Raising rival’s costs in the securities settlement industry*

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Abstract

The competition between a central securities depository (CSD) and a custodian bank is analysed in a Stackelberg model. The CSD sets its prices first, the custodian bank follows. There are many investor banks each of which has to decide whether to use the service of the CSD or of the custodian bank. This decision depends on the prices and the investor bank’s preferences for the inhomogeneous services of the two service providers. Since the custodian bank uses services provided by the CSD as input, the CSD can raise its rival’s costs. However, due to network externalities, the CSD’s equilibrium market share is not necessarily higher than socially optimal. This result has important policy implications that are related to a discussion currently taking place in the securities settlement industry.

Keywords: Securities settlement, network competition, raising rival’s cost

1 Introduction

After two parties have concluded a securities transaction at a financial market, the securities have to be transferred from the seller to the buyer and the related payment has to be transferred from the buyer to the seller. The actual transfer of the securities and the payment is called settlement. Settlement is obviously an essential part of any securities transaction.

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Today, a physical transfer of securities certificates or cash between the seller and the buyer hardly takes place anymore. Instead, assets are transferred electronically by account entries. Both the seller and the buyer need to have a cash account and a securities account with some institutions. In the easiest case, both parties have a cash account with the same institution and also a securities account with the same institution. In this case, the securities account of the seller and the cash account of the buyer are debited and the securities account of the buyer and the cash account of the seller are credited. No other accounts are involved in the settlement. If instead the buyer has his securities account with institution B and the seller has his securities accounts with institution S, settlement of the securities takes place across different settlement service providers.

The simplest sub-case is when B has a securities account with S. If this is the case, then the securities account of the seller with S has to be debited and the securities account of B with S and of the buyer with B have to be credited. I.e. three securities accounts are involved. Matters are getting more complex when B has no securities account with S and vice versa. Then both have to have accounts with intermediating settlement service providers. Similar problems arise if the seller and the buyer do not have cash accounts with the same institution.

Most industrialized countries have established a so called central securities depository (CSD). If an entity wants to issue securities in a certain country, it usually deposits the entire issue with the national CSD, i.e. the CSD is a central storehouse for securities. Investors who want to purchase the security at the primary or later at the secondary market need to have a securities account either with the CSD or with an institution that has a securities account with the CSD or with an institution that has a securities account with an institution that has a securities account with the CSD and so forth. By construction, the number of units of a security deposited in a CSD has to be equal to the number of units of this security on accounts with the CSD.

An institution that holds securities on securities accounts with other institutions mainly on behalf of (small or institutional) investors or on its own behalf is called an investor bank throughout this paper. Institutions that hold securities on accounts with other institutions mainly on behalf of investor banks are called custodian banks. As indicated above, if for example an investor bank that has a securities account with the CSD sells securities to an investor bank that has an account with a custodian bank and if this custodian bank has an account with the CSD, the seller’s account with the CSD has to be debited and the custodian bank’s account with the CSD and the investor bank’s account with the custodian bank have to be credited. I.e. the CSD is not only the central depository, but also the institution in which many trades in securities issued into the CSD are settled.

In the past ten to fifteen years, custodian banking has become increasingly important. Technological progress in information technology and an increasing
globalization contributed to a rapidly growing complexity of financial markets. Many especially smaller investor banks responded to these developments by outsourcing activities to other banks. Some big banks became custodian banks specialized in the securities custody and settlement business. Because of their existence, investor banks have the choice either to hold a securities account directly with the CSD into which a security it wants to trade has been issued or with a custodian bank. I.e. the CSD and the custodian bank compete with each other for investor banks.

When elaborating on the competition between CSDs and custodian banks, a special feature of this competition has to be noted: in order to settle securities stored in a CSD, custodian banks use services provided by the CSD as input, but not vice versa. If both the seller and the buyer in a securities transaction have an account with the custodian bank, the custodian bank can settle the transaction internally as described above without routing it to the CSD. However, if an investor bank that has a securities account with a custodian bank trades with another investor bank that has an account with the CSD, then internalizations of settlement within the custodian bank is not possible. The transaction is settled through the custodian bank’s account with the CSD as described above and the custodian bank has to pay a price to the CSD for having its securities account with the CSD credited or debited. I.e. the CSD can raise the costs of the custodian bank by increasing the price the custodian bank has to pay to the CSD.

In this paper, we present a simple model describing the competition between a CSD and a custodian bank. We assume that there are only one CSD, one custodian bank and a continuum of investor banks. The CSD moves first. It sets a price $q_C$ for opening a securities account with the CSD and another price $p_C$ for debiting or crediting this account, i.e. for the settlement of a securities transaction on this account. Note that we assume that the CSD is not able to price discriminate directly by setting one settlement prices $p_{C,1}$ to be paid by investor banks and another settlement price $p_{C,2}$ to be paid by the custodian bank. This assumption can be justified because in reality competition authorities would probably not allow this kind of direct price discrimination. The custodian bank moves second. It also sets a price $q_A$ for opening and another price $p_A$ for debiting or crediting a securities account with the custodian bank taking into account the prices set by the CSD. Next, each investor bank opens an account with either the CSD or the custodian bank. Finally, banks are randomly matched to trade with each other and the transactions are settled through account with the CSD and the custodian bank.

We show that the CSD can raise the costs of the custodian bank in a very subtle way. Compare the custodian bank with an investor bank that has opened an account with the CSD. Since both have only one securities account with the CSD, both have to pay to the CSD the price $q_C$. But the number of transactions settled on an investor bank’s account with the CSD is low compared to the number of transactions settled on a custodian bank’s account with the CSD unless the custodian bank can by chance internalize the settlement of almost all transactions of its customers. Thus, the relative relevance of the price $p_C$
compared to the price $q_C$ is higher for the custodian bank than for the investor bank. If the CSD raises $p_C$ and simultaneously reduces $q_C$, the overall costs for investor banks for using the service of the CSD may remain unchanged. However, the overall costs for the custodian bank for using the services of the CSD rise. Thus, with this strategy, the CSD can raise its rivals costs without losing investor banks as customers. To the contrary, the custodian bank has to raise its own prices to cover the additional costs it has to pay to the CSD and thus loses investor banks as customers to the CSD.

As can be expected from the reasoning above, we show that in equilibrium the market share of the CSD is higher than the market share of the custodian bank though we assume that the CSD and the custodian bank face the same exogeneous cost and demand parameters (symmetry). Most importantly, we compare the equilibrium market shares with the socially optimal market shares. We show that the CSD’s market share is not always higher than socially optimal. Depending on the parameter constellation, it can be higher than, equal to or lower than the social optimum. I.e. it is often socially desirable to have relatively many investor bank that go to the CSD. The reason is the presence of network externalities. The CSD and the custodian bank are two different settlement networks. Settling transactions across the two networks is socially expensive. It is better to pool to a certain extent many investor banks in one of the two networks, for example in the CSD.

This result has important policy implications that are related to a discussion currently taking place in the securities settlement industry. CSDs argue that the competition between CSDs and custodian banks is distorted in favour of custodian banks since CSDs are regulated by public authorities that aim to reduce the risks of financial instability while the settlement business of custodian banks is not regulated in a similar way. Custodian banks argue that there is also a distortion in favour of CSDs since CSDs have a monopoly as central depository that enables them to raise the costs of custodian banks as described above. They say regulators should therefore treat CSDs in a way that offsets the distortion in favour of CSDs stemming from the CSDs’ monopoly power. However, this reasoning is obviously not supported by our model. In our model, it is true that the CSD can exploit its monopoly position as a central depository by raising the custodian bank’s costs to gain a higher market share. But the equilibrium market share of the CSD without regulatory intervention is not necessarily higher than socially optimal. Thus, our model provides no reason for regulatory interventions favouring custodian banks as long as CSDs are not allowed to price discriminate between custodian banks and investor banks.

Our model is obviously closely related to the theoretical literature on net-

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4 The discussion is closely related to the more prominent discussion between stock exchanges and other financial market participants on the internalizations of securities buy and sell orders by custodian banks. See Euronext (2002) and APCIMS-EASD et al. (2002).
5 See ECSDA (2002). Custodian banks are regulated as banks, but not as settlement service providers.
work industries. This literature can be separated into two branches. The first
branch analyses industries like the electricity or gas industry with only one net-
work provider (the owner of the cable or pipeline network). Firms specialized in
supplying consumers with the commodity conveyed through the network (elec-
tricity or gas) need to buy access to the network from the network provider. The
network provider itself may also offer the commodity to consumers. In this
case, it is both an input supplier for and a direct competitor of the other firms at
the market. In this respect, the competition between the network provider and
other suppliers of the commodity resembles the competition between a CSD and
custodian banks. However, in the gas industry for example, a supplier of gas
has to use more services of the provider of the network the more consumers
the supplier can attract. But the more investor banks a custodian bank can
attract, the more likely is the case that the buyer and the seller of a securities
transaction both have an account with this custodian bank so that the custodian
bank can internalize the settlement without routing it through its account with
the CSD. In the extreme case that all buyers and sellers of securities have an
account with the same custodian bank, the custodian bank can internalize the
settlement of all transactions and no settlement is routed through its account
with the CSD.

