**IV. Conclusion**

This paper discusses the scattering properties of plasmonic antennas with an emphasis on their ability to change the phase and polarization of the scattered light. The former is controlled by tuning the geometries of an antenna for a fix wavelength so different phase shifts between the scattered and incident light are selected on the antenna resonance curve. The polarization of the scattered light is instead controlled by designing antenna spectral responses in two orthogonal directions so the resulting two orthogonal scattered waves can create arbitrary polarization states. The most important feature of our metasurfaces is that they comprise arrays of antennas with subwavelength spacing and with spatially tailored phase and polarization responses. Extraordinary beams with controllable propagation direction, state of polarization, and orbital angular momentum are created using such metasurfaces. We have demonstrated phased antenna arrays that beam light into arbitrary directions, flat lenses that create converging spherical waves, planar wave plates with ultra-broadband performance, and spiral phase masks that generate optical vortex beams. The subwavelength resolution of metasurfaces allows for engineering not only the optical far-field but also the near-field and meso-field.

The efficiency of the demonstrated flat optical components is limited by the antenna scattering amplitude and optical losses due to plasmonic absorption. The scattering amplitude is limited primarily because we only use part of the scattered waves (i.e., the S→A component; see Eq. 5) to synthesize the extraordinary beams, while the rest of the scattered light (i.e., the S+A component) forming the ordinary beams is wasted. This choice is made in our proof-of-principle demonstrations because it is easier to control the phase and polarization of a partial scattered wave compared to controlling those of the entire scattered wave. In our future effort to solve these problems, we will investigate: (a) antennas that allow for control of the phase of the total scattered waves; (b) antennas able to scatter a large percentage of optical power into the cross-polarization direction; and (c) dielectric scatterers with controllable phase and polarization responses and with negligible absorption losses. Furthermore, antennas with reconfigurable optical properties will be investigated, which will enable many important applications such as light detection and ranging (LIDAR) and ultra-thin planar lens for adaptive optics.

**References**


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