Conventional Hot-Wire Anemometer
Advantages

- much smaller probe size ( mm to µm )
- better spatial resolution
- array of the sensors
- higher frequency response
- lower cost
- performance/cost ↑
Fabrication Process (I)

10\(\mu\)m lightly doped epi layer

8\(\mu\)m heavily boron doped etch stop layer

Thermal oxidation of silicon substrate at 1100°C which has highly-boron doped buried etch-stop layer

Low-Pressure Chemical Vapor Deposition (LPCVD) of polysilicon at about 560°C.

Patterning of polysilicon to form the resistor (heater & sensor) as well as interconnection lanes (supports)

High-dose boron implantation of polysilicon for interconnection lanes only. Photoresist PR mask for resistor part.

Low-dose boron implantation of polysilicon including resistor part
Fabrication Process (II)

Aluminum metallization to contact interconnection lanes

LPCVD of low temperature oxide (LTO) to protect structures on frontside of the silicon substrate

Patterning of: LTO and thermal oxide on frontside, polysilicon and thermal oxide on backside of the wafer

Anisotropic etching of the silicon substrate releasing the sandwich polysilicon cantilever

Isotropic etching of silicon to remove the highly-boron doped etch-stop layer
Micro Hot Wire
Operation Principle of Micro Hot Wire

- Hot wire anemometry use the convective heat transfer from from heated sensor to the surrounding fluid to measure the velocity.

- The amount of heat transferred per unit time is inferred from an electric signal that is related to the temperature of the sensor.

- The signal results from the unique relationship between the temperature and resistance of the sensor.

\[
R(T) = R_o [1 + \alpha(T-T_o)]
\]

where \( R_o \) and \( T_o \) are the resistance and temperature at a reference condition, and \( \alpha \): Temperature coefficient of resistance (TCR)

\[
\alpha = \frac{R - R_o}{R_o (T - T_o)} = \frac{\Delta R}{R_o \Delta T}
\]

- Overheat ratio

\[
\alpha_T = \frac{T - T_o}{T_o}
\]

- Resistive overheat ratio

\[
\alpha_R = \frac{R - R_o}{R_o} = \alpha \Delta T = \alpha T_o \alpha_T
\]
**Basic heat transfer mechanisms**

- Heat is introduced into the sensing element by Joule heating and is lost by convection, conduction and radiation.

\[ q_c = hS(T_s - T_f) \]

- Convective heat flow to the fluid

\[ q_s = kA \frac{\partial T}{\partial x} \mid_x \]

- Conduction to the support

\[ Q = mcT \]

- Total electrically produced heat

\[ P = i^2 R = i^2 R_o [1 + \alpha(T - T_o)] \]

- P: electric power

\( h \): convective heat transfer coefficient

\( S \): area

\( T_s \): temperature of sensor

\( T_f \): temperature of fluid environment

\( q_r = \sigma S(T_s^4 - T_f^4) \)

- Heat flow by radiation

\( \sigma \): Stefan-Boltzmann constant (56.7 nWm\(^{-2}\)K\(^{-4}\))

\( S \): radiating area

\( T_s \): temperature of sensor

\( T_f \): temperature of fluid environment

The radiation loss is much less than the convection losses.
**The energy equation**

- The time rate change of energy within the sensing element must equal the difference between the power input ($P$) and the heat leaving the device ($F$).

\[
\frac{dQ}{dt} = P - F \quad \quad F = q_s + q_c + q_r
\]

The time rate change of energy within the sensing element \( Q = cmT_s \)

\[
cm \frac{dT_s}{dt} = P(i, T_s) - F(U, T_s)
\]

**The linearized energy equation:**

\[
T_s = T_s + \Delta T_s \quad \quad P = \bar{P} + \Delta P \quad \quad F = \bar{F} + \Delta F
\]

\[
cm \left( \frac{\partial \bar{P}}{\partial i} |_{r} \Delta i + \frac{\partial P}{\partial T_s} |_{i} \Delta T_s - \left( \frac{\partial F}{\partial U} |_{T_s} \Delta U + \frac{\partial F}{\partial T_s} |_{U} \Delta T_s \right) \right)
\]

\[
cm \frac{d\Delta T_s}{dt} + \left( \frac{\partial F}{\partial T} |_{U} - \frac{\partial P}{\partial T_s} |_{i} \right) \Delta T_s = \frac{\partial P}{\partial i} |_{r} \Delta i - \frac{\partial F}{\partial U} |_{T_s} \Delta U
\]
The Constant Current Anemometer

\[ \Delta i = \frac{i \Delta R}{(R_1 + R_2 + R)} \]