The second branch of literature on network industries looks at competition
between networks. The most prominent example is the competition between
two mobile phone networks. Consumers have the choice between both networks.
If a customer of one network wants to call a customer of another network, the
call has to be routed through a link between the two networks. This situation
obviously resembles the competition between a CSD and a custodian bank.
However, while the competition between mobile phone networks is symmetric,
the competition between the CSD and the custodian bank is asymmetric insofar
as only the custodian bank has an account with the CSD, not vice versa.

The competition between a CSD and a custodian bank in the securities set-
tlement industry therefore deserves a special analysis. However, there is hardly
any theoretical literature on the securities settlement industry. An exception is
Kauko (2002). This paper concentrates on the competition between two CSDs
and appears to be more in line with the above described second branch of lit-
erature on network industries. Other academic papers on securities settlement
are empirical and deal mainly with the costs of domestic versus cross-border
settlement, for example Lamno and Levin (2001) and Schmiedel, Malkamaeki
and Tarkka (2002).

The paper is organized as follows: The assumptions of the model are de-
scribed in section 2. In section 3, we derive the social welfare optimum. The
payoff functions of the players are described in section 4. In the sections 5, 6
and 7, we analyse the equilibrium behaviour of the players. Section 8 is devoted
to an alternative version of our model in which the CSD and the custodian bank
set their prices simultaneously. We show that the simultaneous move game has

\footnote{This literature focuses mainly on access price regulation. See for example Armstrong,

\footnote{See Laffont, Rey and Tirole (1996a and 1996b).}
no equilibrium in pure strategies. This result can be seen as a good justification of our assumption that the CSD moves first.

2 The model

In this section, we describe the assumptions of our model. We assume that there is one CSD, one custodian bank $A$ and a continuum $[0,1]$ of investor banks. Investor bank can maintain a securities account either with the CSD, or with the custodian bank, which in turn must have an account with the CSD. Decisions are taken in three stages.

1) The CSD moves first. It sets a price $q_C$, a bank has to pay if it wants to open a securities account with the CSD and a price $p_C$, a bank has to pay when a securities transaction is settled on its account with the CSD. Note that we do not allow for direct price discrimination, i.e. the CSD is not allowed to charge investor banks a settlement price $p_{C,1}$ and to charge the custodian bank another settlement price $p_{C,2} \neq p_{C,1}$. In reality, competition authorities would most likely not allow price discrimination of this kind. The CSD’s marginal costs of maintaining an account for a bank are $c_a$. When securities are transferred from one account with the CSD to another account with the CSD, the CSD has marginal costs of $2c_a$ (one account has to be debited, the other has to be credited).

2) Custodian bank $A$ moves second. It sets a price $q_A$ an investor bank has to pay if it wants to open a securities account with the custodian bank and a price $p_A$ an investor bank has to pay when a securities transaction is settled on its account with the custodian bank. The custodian bank’s marginal costs of maintaining an account for a bank are $\bar{c}_a$. When securities are transferred from one account with the custodian bank to another account with the custodian bank, the custodian bank has costs of $2\bar{c}_a$ (again, one account has to be debited, the other has to be credited). In most parts of this paper, we will assume $\bar{c}_a = c_a$.
and \( \hat{c}_s = c_s \).

(3) The investor banks move next. Each of these banks has to open a securities account with either the CSD or the custodian bank. Besides the differences in prices, the quality of the services of the CSD on the one hand and the custodian bank on the other hand matters. This is reflected in our model by the assumption that bank \( i \in [0,1] \) has additional costs \( c \left( \frac{1}{2} - i \right) \), \( c > 0 \), when choosing to open an account with the custodian bank. Thus, if the prices of the CSD and the custodian bank were equal \( (q_C = q_A \text{ and } p_C = p_A) \), then all banks in \( [0, \frac{1}{2}] \) would go to the custodian bank and all banks in \( [\frac{1}{2}, 1] \) would go to the CSD. This assumption basically implies that on average the quality of the services provided by the CSD and by the custodian bank is equally high.

After the investor banks have made their decision, a security is issued into the CSD and a number \( \frac{1}{2} \) is drawn randomly. This \( \frac{1}{2} \) is the proportion of investor banks with an account with the CSD that receive one unit of the security at the primary market. On top of this, \( \frac{1}{2} - \alpha \) banks with an account with the custodian bank also receive one unit of the security at the primary market. Thus, one half of all investor banks is able to buy a unit of the security at the primary market.

Denote by \( k \) the proportion of investor banks with an account with the CSD, i.e. \( 1 - k \) is the proportion of investor banks with an account with the custodian bank. Notice that our model setup implies several limitations on possible realizations on the parameter \( \alpha \): \( \alpha \) can neither exceed \( k \) nor \( \frac{1}{2} \). Similarly, \( \frac{1}{2} - \alpha \) cannot exceed \( 1 - k \). Therefore, we need to make restrictions on the distribution of \( \alpha \) and assume that \( \alpha \) is uniformly distributed over the interval

\[ \left[ \max\{0, k - \frac{1}{2}\}; \min\{\frac{1}{2}, k\} \right]. \]

Finally, we assume that the settlement costs of transactions at the primary market can be neglected.

Each investor bank that bought the security at the primary market has to sell its unit and each other investor bank has to buy one unit of the security at the secondary market. We assume that those banks with an account with the custodian bank simply give their sell or buy order to the custodian bank. The custodian bank acts as a broker and executes the orders of its customer banks. It of course internalizes as many trades as possible. Whether there is a net transfer of securites from the CSD to the custodian bank or vice versa, depends on the realization of \( \alpha \): If \( \alpha \leq \frac{1}{2}k \), that is, less than half of the banks with an account with the CSD wish to sell the security, then \( \alpha \) banks with an account with the CSD sell to \( \alpha \) banks with an account with the CSD, the other \( k - 2\alpha \) banks with an account with the CSD buy from \( k - 2\alpha \) banks with an account with the custodian bank. Finally \( \frac{1}{2}(1 - k - (k - 2\alpha)) = \frac{1}{2} - k + \alpha \) banks with an account with the custodian bank sell to \( \frac{1}{2} - k + \alpha \) banks with an account with the custodian bank. If instead \( \alpha > \frac{1}{2}k \), then \( k - \alpha \) banks with an account in the CSD buy from \( k - \alpha \) banks with an account with the CSD, the other \( k - 2(k - \alpha) = 2\alpha - k \) banks with an account with the CSD sell to \( 2\alpha - k \) banks with an account with the custodian bank. Finally, \( \frac{1}{2}(1 - k - (2\alpha - k)) = \frac{1}{2} - \alpha \).
banks with an account with the custodian bank buy from $\frac{1}{2} - \alpha$ banks with an account with the custodian bank.

It is essential to clearly understand how trades are settled. Take for example the case that an investor bank with an account with the CSD sells to another investor bank that also has an account with the CSD. In this case, the account of the former is debited and the account of the latter is credited. The CSD incurs marginal costs of $2c_s$ and both banks pay $p_C$ as price for the settlement to the CSD. If instead a bank with an account with the CSD sells to another bank with an account with the custodian bank, the transaction has to be settled across the two settlement service providers. The account of the seller with the CSD is debited and the (so called omnibus) account of the custodian bank with the CSD and the account of the buyer with the custodian bank are both credited. Here, the CSD faces marginal costs $2c_s$ and the custodian bank $\bar{c}_a$. The seller and the custodian bank both pay $p_C$ to the CSD and the buyer pays $p_A$ to the custodian bank.

3 Social welfare

In this section, we derive the social welfare optimum, i.e. the socially optimal allocation of investor banks to the two settlement service providers. The social costs of settlement for given $\alpha$ and $k$ are

$$S = \begin{cases} 
  k(c_a + c_s) + (1 - k)(\bar{c}_a + \bar{c}_s) + (k - 2\alpha)c_s + \frac{1}{2}k^2c_s(\frac{1}{2} - i)di, & \text{if } \alpha \leq \frac{1}{2}k \\
  k(c_a + c_s) + (1 - k)(\bar{c}_a + \bar{c}_s) + (2\alpha - k)c_s + k^2c_s(\frac{1}{2} - i)di, & \text{if } \alpha \geq \frac{1}{2}k 
\end{cases}$$

Each investor bank has to open one account, either with the CSD or with the custodian. This implies social costs incurred at the CSD of $kc_a$ and social costs incurred at the custodian bank of $(1 - k)c_a$. Also, because investor banks either send or receive exactly once the security, their accounts have to be either debited once or credited once. This implies social costs incurred at the CSD of $kc_a$ and social costs incurred at the custodian bank of $(1 - k)c_a$. Furthermore, if $\alpha \leq \frac{1}{2}k$, a proportion of $(k - 2\alpha)$ transactions has to be settled through the custodian bank’s account with the CSD, i.e. this account has to be credited $(k - 2\alpha)$ times. The social costs for this settlement across the two settlement service providers are $(k - 2\alpha)c_s$ (incurred at the CSD). Analogously, if $\alpha \geq \frac{1}{2}k$, a proportion of $(2\alpha - k)$ transactions has to be settled on the custodian bank’s account with a CSD, i.e. this account has to be debited $(2\alpha - k)$ times. The social costs are $(2\alpha - k)c_s$ (again incurred at the CSD). Finally, the social costs incurred at the investor banks (those that go to the custodian bank) are $\int_k^1 c(\frac{1}{2} - i)di$.

