FUNDING LONG SHOTS

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Abstract

A project with a low probability of success can be difficult to fund even if the expected return is high and the correlation with other investments is low. This paper describes how such projects can be combined in a fund, and securitization techniques used, so that they are attractive to investors. It examines how the proposed structure is affected by the number of projects in the fund, the probability of each project's success, the projects' life, correlations between the success of different projects, and uncertainty about the eventual payoff from a project if it is successful.

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1. Introduction

Some capital investment projects have a low probability of success, but a huge payoff if successful. Examples are biomedical research projects to develop new drugs, clean energy projects, and some mining projects. When these projects have expected returns that are above average, obtaining funding for them is often difficult because the risks are regarded as unacceptable by many investors. However, if handled well, the projects should be attractive to investors because their correlations with other investment opportunities are usually low.

Andrew Lo and his co-researchers at Massachusetts Institute of Technology have done some pioneering work in this area by proposing a new way of funding cancer research.[1] They observed that it is becoming increasingly expensive, lengthy, risky, and complex to move drugs from the research and development stage to the market. They proposed setting up a megafund to diversify risks and use securitization techniques to increase the range of investors that participate. For example, a $30 billion megafund could invest $200 million in each of 150 cancer projects. Similarly to an asset-backed security (ABS), it could have a senior tranche, a mezzanine tranche, and an equity tranche. The senior tranche and mezzanine tranche would be rated AAA and BBB, respectively. The cash flows from successful projects would flow first to the senior tranche. Once investors in the senior tranche had received their promised return, the mezzanine tranche holders would receive the cash flows. Once they had received their promised return, all remaining cash flows would accrue to the equity tranche.

This paper explores the properties of megafunds similar to the one just described. We consider how the funding could work in a wide range of low-success-probability situations. We explore how the risks and returns of tranche holders depend on the number of projects in the fund, the probability of each project's success, the project life, correlations between the success of different projects, and uncertainty about the eventual payoff from a project if it is successful.

[1] See for example Fernández et al (2012) and Fagnan et al (2013)

2. The Megafund Structure

We start by considering an ABS, which has become a well-established instrument for transferring the risks associated with debt instruments. This will enable us to highlight similarities and differences between an ABS and the proposed megafund structure.

In a typical ABS, there is a portfolio of income producing instruments each with a low probability of defaulting. A simple somewhat idealized example is shown in Figure 1. There are 100 loans each with a principal of $1 million and lasting five years. The interest rate on the loans is 6.5% and there a 2% chance of each loan defaulting during the five year period. In the event of a default 80% of the principal will be lost. The loans are sold by the originator to a special purpose vehicle (SPV) and rules are specified for determining how the interest and principal are allocated to tranches. In the example there are three tranches.[2] The senior tranche has a principal of $80 million and is promised a return of 5%; the mezzanine tranche has a principal of $15 million and is promised a return of 8%; the equity tranche has the remaining principal, $5 million, and is promised a return of 26%.

We assume a simple waterfall where interest and principal payments from the borrowers flow in strict priority to the senior tranche, the mezzanine tranche, and the equity tranche. It is easy to see that the senior tranche will have no principal losses if there are no more than 25 defaults. The mezzanine tranche will have no principal losses if there are no more than 6 defaults. The equity tranche suffers losses if any of the loans default.

The rating assigned to a tranche by S&P and Fitch depends on the probability of a loss of principal by the tranche. If the probability is similar to the probability of loss on a AAA-rated bond, the tranche will be rated AAA; if it is similar to the probability of loss on a AA-rated, it will be rated AA; and so on.[3] The probability of a certain number of losses depends on the default correlation. If the loans are independent, the probability of the senior tranche incurring a loss in our example is effectively zero and the probability of the mezzanine tranche incurring

[2] Often many more than three tranches are used. In this example the weighted average of the promised return to investors equals the 6.5% per annum interest on the loans. In practice it would be less than 6.5% to build in a profit for the SPV.
[3] Moody's compares the expected loss on a tranche with the expected loss on bonds to determine the rating.

losses is only 0.4%. As the correlation increases, these probabilities increase. For example, if a Gaussian copula model with correlation parameter 0.2 is used, the probability of the senior tranche incurring losses is 0.08% whereas the probability of the mezzanine tranche incurring losses rises to 7%.

Now consider a $30 billion megafund where 150 projects, each costing $200 million, are funded. Each project lasts 10 years and has a 5% probability of success. If a project is successful it will produce a cash flow of $10 billion in 10 years; otherwise the initial investment is completely lost. Figure 2 shows how the megafund could be structured. As in Figure 1 there are three tranches. In this case, the senior tranche has a principal of $12 billion and is a promised return of 5%; the mezzanine tranche has a principal of $9 billion and is promised a return of 8%. Any additional return after the senior and mezzanine tranches have received their promised returns accrues to the equity tranche.