To reduce the current fluctuations to a relatively small value requires

\[ \frac{\Delta R}{(R_1 + R_2 + R)} \ll 1 \]

So \( R_1 + R_2 \) must be large. The fixed resistor \( R_2 \) should be sufficiently large to eliminate the possibility of accidentally burning out the wire when either the variable resistor \( R_1 \) or the velocity is decreased.

The current through the sensor can be determined by measuring the voltage drop across \( R_2 \).
For constant current mode of operation, $\Delta i=0$, the system is forced by the velocity fluctuations alone.

$$M \frac{d\Delta T_s}{dt} + \Delta T_s = -\frac{M}{cm} \left( \frac{\partial F}{\partial U} \right)_{r} \Delta U = f(t)$$
Time Constant

\[ M \frac{d\Delta T_s}{dt} + \Delta T_s = f(t) \quad f(t) = \frac{M}{cm} \left( \frac{\partial P}{\partial i} \bigg|_T \Delta i - \frac{\partial F}{\partial U} \bigg|_T \Delta U \right) \]

Solution:

\[ \Delta T_s = \frac{1}{M} \int_{-\infty}^{t} \exp\left(-\frac{t-\tau}{M}\right) f(\tau) d\tau \]

To determine time constant M, an instantaneous change in current or velocity

\[
\Delta T(t) = \begin{cases} 
0 & \text{for } t < 0, \\
fo(1 - e^{-tM}) & \text{for } t \geq 0,
\end{cases}
\]

\[
M = \frac{cm}{\left(\frac{\partial F}{\partial T_s} \bigg|_U - \frac{\partial P}{\partial T_s} \bigg|_i\right)}
\]
Time Constant of a Micro Hot Wire (I)

(a) Unsteady heat balance over a micro hot wire:

\[ \frac{dQ}{dt} = P - F \]

\[ Q = mcT_s = \rho hw c T_s l \]

\[ P = i^2 R = i^2 R_o [1 + \alpha(T_s - T_o)] \]

\[ F = \int_{\text{sur}} h_c (T_s - T_f) dS = 2h_c (w + h)(T_s - T_f)l \]

\[ \rho hw c l \frac{dT_s}{dt} = i^2 R_o [1 + \alpha(T_s - T_f)] - 2h_c (w + h)(T_s - T_f)l \]

\[ \rho hw c l \frac{dT_s}{dt} + [2h_c (w + h)l - i^2 R_o \alpha] T_s = f_1(t) \]

\[ \frac{dT_s}{dt} + \frac{[2h_c (w + h)l - i^2 R_o \alpha]}{\rho hw c l} T_s = \frac{dT_s}{dt} + \frac{T_s}{M} = f_1(t) \]

\[ f_1(t) = i^2 R_o (1 - \alpha T_f) + 2h_c (w + h)(T_f)l \]

\[ M = \frac{\rho hw c l}{[2h_c (w + h)l - i^2 R_o \alpha]} \]
Time Constant of a Micro Hot Wire (II)

(b) steady state heat balance over a micro hot wire:

\[ \overline{P} = \overline{F} \]

thus:

\[ i^2 R_0 [1 + \alpha(T_s - T_f)] = 2h_c(w + h)(T_s - T_f)l \]

\[ 2h_c(w + h)l = \frac{i^2 R_0 [1 + \alpha(T_s - T_f)]}{(T_s - T_f)} \]