Taking expectations with respect to $\alpha$ easily leads to

$$E(S) = \begin{cases} 
  k(c_a + c_s) + (1 - k)(\bar{c}_a + \bar{c}_s) + \frac{1}{2}kc_s(1 - k), & \text{if } k \leq \frac{1}{2} \\
  k(c_a + c_s) + (1 - k)(\bar{c}_a + \bar{c}_s) + \frac{1}{2}(1 - k)c_s - \frac{1}{2}k(1 - k), & \text{if } k \geq \frac{1}{2} 
\end{cases}$$

Here, the third term $(\frac{1}{2}kc_s$ resp. $\frac{1}{2}(1 - k)c_s$) represents the expected social costs of settling transactions across the two service providers, i.e. on the account of

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When \( c_s \) is low relative to \( c \), investor bank’s preferences are crucial for the allocation of banks across institutions, and \( k_{opt} = \frac{1}{2} \). For high \( c_s \), the cost of settling across institutions, the investor banks’ preferences become less important relative to the costs of settling across institutions and \( k_{opt} \) moves away from \( \frac{1}{2} \).

the custodian bank with the CSD. This term is decreasing as \( k \) moves away from its midpoint \( \frac{1}{2} \). The reason is that settlement across institutions is costly. If many investor banks are concentrated in either the CSD or in the custodian bank (in other words, \( k \) is close to either 1 or 0), then most trades can be settled internally and these costs are avoided. In a sense, the term can be interpreted as network externalities that arise from concentrating accounts in one institution.

To derive the socially optimal \( k \), \( E(S) \) has to be minimised with respect to \( k \) and subject to \( 0 \leq k \leq 1 \). In the appendix, we prove

**Proposition 1** To avoid corner solutions, assume \( c_o + \frac{3}{2} c_s - \frac{3}{2} c \leq \bar{c}_o + \bar{c}_s \leq c_o + c_s \) or \( c_o + c_s \leq \bar{c}_o + \bar{c}_s \leq c_o + \frac{1}{2} c_s + \frac{1}{2} c \). The social optimum is then given by \( k = k_{opt} \) with

\[
k_{opt} = \begin{cases} 
\frac{1}{2} c + \bar{c}_o + \bar{c}_s - c_o - \frac{3}{2} c_s & \text{if } \bar{c}_o + \bar{c}_s < c_o + c_s \\
\frac{1}{2} c - c_o - \frac{3}{2} c_s & \text{if } \bar{c}_o + \bar{c}_s = c_o + c_s \\
\frac{1}{2} c + \bar{c}_o + \bar{c}_s - c_o - \frac{1}{2} c_s & \text{if } \bar{c}_o + \bar{c}_s > c_o + c_s 
\end{cases}
\]

From now on, we focus on the case \( \bar{c}_o + \bar{c}_s = c_o + c_s \) to reduce the mathematical complexity of the analysis. Note that this is the only parameter constellation with two socially optimal allocations according to proposition 1, namely \( k_{opt}^1 = \frac{1}{2} c - c_o < 1/2 \) and \( k_{opt}^2 = \frac{1}{2} c + c_s > 1/2 \). Both have the same distance from \( k = 1/2 \) and are in this sense symmetric (\( k_{opt}^1 = 1 - k_{opt}^2 \)). Only for \( c_s = 0 \), we
have \(k^1_{opt} = k^2_{opt} = \frac{1}{2}\). The reason is the following: If \(c_s = 0\), the settlement of transactions across the two service providers causes no social costs and does not need to be avoided. Since \(\bar{c}_a + \bar{c}_s = c_a + c_s\), the only relevant cost parameter is \(c\), which determines the investor banks’ preferences for settlement. Accordingly, it is socially optimal that any investor bank \(i \in [0, \frac{1}{2}]\) goes to the custodian bank and any investor bank \(i \in [\frac{1}{2}, 1]\) goes to the CSD. If now \(c_s\) increases, it is more and more important to avoid settlement across the two service providers, i.e. to pool many investor banks in one service provider. By doing so, network externalities can be exploited. If \(c_s\) is sufficiently high and \(c\) sufficiently low, it is optimal that all investor banks have accounts with the same institution (\(k = 0\) or \(k = 1\)) so that settlement across service providers does not take place at all. Figure 2 displays expected social costs for different realizations of \(c_s\).

To avoid corner solutions with \(k^1_{opt} = 0\) and \(k^2_{opt} = 1\), we assume for the rest of the paper \(c \geq c_s\).

4 The payoff functions

We now present the payoff functions of the players for given \(k\) and prices \(p_A\), \(q_A\), \(p_C\) and \(q_C\). The payoff function of some investor bank \(i \in [0, 1]\) is simply

\[
\pi_i = \begin{cases} 
-q_A + p_A + c(\frac{1}{2} - i) & \text{if } i \text{ is customer of the custodian bank} \\
-q_C + p_C & \text{if } i \text{ is customer of the CSD}
\end{cases}
\]

Now consider the custodian bank. For each investor bank that maintains an account with it and uses it to make one transaction, the custodian bank receives \(q_A + p_A\), but incurs settlement costs of \(c_a + c_s\). Moreover, for those \(k - 2\alpha\) (resp. \(2\alpha - k\)) transactions where the receiver (the sender) is customer of the CSD, the custodian bank needs to pay a fee \(p_C\) to the CSD for settlement. All other trades are settled internally. The profit of the custodian bank is thus given by

\[
\pi_A = \begin{cases} 
(1-k)(q_A - c_a + p_A - c_s) - (k - 2\alpha)p_C & \text{if } \alpha < \frac{1}{2}k \\
(1-k)(q_A - c_a + p_A - c_s) - (2\alpha - k)p_C & \text{if } \alpha > \frac{1}{2}k
\end{cases}
\]

Taking expectations with respect to \(\alpha\), we obtain the custodian bank’s payoff function

\[
E[\pi_A] = \begin{cases} 
(1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}k p_C & \text{if } k \leq \frac{1}{2} \\
(1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}(1-k)p_C & \text{if } k \geq \frac{1}{2}
\end{cases}
\]  

(1)

Again, \(\frac{1}{2}k\) is the expected number of trades to be settled through the custodian bank’s account with the CSD if \(k \leq \frac{1}{2}\) (and \(\frac{1}{2}(1-k)\) if \(k \geq \frac{1}{2}\)). Obviously, it peaks at \(k = 1/2\), that is, when just one half of all investor banks have an account with either institution. In the other extreme, for \(k = 0\), all trades can be settled internally on accounts with the custodian bank. Similarly, if \(k = 1\), all banks maintain an account with the CSD, so the custodian bank settles no trades.
Finally, consider the CSD. Similar considerations as above lead to the CSD’s profit
\[
\pi_C = \begin{cases} 
k(q_C - c_a + p_C - c_a) + (k - 2\alpha)(p_C - c_a), & \text{if } \alpha < \frac{1}{2}k \\
 k(q_C - c_a + p_C - c_a) + (2\alpha - k)(p_C - c_a), & \text{if } \alpha > \frac{1}{2}k 
\end{cases}
\]
Taking expectations with respect to \(\alpha\) gives the CSD’s payoff function
\[
E[\pi_C] = \begin{cases} 
\frac{1}{2}k(q_C - c_a + p_C - c_a) + \frac{1}{2}k(p_C - c_a), & \text{if } k \leq \frac{1}{2} \\
 k(q_C - c_a + p_C - c_a) + \frac{1}{2}(1 - k)(p_C - c_a), & \text{if } k \geq \frac{1}{2} 
\end{cases}
\] (2)
Before proceeding with solving the game, we would like to point out one feature: Suppose that \(k > \frac{1}{2}\), so most investor banks settle directly with the CSD, and \(p_C\) is high. In this case, note that a decrease in \(k\), i.e. an increase in the number of customers of the custodian bank, could reduce the custodian banks expected profit \(E[\pi_A]\) even if \(q_A \geq c_a\) and \(p_A \geq c_a\). The economic reason is simple: If \(k > \frac{1}{2}\), then a decrease in \(k\) increases the expected number of settlements that have to be routed through the custodian bank’s account with the CSD. If the price \(p_C\) the custodian bank has to pay for such settlements is high enough, a higher number of customers of the custodian bank could mean a lower expected profit for the custodian bank. If at the same time \(p_C > c_a\) and \(q_C > c_a\), then the CSD’s expected profit \(E[\pi_C]\) increases for similar reasons.