As in the case of the ABS, we assume a strict priority rule so that cash flows from successful investments flow first to the senior tranche, then to the mezzanine tranche and then to the equity tranche. We define a “hit” as a successful project. In our example, if there is more than one hit, the senior tranche receives its promised return. If there are more than three hits the mezzanine tranche receives its promised return. When projects are independent, the probability of the senior tranche receiving its promised return is 99.6% while the probability of the mezzanine tranche doing so is 95%. Later we show that, as the correlation between the probability of success of different projects increases from zero, the structure rapidly becomes much less attractive to the senior and mezzanine tranche holders.

The financial engineering in the megafund is analogous to that in an ABS. In an ABS losses move up through the waterfall. They are borne first by the equity tranche then by the mezzanine tranche and finally by the senior tranche. In the megafund gains from hits move down through the waterfall. They accrue first to the senior tranche then to the mezzanine tranche and finally to the equity tranche. The megafund we have outlined is structurally the same as an ABS where the probability of default is 95% and the principal of each loan is the $10 billion

3. Analysis: Independent Projects with no Payoff Uncertainty

In this section we show how the megafund structure can be analyzed in the simple (somewhat stylized) case where the projects in the underlying portfolio are independent and there is no uncertainty about the final payoff if a project is successful. Projects either provide zero future cash flow or a known positive future cash flow and the probabilities of these two outcomes are assumed known in advance. Later we will relax these assumptions. Define:

- n: Number of projects
- p: Probability of that a project will succeed
- T: Life of each project (years)
- S: Cash flow at time T from each successful project (millions of dollars)
- h: Number of hits (i.e., successful projects)
- $q(h)$: Probability of h hits
- I: Investment required per project (millions of dollars)
- β_j: Proportion of portfolio financed by the tranche which is jth in order of seniority (j = 1,2 ,3)
- α_j: Promised return for tranche which is jth in order of seniority (j = 1,2)
- $C_j(h)$: Cash inflow at time T to tranche which is jth in order of seniority when there are h hits (j = 1,2 ,3)
- R_j: Expected return to tranche which is jth in order of seniority (j = 1,2 ,3)
- P_j: Probability that promised return is realized by the tranche which is jth in order of seniority (j = 1,2)
- Q_j: Probability of loss by the tranche which is jth in order of seniority (j = 1,2 ,3)

From the binomial distribution,

$$q(h) = \frac{n!}{(n-h)!h!} p^h (1-p)^{n-h}$$

Define

$$\begin{aligned} K_1 &= \beta_1 nI(1+\alpha_1)^T \\ K_2 &= \beta_1 nI(1+\alpha_1)^T + \beta_2 nI(1+\alpha_2)^T \end{aligned} \tag{1}$$

$$L_1 = \beta_1 nI$$

$$L_2 = \beta_1 nI(1+\alpha_1)^T + \beta_2 nI \qquad (2)$$

$$L_3 = \beta_1 nI(1+\alpha_1)^T + \beta_2 nI(1+\alpha_2)^T + \beta_3 nI$$

Let $u(j)$ be the minimum number of hits necessary for tranche j to receive its promised return (j=1, 2) and $v(j)$ as the maximum number of hits for which tranche j loses some principal (j=1, 2, 3). It follows that

$$u(j) = \text{int}\left(\frac{K_j}{S}\right) + 1 \qquad v(j) = \text{int}\left(\frac{L_j}{S}\right)$$

and

$$P_j = \sum_{h=u(j)}^{n} q(h) \qquad Q_j = \sum_{h=0}^{v(j)} q(h)$$

Furthermore

$$C_1(h) = \min(hS, K_1)$$

$$C_2(h) = \min(hS, K_2) - C_1(h)$$

$$C_3(h) = \max(hS - C_1(h) - C_2(h),\ 0)$$

and the expected return from each tranche is given by[4]

$$R_j = \left[\frac{\sum C_j(h)q(h)}{nI\alpha_j}\right]^{1/T} - 1 \qquad (3)$$

[4] Here and elsewhere the expected return could be calculated as a probability-weighted average of the return from each outcome. However, the approach we use seems to make more sense as the return when the whole of the initial investment is lost is minus 100% per year with annual compounding or minus infinity with continuous compounding. This has an unreasonably large effect on the resulting overall expected return when a probability-weighted return calculation is used. Indeed, even a small probability of losing the initial principal would give a continuously compounded expected return of minus infinity for that calculation.