(c) Estimation of the time constant, M:

\[ M = \frac{\rho hwcl}{[2h_c(w + h)l - i^2 R_0 \alpha]} = \frac{\rho hwcl}{[\frac{i^2 R_0 [1 + \alpha(T_s - T_f)]}{(T_s - T_f)} - i^2 R_0 \alpha]} = \frac{i^2 R_0}{(T_s - T_f)} [[1 + \alpha(T_s - T_f) - \alpha(T_s - T_f)] \]

\[ M = \frac{\rho hwcl(T_s - T_f)}{i^2 R_0} = \frac{\rho c}{R_0}(hwl) \frac{(T_s - T_f)}{i^2} \]

\[ \alpha_T = \frac{(T_s - T_f)}{T_f} \]

\[ \alpha_R = \frac{(R - R_o)}{R_o} = \alpha_T \alpha_T \]

\[ (T_s - T_f) = \alpha_T T_f = \alpha_R / \alpha \]
Time Constant of a Micro Hot Wire (III)

\[ M = \frac{\rho hwcl(T_s - T_f)}{i^2 R_0} = \left( \frac{\rho c}{R_0 \alpha} \right)(hw/l)\left( \frac{\alpha_r}{i^2} \right) \]

material  geometry  Operating  conditions

For polysilicon sensor:

\[ \rho = 2.3 \text{ gm/cm}^3 \]
\[ c = 0.7 \text{ Joule/gmK} \]
\[ R_0 = 20 \Omega \]
\[ \alpha = 1\% / K \]
\[ h = 0.5 \mu m \]
\[ w = 1 \mu m \]
\[ l = 100 \mu m \]
\[ \alpha_r = 0.5 \]
\[ i = 10 mA \]
The Constant Temperature Anemometer (CTA)

(1) The basic elements of a constant temperature anemometer circuit are a differential dc amplifier, the sensing element and a reference voltage.

(2) The sensor is placed in one leg of a Wheatstone bridge. The current through the element gives a voltage $e_1$.

(3) A reference voltage is used by the feedback amplifier to determine how much the resistance, and hence temperature, of the sensor has change. This is obtained by $e_2$.

(4) These two voltages forms the inputs to the differential amplifier. The amplifier’s output current is inversely proportional to the resistance change in the sensor.

(5) The current is feedback into the top of the bridge and restores the sensor’s resistance and temperature to their original values.
The constant temperature mode of operation of a hot-wire means that the temperature or resistance fluctuations of the sensing element are negligible compared to the current fluctuations.

\[ \frac{\Delta R}{R} \ll \frac{\Delta i_1}{i_1} \]

\[ cm \frac{dT_s}{dt} = P(i_1, T_s) - F(U, T_s) = 0 \quad \Rightarrow \quad P(i_1, T_s) = F(U, T_s) \]

The dynamic response of the constant temperature anemometer are determined by three basic equations that (1) the bridge, (2) the sensing element, and (3) the amplifier.

(1) bridge:

\[ e_0(t) = [\overline{R}(t) + R_1] i_1(t) \quad \Rightarrow \quad \overline{e}_0 + \Delta e_0 = [\overline{R} + \Delta R + R_1](\overline{i}_1 + \Delta i_1) \]

Neglect the nonlinear 2nd order terms and

\[ \overline{e}_0 = [\overline{R} + R_1] \overline{i}_1 \]

\[ \Delta e_0(t) = [\overline{R} + R_1] \Delta i_1(t) + \overline{i}_1 \Delta R(t) \]

\[ e_1 = i_1 R = e_0 R / (R + R_1) \quad \Rightarrow \quad \Delta e_1(t) = e_1(t) - e_2(t) = \frac{R_2 R(t) - R_1 R_3}{(R_2 + R_3)(R_1 + R(t))} e_0(t) \]

\[ e_2 = i_2 R_3 = e_0 R_3 / (R_2 + R_3) \]
(a) Initially two sides of the bridge have equal voltage, the differential input to the amplifier is zero, giving a zero output.