5 The decision of the investor banks
We solve the model backwards and start with stage (3). Here, each bank \(i \in [0, 1]\) has to decide with which service provider it wants to have an account given the prices \(q_C, p_C, q_A, p_A\). Bank \(i\) is indifferent if and only if
\[
q_C + p_C = c\left(\frac{1}{2} - i\right) + q_A + p_A
\]
\[
i = \frac{q_A + p_A - q_C - p_C}{c}
\]
The proportion of banks with an account with the CSD is thus
\[
k = \frac{q_A + p_A - q_C - p_C}{c}
\] (3)
Here and throughout the paper, we assume that corner solutions with \(k = 0\) or \(k = 1\) will not occur in equilibrium.
If \(q_A + p_A = q_C + p_C\), we of course have \(k = \frac{1}{2}\). If the CSD and the custodian bank charge different prices, i.e. \(q_A + p_A > (or <) q_C + p_C\), we obviously have \(k > (or <) \frac{1}{2}\) so that more investor banks settle at the institution with lower prices. Finally, notice the impact of the investors’ cost \(c\) of having to settle at a different institution than the preferred one: If \(c\) increases, then \(k\) moves closer to \(\frac{1}{2}\) since it gets more expensive for banks in \([0, \frac{1}{2}]\) and less expensive for banks in \([\frac{1}{2}, 1]\) to use the custodian bank. Thus, \(k\) is decreasing (increasing) in \(c\) if \(q_A + p_A > (or <) q_C + p_C\).
6 The decision of the custodian bank

The payoff function of the custodian bank is obviously a function of \( q_A + p_A \). Thus, to derive the best response correspondence of the custodian bank on given prices \( q_C \) and \( p_C \), \( E[\pi_A] \) is to be maximized with respect to \( q_A + p_A \) only (not with respect to \( q_A \) and \( p_A \)) and subject to \( q_A + p_A \geq 0 \) and \( 0 \leq k \leq 1 \). In the appendix, we prove

**Proposition 2** The best response correspondence of the custodian bank is given by the following:

(i) If \( 2q_C + p_C \geq 2(c_a + c_s) - c \) and \( 2q_C + 3p_C \leq 3c + 2(c_a + c_s) \), then

\[
q_A + p_A \begin{cases} 
\frac{1}{2}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{3}p_C \equiv X & \text{if } q_C + p_C > \hat{c} \\
\frac{1}{2}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C \equiv Y & \text{if } q_C + p_C = \hat{c} \\
\frac{1}{2}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{4}p_C \equiv Y & \text{if } q_C + p_C < \hat{c} 
\end{cases}
\]

(ii) If \( 2q_C + p_C < 2(c_a + c_s) - c \), then

\[
q_A + p_A \begin{cases} 
V & \text{if } 2q_C + 3p_C > 3c + 2(c_a + c_s) \text{ and } q_C + p_C > \hat{c} \\
W & \text{otherwise}
\end{cases}
\]

(iii) If \( 2q_C + 3p_C > 3c + 2(c_a + c_s) \), then

\[
q_A + p_A \begin{cases} 
W & \text{if } 2q_C + p_C < 2(c_a + c_s) - c \text{ and } q_C + p_C < \hat{c} \\
V & \text{otherwise}
\end{cases}
\]

with \( \hat{c} \equiv \frac{1}{2}c + c_a + c_s \), \( V \equiv q_C + p_C - \frac{1}{2}c \) and \( W \equiv q_C + p_C + \frac{1}{2}c \).

Though the best response correspondence looks rather complex, it is intuitive. Firstly note that \( q_A + p_A \) is increasing in \( q_C \) and \( p_C \) as it is normal for price competition. Secondly, if \( q_C \) and \( p_C \) are sufficiently high, the custodian bank chooses \( q_A + p_A = V \), i.e. all investor banks go to the custodian bank \((k = 0)\). If instead \( q_C \) and \( p_C \) are sufficiently small, the custodian bank chooses \( q_A + p_A = W \), i.e. all investor banks go to the CSD \((k = 1)\).

Furthermore, it is important to note that in \( q_C + p_C = \frac{1}{2}c + (c_a + c_s) \) the best response correspondence is discontinuous (at least if \( p_C > 0 \)). The custodian bank is indifferent between \( q_A + p_A = X \) \((k = \frac{1}{2} - \frac{p_C}{q_C} < \frac{1}{2})\) and \( q_A + p_A = Y \) \((k = \frac{1}{2} + \frac{p_C}{q_C} > \frac{1}{2})\) or between \( q_A + p_A = V \) \((k = 0)\) and \( q_A + p_A = W \) \((k = 1)\). This is due to the fact that the custodian bank’s payoff function \( E[\pi_A] \) is not quasi-concave in \( q_A + p_A \). For this reason, there is no equilibrium in the simultaneous move game, i.e. if the CSD and the custodian bank choose their prices simultaneously. (See section 8 below.)

7 The equilibrium

The CSD moves first so that it maximizes \( E[\pi_C] \) with respect to \( q_C \) and \( p_C \) taking into account both \( q_A + p_A \) as given in proposition 2 and \( k \) as derived in
section 5. We show in the appendix that the maximization leads to the following proposition:

**Proposition 3** (1) If \( c_0 + \frac{1}{2}c_s \leq c \), then the CSD chooses \( q_C = 0 \) and \( p_C = \frac{5}{6}c + c_0 + c_s \). In this case, the prices of the custodian bank are given by any \( q_A + p_A = \frac{5}{6}c + \frac{5}{6}(c_0 + c_s) \), \( q_A \geq 0 \) and \( p_A \geq 0 \). Finally, we have

\[
k = \frac{5}{6}c + \frac{5}{6}(c_0 + c_s) \equiv k_1
\]

(2) If \( c \leq c_0 + \frac{1}{2}c_s \), then the CSD chooses \( p_C = \frac{5}{6}c + \frac{1}{2}c_s \) and \( q_C = c_0 + \frac{1}{2}c_s - c \). In this case, the prices of the custodian bank are given by any \( q_A + p_A = \frac{5}{6}c + c_0 + \frac{5}{6}c_s \), \( q_A \geq 0 \) and \( p_A \geq 0 \). Finally, we have

\[
k = \frac{7c + \frac{1}{2}c_s}{c} \equiv k_2
\]

From this proposition, we can draw some remarkable conclusions. Firstly, we see that \( k > \frac{1}{2} \), i.e. \( q_C + p_C < q_A + p_A \) and the CSD is able to attract more customers than the custodian bank although we have assumed symmetry in marginal costs (\( \bar{c}_a = c_0 \) and \( \bar{c}_a = c_s \)) and in the quality of the settlement services of the two service providers. The main reason is that the CSD can raise the costs of the custodian bank without raising the costs of its other customers in the following way: The CSD can increase \( p_C \) and simultaneously decrease \( q_C \) by the same amount. The costs of investor banks with the CSD remain unchanged since these banks have one account with the CSD and want to have only one trade settled. However, the costs of the custodian bank increase since it has also only one account, but many trades settled through its account with the CSD.

The resulting higher costs of the custodian bank normally force the custodian bank to increase its prices so that it loses customers to the CSD. If \( c \) is high (case (1) in the proposition), then in equilibrium we have \( q_C = 0 \). In other words, the CSD applies the above described strategy of raising-rival’s-costs as far as possible. Note that \( k = \frac{5}{6} \) \( > \frac{1}{2} \) even if \( c \to \infty \). This is because if \( c \) approaches infinity, the prices of both service providers do the same.

If \( c \) is low (case (2) in the proposition), we find \( q_C > 0 \), i.e. the CSD has less incentives to raise the cost of the custodian bank. The reason appears to be the following: If \( c \) is low and \( q_C + p_C \) is only slightly lower than \( q_A + p_A \), it is attractive to choose the CSD even for those investor banks that are located close to the bottom of the interval \([0, 1]\). In this case, it is sufficient for the CSD to raise the costs of the custodian bank only little. Secondly, we find that \( k \) is decreasing in \( c \). This makes sense given that we have \( k > \frac{1}{2} \) in equilibrium (recall the results of section 5). If \( c \) increases, banks in \([0, \frac{1}{2}]\) increasingly prefer to go to the custodian bank and banks in \([\frac{1}{2}, 1]\) increasingly prefer to go to the CSD, provided prices remain unchanged.

---

9 Another reason may be the first mover advantage of the CSD.
10 \( k \) is a continuous function of \( c \), but not differentiable in \( c = c_0 + \frac{1}{2}c_s \).
Consequently, if $k > 1/2$ and $c$ increases, then the CSD will lose customers. Finally, $k$ is increasing in $c_a$ and $c_s$, i.e. technological progress leads to a lower $k$, thus to more custodian banking.

We now compare the equilibrium with the results of our welfare analysis (proposition 1). As discussed in section 3, there are two welfare maximisers, namely $k_{opt}^1 = \frac{1}{2} \frac{c - c_a}{c}$, $k_{opt}^2 = \frac{1}{2} \frac{c + c_s}{c} > 1/2$. Since both equilibrium solutions $k_1$ and $k_2$ are above $1/2$, it is obvious that in equilibrium we have

$$k > k_{opt}^1.$$  

Since $c \geq c_s$, we also have $k_2 \geq k_{opt}^2$. However, we can not say that the equilibrium value of $k$ is always higher than the socially optimal value, because it is not always true that $k_1 \geq k_{opt}^1$. Indeed, it is easy to check that $k_1 \geq k_{opt}^1 \Leftrightarrow c \geq 2(c_a - c_s)$. Since $c < 2(c_a - c_s) \Leftrightarrow c_a + \frac{1}{2} c < c_a$, we know because of $c \geq c_s$ that $c < 2(c_a - c_s)$ implies $c_a + \frac{1}{2} c < c$. Thus, we get

$$k < k_{opt}^2 \Leftrightarrow c < 2(c_a - c_s).$$

If and only if $c$ is sufficiently low, we have $k < k_{opt}^2$ in equilibrium. Since $k$ can be higher than, equal to or lower than $k_{opt}^2$, there is no case for regulatory interventions aiming at reducing the CSD’s market share.

### 8 The simultaneous move game

In this final section, we show that our model would have no equilibrium if we were assuming that the CSD and the custodian bank set their prices simultaneously. Firstly note as mentioned earlier that the payoff function $E[\pi_A]$ of the custodian bank is not quasi concave in $q_A + p_A$. For given prices $q_C$ and $p_C$ of the CSD, $E[\pi_A]$ has a local maximum with $q_A + p_A$ somewhere between $q_C + p_C - \frac{1}{2} c$ and $q_C + p_C$ and another local maximum with $q_A + p_A$ somewhere between $q_C + p_C$ and $q_C + p_C + \frac{1}{2} c$. Thus, the sufficient conditions for the existence of a Nash equilibrium are not satisfied. However, that does not necessarily imply that there is no Nash equilibrium.

To show that there is indeed no Nash equilibrium, we have to derive the best response function of the CSD which is given by

**Proposition 4** The best response function of the CSD is

$$p_C \begin{cases}  
\in [0, q_A + p_A - \frac{1}{2} c] & \text{if } \frac{5}{2} c + 2 c_a + c_s \leq q_A + p_A \\
= \frac{3}{2} c + \frac{1}{2} (q_A + p_A) + c_a + \frac{1}{2} c_s & \text{if } \frac{3}{2} c + 2 c_a + c_s \leq q_A + p_A \leq \frac{5}{2} c + 2 c_a + c_s \\
= q_A + p_A & \text{if } \frac{1}{2} c + \frac{1}{2} c_s + c_a \leq q_A + p_A \leq \frac{3}{2} c + 2 c_a + c_s \\
= \frac{1}{2} c + \frac{1}{2} (q_A + p_A) + \frac{1}{2} c_s & \text{if } \frac{1}{2} c + c_s + \frac{1}{2} c \leq q_A + p_A \leq \frac{1}{2} c + \frac{3}{2} c_a + c_s \\
\in [0, \frac{1}{2} c + q_A + p_A] & \text{if } q_A + p_A \leq \frac{1}{2} c_a + c_s - \frac{1}{2} c \\
\end{cases}$$

$$q_C \begin{cases}  
= q_A + p_A - \frac{1}{2} c - p_C & \text{if } \frac{5}{2} c + 2 c_a + c_s \leq q_A + p_A \\
= 0 & \text{if } \frac{5}{2} c_a + c_s - \frac{1}{2} c \leq q_A + p_A \leq \frac{5}{2} c + 2 c_a + c_s \\
= q_A + p_A + \frac{1}{2} c - p_C & \text{if } q_A + p_A \leq \frac{5}{2} c_a + c_s - \frac{1}{2} c \\
\end{cases}$$
We see from this proposition that in the simultaneous move game the CSD still has clear incentives to set \( q_C = 0 \) and \( p_C \) relatively high to raise the custodian bank’s costs without raising the overall costs of the investor banks. Furthermore, note that \( p_C \) is increasing in \( q_A + p_A \) and continuous in \( q_A + p_A \), while the custodian banks best response correspondence \( q_A + p_A \) is discontinuous according to proposition 2. As mentioned before, this is the reason why the simultaneous move game has no equilibrium as stated in

**Proposition 5** If the CSD and the custodian bank set their prices simultaneously, then there is no equilibrium in pure strategies.

With numeric examples, it is easy to show that in a diagramm with \( q_A + p_A \) on the horizontal and \( p_C \) on the vertical axis, the best response of the custodian bank runs right of the best response of the CSD as long as \( p_C < \frac{1}{2}c + c_a + c_s \) (and \( q_C = 0 \)). For \( p_C > \frac{1}{2}c + c_a + c_s \) (and \( q_C = 0 \)) however, it runs left of the CSD’s best response. I.e. it jumps from the right to the left of the CSD’s best response in \( p_C = \frac{1}{2}c + c_a + c_s \). For that reason, there is no interception of the two best responses and no equilibrium in pure strategies.

### 9 Concluding remarks

We have discussed a simple model of the competition between a CSD and a custodian bank. It has been shown that the CSD is able to exploit its monopoly position as central depository by raising the custodian bank’s costs to attract more investor banks as customers. However, we have also shown that the market share of the CSD in equilibrium is not necessarily higher than its socially optimal market share.

Our analysis appears to be relevant in two ways. Firstly, it contributes to the current discussion between market participants, especially CSDs and custodian banks, on whether and how competition in the securities settlement industry is distorted. Since we have shown that the CSD’s equilibrium market share is not necessarily higher than socially optimal, the model provides no case for regulatory interventions aiming at reducing the CSD’s market share. Secondly, it is theoretically relevant in that it considers a case of asymmetric network competition which has not been analyzed yet.

Extensions of the model could go into different directions. Firstly, it might be interesting to allow the custodian bank to price discriminate by charging investor banks with an account with the custodian bank a higher price if they trade with an investor bank with an account with the CSD. This idea is in line with Laffont, Rey and Tirol (1996b). Secondly, one may want to analyze our model with the assumption that the CSD can charge a progressive settlement price, i.e. the more trades a customer of the CSD wants to have settled through accounts with the CSD, the higher the price per settlement this customer has to pay. Since the custodian bank is the customer of the CSD with the highest number of trades to be settled through accounts with the CSD, this would clearly adversely effect the custodian bank and give the CSD power to raise its
rival’s cost even higher. Finally, it might be interesting to assume that there are more than one custodian banks so that there is competition between several custodian banks. If the services of all custodian banks are perfect substitutes for all investor banks, the price competition between custodian banks would reduce settlement prices significantly.

10 Appendix

Proof of proposition 1:
Let \( E[S^u] = k(c_a + c_s) + (1-k)(\bar{c}_a + \bar{c}_s) + \frac{1}{2}kc_a - \frac{1}{2}kc(1-k) \) be the upper branch of \( E[S] \). This is a convex function in \( k \) with a minimum in

\[
k = \frac{\frac{1}{2}c + \bar{c}_a + \bar{c}_s - c_a - \frac{3}{2}c_s}{c} \equiv k_1
\]

with

\[
E[S^u]^* = \bar{c}_a + \bar{c}_s - \frac{1}{2}\left(\frac{1}{2}c + \bar{c}_a + \bar{c}_s - c_a - \frac{3}{2}c_s\right)
\]

Let \( E[S^L] = k(c_a + c_s) + (1-k)(\bar{c}_a + \bar{c}_s) + \frac{1}{2}(1-k)c_a - \frac{1}{2}kc(1-k) \). This is also a convex function in \( k \) with a minimum in

\[
k = \frac{\frac{1}{2}c + \bar{c}_a + \bar{c}_s - c_a - \frac{1}{4}c_a}{c} \equiv k_2
\]

with

\[
E[S^L]^* = \bar{c}_a + \bar{c}_s + \frac{1}{2}c_s - \frac{1}{2}\left(\frac{1}{2}c + \bar{c}_a + \bar{c}_s - c_a - \frac{3}{4}c_s\right)
\]

It is easy to show that

\[
E[S^u]^* \geq E[S^L]^*
\]

\[
\leftrightarrow \bar{c}_a + \bar{c}_s \geq c_a + c_s
\]

Assume \( c_a + c_s \leq \bar{c}_a + \bar{c}_s \leq c_a + \frac{3}{2}c + \frac{1}{2}c \). In this case, we have \( E[S^u]^* \geq E[S^L]^* \) and \( 0 \leq k_1 \leq \frac{1}{2} \), i.e. \( k_{opt} = k_1 \). Assume \( c_a + \frac{3}{2}c_a - \frac{1}{2}c \leq \bar{c}_a + \bar{c}_s \leq c_a + c_s \). In this case, we have \( E[S^u]^* \leq E[S^L]^* \) and \( \frac{1}{2} \leq k_2 \leq 1 \), i.e. \( k_{opt} = k_2 \). This completes the proof of the proposition.