Balanced bridge: \[ RR_2 = R_1 R_3 \]

\[ e_1(t) - e_2(t) = \frac{R_2 R(t) - R_1 R_3}{(R_2 + R_3)(R_1 + R(t))} e_0(t) = 0 \]

(b) Under balanced conditions, an offset voltage is added to the amplifier. This voltage establishes a mean current through the sensor, which makes the bridge seem to have a slightly higher resistance in one leg. Thus one can consider the mean resistance in the sensor to be

\[ R(t) = \bar{R}(1 + \delta) \]

\( \delta \) is unbalanced parameter which determines the sensor operating temperature and overheat.

\[ \delta = \frac{R_2 \bar{R} - R_1 R_3}{R_2 \bar{R}} \]

\[ e_{\text{offset}} = e_1 - e_2 = \frac{\delta R_2 \bar{R}}{(R_1 + \bar{R})(R_2 + R_3)} e_0 \]
(c) For completeness, a test signal is introduced through a resistor \( R_t \). If \( R_t \) is large compared to the other resistors in the bridge, the voltage into the amplifier is given by \( g_t e_t(t) \), where \( e_t(t) \) is the test signal generator voltage, and \( g_t \) is the attenuation due to \( R_t \) and the bridge.

\[
e_t(t) = \frac{R_2 R(t) - R_1 R_3}{(R_2 + R_3)(R_1 + R(t))} e_0(t) + \frac{\delta R_2 R}{(R + R_1)(R_2 + R_3)} e_0 + g_t e_t
\]

\[
e_t(t) = \bar{e}_t + \Delta e_t(t)
\]

\[
\bar{e}_t + \Delta e_t(t) = \frac{R_2 (\bar{R} + \Delta R(t)) - R_1 R_3}{R_2 + R_3}[\bar{e}_0 + \Delta e_0(t)] + \frac{\delta R_2 R}{R + R_1}(\bar{R} + \Delta R(t))(R_2 + R_3) e_0 + \Delta e_0(t) + g_t \bar{e}_t + \Delta e_t(t)
\]

Neglect the nonlinear 2\(^{nd}\) order terms and sub

\[
\bar{e}_t = \frac{R_2 \bar{R} - R_1 R_3}{R_2 + R_3}[\bar{e}_0 + \Delta e_0(t)] + \frac{\delta R_2 R}{R + R_1}(R_2 + R_3) e_0 + g_t \bar{e}_t
\]

\[
\Delta e_t(t) = \frac{e_0 R_2 \Delta R(t)}{R_2 + R_3}(R_1 + \bar{R}) + \frac{\delta R_2 R}{R + R_1}(R_2 + R_3) \Delta e_0(t) + g_t \Delta e_t(t)
\]

Under balanced conditions:

\[
\frac{R_2}{R_2 + R_3} = \frac{R_1}{R + R_1}
\]

\[
\Delta e_t(t) = \frac{e_0 R_1 \Delta R(t)}{(R_1 + \bar{R})^2} + \frac{\delta R_1 R}{(R + R_1)^2} \Delta e_0(t) + g_t \Delta e_t(t)
\]
(2) the sensing element:

\[
cm \frac{d\Delta T_s}{dt} + \left( \frac{\partial F}{\partial T_s} \bigg|_U - \frac{\partial P}{\partial T_s} \bigg|_i \right) \Delta T_s = \frac{\partial P}{\partial i} \bigg|_r \Delta i - \frac{\partial F}{\partial U} \bigg|_r \Delta U
\]

\[
P = i^2 R = i^2 R_0 \left[1 + \alpha(T_s - T_0) \right]
\]

\[
\frac{\partial P}{\partial T_s} \bigg|_i = i^2 R_0 \alpha
\]

\[
\frac{\partial F}{\partial T_s} \bigg|_U = 0
\]

\[
\frac{\partial P}{\partial i} \bigg|_r = 2iR
\]

\[
\Delta T_s = \Delta R / \left( \alpha R_0 \right)
\]

\[
\frac{cm}{(i^2 R_0)} \frac{d\Delta R}{dt} + \Delta R = \frac{2\alpha R}{i} \Delta i - \alpha \frac{\partial F}{\partial U} \bigg|_r \Delta U
\]

\[
M \frac{d\Delta R}{dt} + \Delta R = a \Delta i + b \Delta U
\]

\[
M = \frac{cm \alpha R}{i^2 R_0 \alpha}
\]

\[
a = \frac{2\alpha R}{i}
\]