\( \Box \)

Proof of proposition 2:
For given prices \( q_C \) and \( p_C \), we have to maximise \( E[\pi_A] \) as given in equation 1 with respect to \( q_A + p_A \) subject to \( q_A + p_A \geq 0 \) and \( 0 \leq k \leq 1 \), where \( k \) is given by equation 3.

Let \( E[\pi_A^L] = (1-k)(q_A - c_a + p_A - c_s) - \frac{1}{2}kp_C \) be the upper branch of \( E[\pi_A] \). In a first step, we maximise \( E[\pi_A^L] \) subject to \( q_A + p_A \geq 0 \) and \( 0 \leq k \leq \frac{1}{2} \). We easily get the following maximiser and maximum:

\[
q_A + p_A = \begin{cases} 
q_C + \frac{1}{2}c & \text{if } q_C + \frac{1}{2}c < c_a + c_s + \frac{1}{2}c \\
\frac{3}{4}c + \frac{1}{4}(c_a + c_s) + \frac{1}{2}qc + \frac{1}{2}p_C & \text{if } c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{4}p_C \leq c_a + c_s + \frac{3}{2}c \\
q_C + p_C - \frac{1}{2}c & \text{if } q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c 
\end{cases}
\]
and

\[
E[\pi_A^U] = \begin{cases} 
\frac{1}{2}q_C + \frac{1}{2}p_C - \frac{1}{2}(c_a + c_s) \equiv E_1 & \text{if } q_C + \frac{1}{2}p_C < c_a + c_s + \frac{1}{2}c \\
\frac{1}{2}q_C + \frac{1}{2}p_C - \frac{1}{2}(c_a + c_s) \equiv E_2 & \text{if } c_a + c_s + \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{3}{2}c \\
q_C + p_C - \frac{1}{2}c - (c_a + c_s) \equiv E_3 & \text{if } q_C + \frac{1}{2}p_C > c_a + c_s + \frac{3}{2}c 
\end{cases}
\]

Let \( E[\pi_A^U] = (1 - k)(q_A - c_a + p_A - c_s) - \frac{k}{2}(1 - k)p_C \) be the lower branch of \( E[\pi_A] \). We maximise \( E[\pi_A^U] \) subject to \( q_A + p_A \geq 0 \) and \( \frac{1}{2} \leq k \leq 1 \). Again we easily get the maximiser and maximum:

\[
g_a + p_a = \begin{cases} 
q_C + p_C + \frac{1}{2}c & \text{if } q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \\
\frac{1}{2}q_C + \frac{1}{2}p_C - \frac{1}{2}(c_a + c_s) & \text{if } c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \\
q_C + p_C & \text{if } q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c 
\end{cases}
\]

and

\[
E[\pi_A^U] = \begin{cases} 
0 \equiv E_4 & \text{if } q_C + \frac{1}{2}p_C < c_a + c_s + \frac{1}{2}c \\
\frac{1}{2}q_C + \frac{1}{4}(c_a + c_s) + \frac{1}{8}(c_a + c_s) & \text{if } c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \\
\frac{1}{2}q_C + \frac{1}{4}(c_a + c_s) - \frac{1}{4}(c_a + c_s) & \text{if } c_a + c_s + \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \\
q_C + \frac{3}{2}p_C & \text{if } q_C + \frac{3}{2}p_C > c_a + c_s + \frac{1}{2}c 
\end{cases}
\]

We now have to consider the following nine cases:

A) \( q_C + \frac{3}{2}p_C < c_a + c_s + \frac{3}{2}c \) and \( q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \). It follows immediately that \( q_C + p_C < c_a + c_s \), i.e. \( E_1 < 0 = E_4 \). The best response is thus given by \( q_a + p_a = q_C + p_C + \frac{1}{2}c \).

B) \( q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c \) and \( c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \).

Because \( E_5 = \frac{1}{2}E_1 + \frac{1}{2}E_2 \), we know that \( E_5 \leq E_1 < E_2 - \frac{1}{2}c \leq 0 \). This is only possible if \( E_4 \equiv 0 \), i.e. \( q_C + \frac{1}{2}p_C = c_a + c_s + \frac{1}{2}c \), which is in contradiction to \( q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c \). The best response is thus given by \( q_a + p_a = \frac{1}{2}c + \frac{1}{4}(c_a + c_s) + \frac{1}{8}(q_C + \frac{3}{2}p_C) \).

C) \( q_C + \frac{3}{2}p_C < c_a + c_s + \frac{1}{2}c \) and \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{1}{2}c \). This case is not possible.

D) \( c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c \) and \( q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \).

First note that \( \frac{1}{2}c - \frac{1}{2}(c_a + c_s) = \frac{1}{2}q_C + \frac{3}{2}p_C \geq 0 \) so that \( E_2 \) is maximised by minimising \( (c_a + c_s) \). We now have to consider two sub-cases: (i) \( 2c \geq p_C \). Since \( c_a + c_s \geq q_C + \frac{3}{2}p_C + \frac{1}{2}c \), we get \( E_2 < \frac{1}{2}p_C(\frac{1}{2}p_C - 1) \leq 0 \). (ii) \( 2c < p_C \). Since \( c_a + c_s < q_C + \frac{3}{2}p_C - \frac{1}{2}c \), we get \( E_2 < c - \frac{1}{2}p_C < 0 \). I.e. we have \( E_2 < E_4 \) and the best response is \( q_a + p_a = q_C + p_C + \frac{1}{2}c \).

E) \( c_a + c_s + \frac{1}{2}c \leq q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c \) and \( c_a + c_s + \frac{3}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \).

It is easy to see that \( E_2 \geq E_5 \) \( \Leftrightarrow q_C + p_C \geq \frac{1}{2}c + c_a + c_s \). The best response is therefore \( q_a + p_a = \frac{1}{2}c + \frac{1}{4}(c_a + c_s) + \frac{1}{8}(q_C + \frac{3}{2}p_C) \) if \( q_C + p_C \geq \frac{1}{2}c + c_a + c_s \) and \( q_a + p_a = \frac{1}{4}c + \frac{1}{4}(c_a + c_s) + \frac{1}{8}(q_C + \frac{3}{2}p_C) \) if \( q_C + p_C \leq \frac{1}{2}c + c_a + c_s \).

F) \( c_a + c_s + \frac{3}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{3}{2}c \) and \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{1}{2}c \).

Because \( E_2 = \frac{1}{2}E_1 + \frac{1}{2}E_2 + \frac{1}{2}(E_2 - \frac{3}{2}p_C) \), we know that \( E_2 \leq E_5 \) \( \Leftrightarrow \frac{1}{2}E_1 - \frac{3}{2}p_C \leq 0 \). This is only possible if \( E_5 \equiv \frac{1}{2}E_1 - \frac{3}{2}p_C \), i.e. \( q_C + \frac{3}{2}p_C = c_a + c_s + \frac{1}{2}c \) which is in contradiction to \( q_C + \frac{1}{2}p_C < c_a + c_s + \frac{1}{2}c \). Thus we get \( E_2 > E_5 \) and \( q_a + p_a = \frac{1}{2}c + \frac{1}{4}(c_a + c_s) + \frac{1}{8}(q_C + \frac{3}{2}p_C) \) as best response.

G) \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \) and \( q_C + \frac{1}{2}p_C < c_a + c_s - \frac{1}{2}c \). Here we have \( E_5 \geq E_4 \) \( \Leftrightarrow q_C + p_C \geq \frac{3}{2}c + c_a + c_s \). The best response is therefore
\( q_A + p_A = q_C + p_C - \frac{1}{2}c \) if \( q_C + p_C \geq \frac{1}{2}c + c_a + c_s \) and \( q_A + p_A = q_C + p_C + \frac{1}{2}c \) if \( q_C + p_C \leq \frac{1}{2}c + c_a + c_s \).

H) \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \) and \( c_a + c_s - \frac{1}{2}c \leq q_C + \frac{1}{2}p_C \leq c_a + c_s + \frac{1}{2}c \).

First note that \( \frac{1}{2}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{2}p_C \geq 0 \). Moreover, we have \( \frac{1}{2}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{2}p_C \geq c \) if \( q_C + \frac{1}{2}p_C \geq c_a + c_s + \frac{1}{2}c \). Thus, we have \( E_5 < \frac{1}{2}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{2}p_C \). Since \( \frac{1}{2}c - \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{1}{2}p_C \geq E_3 \Leftrightarrow q_C + \frac{3}{2}p_C \leq c_a + c_s + \frac{3}{2}c \) which is in contradiction to above, we know that \( E_3 > E_5 \) and get as best response \( q_A + p_A = q_C + p_C - \frac{1}{2}c \).

1) \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \) and \( q_C + \frac{1}{2}p_C > c_a + c_s + \frac{1}{2}c \). Since \( E_3 \leq E_6 \Leftrightarrow q_C + \frac{3}{2}p_C \leq c_a + c_s + e \) which is in contradiction to \( q_C + \frac{3}{2}p_C > c_a + c_s + \frac{3}{2}c \), we get \( E_3 > E_6 \) and \( q_A + p_A = q_C + p_C - \frac{1}{2}c \) as best response.

It is easy to verify that these results finalize the proof of proposition 2.

\( \Psi \)

Proof of proposition 3:

(I) To begin with, we determine the best the CSD can do under the restrictions \( q_C \geq 0, p_C \geq 0, q_C + p_C \leq c_a + c_s + \frac{1}{2}c, 2q_C + p_C \geq 2(c_a + c_s) - c, 2q_C + 3p_C \leq 2(c_a + c_s) + 3c \). Here, the custodian bank chooses according to proposition 2 \( q_A = \frac{5}{8}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{2}p_C \) so that \( k = \frac{1}{2}c + \frac{1}{2}(c_a + c_s) - \frac{3}{2}q_C - \frac{1}{2}p_C \geq \frac{1}{2}c \). Thus, we have to solve

\[
\begin{align*}
\text{Max } E[\pi_C] &= k(q_C - c_a + p_C - c_s) + \frac{1}{2}(1 - k)(p_C - c_s) \\
\text{s.t. } q_C &\geq 0, p_C \geq 0, q_C + p_C \leq c_a + c_s + \frac{1}{2}c, \\
2q_C + p_C &\geq 2(c_a + c_s) - c, 2q_C + 3p_C \leq 2(c_a + c_s) + 3c.
\end{align*}
\]

We proceed as follows: We ignore the last two constraints and then show that the solution of the reduced problem satisfies the last two constraints, i.e. these constraints are not binding. It is easy to show that the solution of the reduced problem is the following:

(ii) If \( c \geq c_a + \frac{1}{2}c_s \), then \( q_C = 0 \) and \( p_C = \frac{1}{2}c + c_a + c_s \). In this case, we have \( 2q_C + p_C \leq 2(c_a + c_s) - c \Leftrightarrow \frac{1}{2}c + c < c_a + \frac{1}{2}c_s + \frac{1}{2}c \) and \( 2q_C + 3p_C > 2(c_a + c_s) \). Furthermore, we get \( q_A + p_A = \frac{5}{8}c + \frac{1}{2}(c_a + c_s), k = \frac{1}{2}(\frac{5}{8}c + \frac{1}{2}(c_a + c_s)) \) and \( E[\pi_C] = \frac{1}{2}(\frac{5}{8}c + \frac{1}{2}(c_a + c_s))\frac{1}{2}c - \frac{1}{2}(c_a + c_s) = \frac{1}{2}(\frac{5}{8}c - \frac{1}{2}c_a)^2 + \frac{1}{2}c \equiv \tilde{E}_1(i) \).

(ii) If \( c \leq c_a + \frac{1}{2}c_s \), then \( q_C = c_a + \frac{1}{2}c_s - c \) and \( p_C = \frac{3}{2}c + \frac{1}{2}c_s \). In this case, we have \( 2q_C + p_C \leq 2(c_a + c_s) - c \Leftrightarrow c < c_a + \frac{1}{2}c_s + \frac{1}{2}c \) and \( 2q_C + 3p_C > 2(c_a + c_s) \). Furthermore, we get \( q_A + p_A = \frac{5}{8}c + c_a + \frac{1}{2}c_s, k = \frac{1}{2}(\frac{5}{8}c + \frac{1}{2}(c_a + c_s)) \) and \( E[\pi_C] = \frac{1}{2}(\frac{5}{8}c - \frac{1}{2}c_a)^2 + \frac{1}{2}c \equiv \tilde{E}_1(ii) \).

(II) Now we show that the best the CSD can do under the restrictions \( q_C \geq 0, p_C \geq 0, q_C + p_C \geq c_a + c_s + \frac{1}{2}c, 2q_C + p_C \geq 2(c_a + c_s) - c, 2q_C + 3p_C \leq 2(c_a + c_s) + 3c \) would make the CSD worse off. Here, the custodian bank chooses according to proposition 2 \( q_A + p_A = \frac{5}{8}c + \frac{1}{2}(c_a + c_s) + \frac{1}{2}q_C + \frac{3}{2}p_C \) so that
\[ k = \frac{1}{c}(\frac{3}{4}c + \frac{1}{2}(c_a + c_s) - \frac{1}{2}q_C - \frac{3}{8}p_C) \leq \frac{1}{2} \quad \text{and} \quad E[\pi_C] = \frac{1}{c}3c + \frac{1}{2}(c_a + c_s) - \frac{1}{2}(q_C + \frac{3}{2}p_C)]q_C + \frac{3}{2}p_C - c - \frac{3}{2}c_a] \]

We ignore the constraints and maximise \( E[\pi_C] \) with respect to \( q_C + \frac{3}{2}p_C \). It is very easy to check that the maximum is given by \( E[\pi_C] = \frac{1}{c}(\frac{3}{4}c - \frac{1}{4}c_a)^2 \equiv E_{II} \).

It is clear that the maximum under the constraints of (II) cannot be higher than \( E_{II} \). Thus, we only have to compare

(a) \( E_{II} \) with \( E_{I(ii)} \) under \( c \geq c_a + \frac{1}{2}c_s \). It is easy to see that \( E_{II} \geq E_{I(ii)} \iff d[c + 2c_a + 2c_s] \leq [c_a + \frac{1}{2}c_s]^2 \) which is in contradiction to \( c \geq c_a + \frac{1}{2}c_s \) so that \( E_{II} < E_{I(ii)} \).

(b) \( E_{II} \) with \( E_{I(iii)} \) under \( c \leq c_a + \frac{1}{2}c_s \). It is easy to see that \( E_{II} \geq E_{I(iii)} \iff c + \frac{1}{2}c_s \leq 0 \) so that \( E_{II} < E_{I(iii)} \).

(III) Now we show that the best the CSD can do under the restrictions that \( q_A + p_A = V \) would make the CSD worse off compared to the maximum derived under (I). If \( q_A + p_A = V \), then \( E[\pi_C] = 0 \equiv E_{II} \). Thus we have to compare

(a) \( E_{III} = 0 \) with \( E_{I(i)} \) under \( c \geq c_a + \frac{1}{2}c_s \). As long as \( \frac{1}{4}c - \frac{1}{2}c_a \geq 0 \), we have \( E_{I(i)} > 0 \), i.e. \( E_{III} < E_{I(i)} \). If \( \frac{1}{4}c - \frac{1}{2}c_a < 0 \), then \( E_{I(i)} \geq \frac{1}{4}c - \frac{1}{2}c_a + \frac{1}{2}c + \frac{1}{2}c_a > 0 \). Thus, we again have \( E_{III} < E_{I(i)} \).

(b) \( E_{III} \) with \( E_{I(ii)} \) under \( c \leq c_a + \frac{1}{2}c_s \). Since we obviously have \( E_{I(iii)} > 0 \), we immediately get \( E_{III} < E_{I(ii)} \).

(IV) Finally, we show that the best the CSD can do under the restrictions that \( q_A + p_A = W \) would make the CSD worse off compared to the maximum derived under (I). If \( q_A + p_A = W \), then \( E[\pi_C] = q_C + p_C - c_a - c_s \).

(a) \( c \geq c_a + \frac{1}{2}c_s \). According to proposition 2, we have \( q_A + p_A = W \) only if \( 2q_C + p_C < 2(c_a + c_s) - c \). Thus, the best the CSD can do is to maximise \( q_C + p_C - c_a - c_s \) subject to \( q_C \geq 0 \), \( p_C \geq 0 \), \( 2q_C + p_C < 2(c_a + c_s) - c \), i.e. to choose \( q_C = 0 \), \( p_C = 2(c_a + c_s) - c \) with \( E[\pi_C] = c_a + c_s - c \equiv E_{IV(a)} \).