\[
b = -\alpha \frac{\partial F}{\partial U} \bigg|_r
\]
The dynamic response of the constant temperature anemometer

\[ \Delta e_0(t) = [\bar{R} + R_1] \Delta i_1(t) + \bar{i}_1 \Delta R(t) \]  \hspace{1cm} (1)

\[ \Delta e_i(t) = \frac{e_0 R_i \Delta R(t)}{(R_i + \bar{R})^2} + \frac{\delta R_i \bar{R}}{(R + R_1)^2} \Delta e_0(t) + g_i \Delta e_i(t) \]  \hspace{1cm} (2)

\[ M \frac{d\Delta R}{dt} + \Delta R = a \Delta i_1(t) + b \Delta U \]  \hspace{1cm} (3)

\[ \mu \frac{d\Delta e_0}{dt} + \Delta e_0 = -A_0 \Delta e_i \]  \hspace{1cm} (4)

Combine (1) and (3) to eliminate \( \Delta i_1(t) \), (2) to eliminate \( \Delta R(t) \) and (4) becomes

\[ \frac{d^2 \Delta e_0}{dt^2} + 2\zeta \omega_0 \frac{d\Delta e_0}{dt} + \omega_0^2 \Delta e_0 = \omega_0^2 \{S_U \Delta U(t) + S_T \left[ M \frac{d\Delta e_i}{dt} + (1 + \frac{2R\alpha_R}{R+R_1}) \Delta e_i \right] \} \]
\[
\frac{d^2 \Delta e_0}{dt^2} + 2 \zeta \omega_0 \frac{d \Delta e_0}{dt} + \omega_0^2 \Delta e_0 = \omega_0^2 \{ S_U \Delta U(t) + S_f \left[ M \frac{d \Delta e_f}{dt} + (1 + \frac{2R \alpha_R}{R+R_1}) \Delta e_r \right] \}
\]

\( \omega_0 = (2 \alpha_R K_0 / \mu M)^{1/2} \): natural frequency

\( \zeta = \frac{1}{2} (1 + K_0 \delta)(M / 2 K_0 \mu \alpha_R)^{1/2} \): damping coefficient

\( S_U = \frac{\overline{R} + R_1}{2 \overline{R} \alpha_R} \frac{\partial F}{\partial U} \bigg|_T \): sensitivity to the velocity

\( S_f = \frac{(\overline{R} + R_1)^2}{2 \overline{R} \alpha_R^2} g_f \): sensitivity to the test signal

\( K_0 = \frac{A_0 \overline{R} R_1}{(\overline{R} + R_1)^2} \): system gain parameter
For the following parameters:

\[ M = 2 \times 10^{-6} \text{ sec} : \text{micro hot wire time constant} \]

\[ \mu = 10^{-5} \text{ sec} : \text{amplifier time constant} \]

\[ \alpha_R = 0.5 \]

\[ A_0 = 10^3 : \text{amplifier gain} \]

\[ K_0 = \frac{A_0 \bar{R}R_1}{(R + R_1)^2} = 10^3 : \text{system gain parameter} \]

\[ \frac{\bar{R}R_1}{(R + R_1)^2} = 1 \]

\[ \omega_0 = \frac{2 \cdot 0.5 \cdot 10^3}{(10^{-5} \cdot 2 \times 10^{-6})^{1/2}} = 2.2 \times 10^8 \text{ sec}^{-1} \]

The characteristic frequency is:

\[ f_0 = \frac{\omega_0}{2\pi} = 35 \text{ MHz} \]
Frequency Resolution

First order system:

\[ M \frac{d\Delta T_s}{dt} + \Delta T_s = f(t) \]

M – time constant (thermal lag)

f – forcing function

e.g. constant current hot-wire

Second order system:

\[ \frac{1}{\omega_0^2} \frac{d^2 \Delta e_0}{dt^2} + \frac{2\zeta}{\omega_0} \frac{d\Delta e_0}{dt} + \Delta e_0 = Sf \]

\[ \omega_0 \] – resonance frequency

\[ \zeta \] – damping coefficient

S - sensitivity

f – forcing function

e.g. constant temperature hot-wire