(a) If \( \frac{1}{2}c - \frac{1}{2}c_a \geq 0 \), we have \( E_{I(i)} > \frac{1}{2}(\frac{1}{4}c - \frac{1}{2}c_a) + \frac{1}{2}c + \frac{1}{2}c_a \). With \( c \geq c_a + \frac{1}{2}c_a \) we easily get \( \frac{1}{2}(\frac{1}{4}c - \frac{1}{2}c_a) < E_{IV(a)} \Rightarrow c < c_a \) which is in contradiction to our assumption \( c \geq c_a \). (b) If \( \frac{1}{4}c - \frac{1}{2}c_a = 0 \), we have \( E_{I(i)} = \frac{1}{4}c \) and \( E_{IV(a)} = c_a - \frac{1}{4}c \) so that \( E_{I(i)} > E_{IV(a)} \) as long as \( c > c_a \). For \( c = c_a \), the cases (I) and (IV)(a) are equivalent and do not need to be distinguished. (c) If \( \frac{1}{4}c - \frac{1}{2}c_a < 0 \), we have \( E_{I(i)} > \frac{1}{4}c \) and \( E_{IV(a)} \leq \frac{1}{4}c \) so that again \( E_{I(i)} > E_{IV(a)} \).

(b) \( c \leq c_a + \frac{1}{2}c_s \). According to proposition 2, we have \( q_A + p_A = V \) only in two cases: (a) \( 2q_C + p_C < 2(c_a + c_s) - c \) and \( 2q_C + 3p_C < 2(c_a + c_s) + 3c \). This implies \( q_C + p_C < c_a + c_s + \frac{1}{4}c \), i.e. \( E[\pi_C] < \frac{1}{4}c \leq E_{I(ii)} \).

Proof of proposition 4:

For given prices \( q_A + p_A \), we have to maximise \( E[\pi_C] \) as given in equation 2 with respect to \( q_C \) and \( p_C \) subject to \( q_C \geq 0 \), \( p_C \geq 0 \) and \( 0 \leq k \leq 1 \), where \( k \) is given by equation 3.
Let \( E[\pi_C^L] = k(q_C - c_a + p_C - c_a) + \frac{1}{2}(1-k)(p_C - c_a) \) be the lower branch of \( E[\pi_C] \). In a first step, we maximise \( E[\pi_C^L] \) subject to \( q_C \geq 0, p_C \geq 0 \) and \( \frac{1}{2} \leq k \leq 1 \). We get the following maximiser and maximum:

\[
P_C \begin{cases} \frac{q_A + p_A}{2} + c_a + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) & \text{if } q_A + p_A < 2c_a + c_s + \frac{3}{2}c \\ \frac{q_A + p_A}{2} + c_a + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) - \frac{1}{2} (q_A + p_A) & \text{if } q_A + p_A \geq 2c_a + c_s + \frac{3}{2}c \end{cases}
\]

\[
q_C \begin{cases} 0 & \text{if } q_A + p_A < 2c_a + c_s + \frac{3}{2}c \\ q_A + p_A - \frac{1}{2} c - p_C & \text{if } q_A + p_A \geq 2c_a + c_s + \frac{3}{2}c \end{cases}
\]

and

\[
E[\pi_C^L] = \begin{cases} \frac{3}{2}(q_A + p_A) - \frac{1}{2} c_a - \frac{3}{2} c_a & \equiv \tilde{E}_1 & \text{if } q_A + p_A < 2c_a + c_s + \frac{3}{2}c \\ \frac{1}{2} q_A + p_A + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) & \equiv \tilde{E}_2 & \text{if } q_A + p_A \geq 2c_a + c_s + \frac{3}{2}c \end{cases}
\]

Let \( E[\pi_C^U] = k(q_C - c_a + p_C - c_a) + \frac{1}{2}k(p_C - c_a) \) be the upper branch of \( E[\pi_C] \). We maximise \( E[\pi_C^U] \) subject to \( q_C \geq 0, p_C \geq 0 \) and \( 0 \leq k \leq \frac{1}{2} \). We get the following maximiser and maximum:

\[
P_C \begin{cases} \frac{q_A + p_A}{2} & \text{if } q_A + p_A \geq \frac{2}{3}c_a + c_s + \frac{2}{3}c \\ \frac{q_A + p_A}{2} + c_a + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) & \text{if } \frac{2}{3}c_a + c_s + \frac{2}{3}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{2}{3}c \end{cases}
\]

\[
q_C \begin{cases} 0 & \text{if } q_A + p_A < \frac{2}{3}c_a + c_s + \frac{2}{3}c \\ q_A + p_A + \frac{1}{2} c - p_C & \text{if } q_A + p_A \geq \frac{2}{3}c_a + c_s + \frac{2}{3}c \end{cases}
\]

and

\[
E[\pi_C^U] = \begin{cases} \frac{3}{2}(q_A + p_A) - \frac{1}{2} c_a - \frac{3}{2} c_a & \equiv \tilde{E}_4 & \text{if } q_A + p_A \geq \frac{2}{3}c_a + c_s + \frac{2}{3}c \\ \frac{1}{2} q_A + p_A + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) & \equiv \tilde{E}_5 & \text{if } \frac{2}{3}c_a + c_s + \frac{2}{3}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{2}{3}c \end{cases}
\]

\[
0 & \equiv \tilde{E}_6 & \text{if } q_A + p_A < \frac{2}{3}c_a + c_s + \frac{2}{3}c
\]

We now have to consider the following five cases:

A) \( q_A + p_A > 2c_a + c_s + \frac{5}{2}c \). We easily find that \( \tilde{E}_3 > \tilde{E}_4 \), i.e. the best response is \( p_C \in [0, q_A + p_A - \frac{1}{2}c] \), \( q_C = q_A + p_A - \frac{1}{2}c - p_C \).

B) \( 2c_a + c_s + \frac{3}{2}c \leq q_A + p_A \leq 2c_a + c_s + \frac{5}{2}c \). Here we have \( \tilde{E}_2 > \tilde{E}_4 \), i.e. \( p_C = \frac{1}{2} q_A + p_A + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A) \), \( q_C = 0 \).

C) \( \frac{2}{3}c_a + c_s + \frac{5}{2}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{3}{2}c \). Here we have \( \tilde{E}_3 = \tilde{E}_4 \), i.e. \( p_C = q_A + p_A, q_C = 0 \).

D) \( \frac{2}{3}c_a + c_s + \frac{3}{2}c \leq q_A + p_A \leq \frac{2}{3}c_a + c_s + \frac{1}{2}c \). Here we have \( \tilde{E}_1 < \tilde{E}_5 \), i.e. \( p_C = \frac{1}{2} q_A + p_A + \frac{1}{2} c_a + \frac{1}{2} (q_A + p_A), q_C = 0 \).

E) \( q_A + p_A \leq \frac{2}{3}c_a + c_s - \frac{1}{2}c \). Here we have \( \tilde{E}_1 < \tilde{E}_5 \), i.e. \( p_C \in [0, q_A + p_A - \frac{1}{2}c] \), \( q_C = q_A + p_A - \frac{1}{2}c - p_C \).

This finalises the proof.

\( \forall \)
Proof of proposition 5:

To proof this proposition, one would need to consider 20 cases. Since the proof is simple, but tedious, we discuss only the first two cases:

(1) \(q_A + p_A = \frac{2}{3}c + \frac{1}{3}(a + c_s) + \frac{1}{3}q_C + \frac{1}{3}p_C\) and \(q_C + p_C = q_A + p_A - \frac{1}{3}c\). This implies \(q_C + p_C + \frac{1}{3}c = \frac{2}{3}c + \frac{1}{3}(a + c_s) + \frac{1}{3}q_C + \frac{1}{3}p_C\), i.e. \(2q_C + 3p_C = 2(a + c_s) - c\). Since \(q_A + p_A = \frac{2}{3}c + \frac{1}{3}(a + c_s) + \frac{1}{3}q_C + \frac{1}{3}p_C\) requires \(2q_C + p_C \geq 2(a + c_s) - c\), this is only possible if \(p_C = 0\), i.e. \(q_C = a + c_s - \frac{1}{2}c\). Since \(q_A + p_A = \frac{2}{3}c + \frac{1}{3}(a + c_s) + \frac{1}{3}q_C + \frac{1}{3}p_C\) also requires \(q_C + p_C \geq \frac{1}{2}c + a + c_s\), this is not possible.

(2) \(q_A + p_A = \frac{2}{3}c + \frac{1}{3}(a + c_s) + \frac{1}{3}q_C + \frac{1}{3}p_C\), \(q_C = 0\) and \(p_C = \frac{3}{2}c + \frac{1}{2}(q_A + p_A) + a + \frac{1}{3}c_s\). This implies \(q_A + p_A = \frac{1}{3}c + \frac{1}{6}c_s + \frac{5}{6}c_s\). This is not possible since \(q_C = 0\) and \(p_C = \frac{3}{2}c + \frac{1}{2}(q_A + p_A) + a + \frac{1}{3}c_s\) requires \(q_A + p_A \geq 2c + a + \frac{1}{3}c\).

References