4. Base Case

The base case is that considered in Figure 2: n = 150, T = 10, I = 200, p = 0.05, S= 10,000, β_1 = 0.4, β_2 = 0.3, β_3 = 0.3, α_1=0.05, α_2=0.08. The projects are assumed to be independent and there is no uncertainty about the final payoff.

The expected final cash flow from each project in the base case is 0.05 times $10 billion or $500 million. Because the initial investment is $200 million, the expected return from each project with annual compounding is

$$\left(\frac{500}{200}\right)^{1/10} - 1 = 9.60\%$$

Table 1 shows the risks being taken by each tranche and the expected return. For a tranche to be rated AAA the probability of loss over ten years must be about 0.5% or better; for a tranche to be rated BBB the probability of loss over ten years must be about 5%. Table 1 therefore shows that in the base case the senior tranche would be rated AAA and the mezzanine tranche would be rated BBB.

5. Impact of Parameters

In this section we consider the impact of making changes to the parameters defining our base case.

5.1 Number of Projects

To test the effect of the number of projects on the risks being taken by tranche holders we (a) doubled the size of the megafund increasing the number of projects from 150 to 300 and (b) halved the size of the megafund decreasing the number of projects from 150 to 75. All other aspects of the structure remained the same. The results are shown in Table 2. As the number of projects is increased, a "law of large numbers" comes into play. For example, with 75 projects the probably that the number of hits will be between 4% and 6% is 40.9%; with 300 projects, this rises to 64.7%; with 1000 projects it rises to 87.3%.

Tables 1 and 2 shows that, when there are 75 rather than 150 projects, the expected return for the senior and mezzanine tranches decrease and the expected return for the equity tranche

increases. In this case, a AAA-rated tranche cannot be constructed as there is a 2.13% chance of no hits and therefore a loss. When there are 300 rather than 150 projects, the expected return for the senior and mezzanine tranches increase and the expected return for the equity tranche decreases. In this case, the size of a AAA-rated senior tranche could be increased from 40% to about 90%.

5.2 Probability of Success

In the base case p is 5%. We tested the effect of changing p and S while keeping $pS = 500$, as in the base case, so that the expected return from a project stayed at 9.60%. The other parameters describing the base case were not changed. The results for p equal to 2.5% and 10% are shown in Table 3. As the probability of success increases, the expected return to the senior and mezzanine tranches increase and that to the equity tranche decreases. When the probability of success is 2.5%, a AAA-rated tranche cannot be constructed as there is a 2.24% probability of no hits and a therefore a loss. When the probability of success is 10%, the size of the AAA tranche could be increased to over 90%.

5.3 Life of Project

In the base case, T=10. We tested the effect of changing T and S so that the expected return on a project stayed at 9.6%. This meant that S is $10{,}000\times 1.096^{T/10}$. The results are summarized in Table 4. A further analysis shows that the size of the AAA tranche when the life is 5 years must be reduced to about 20%. But when it is 20 years, the size of the AAA-rated tranche can be increased to over 90%.

It is perhaps a little surprising that the senior and mezzanine tranches become safer, and their returns increase, as the life of the project increases. The reason is that we are assuming that the promised returns are independent of the project life. A return of 9.6% on a low principal will eventually dominate a return of 5% or 8% on a higher principal as life is increased. Hence, even one hit is capable of satisfying the senior tranche and the mezzanine tranche if the projects last sufficiently long.

5.4 Success Correlation

So far we have assumed that project successes are independent events. We will use the term "success correlation" to describe the extent to which successes cluster. To test the impact of success correlation we used a one-factor Gaussian copula model. Defining ρ as the correlation parameter in the Gaussian copula model, the probability of success is

$$p(F) = N\left(\frac{N^{-1}(p) - \sqrt{\rho}F}{\sqrt{1-\rho}} \right)$$

where N is the cumulative normal distribution function and the factor, F, is a random sample from a standard normal distribution. Gaussian quadrature was used to generate weights w_k and factor values F_k ($1 \leq k \leq m$). For a given factor value, F, we can calculate the probability of success and use the formulas in Section 4 to calculate $P_j(F)$, the probability of the promised return being achieved for tranche j; $Q_j(F)$ the probability of a loss of principal for tranche j; and $C_j(F)$ the expected cash flow for tranche j. The unconditional values of these are calculated as

$$\sum_{k=1}^{m} w_k P_j(F_k) \qquad \sum_{k=1}^{m} w_k Q_j(F_k) \qquad \sum_{k=1}^{m} w_k C_j(F_k)$$

and the expected return to tranches is calculated using equation (3)

The results for correlation parameters of 0.1 and 0.2 are shown in Table 5. The riskiness of the senior and mezzanine tranches increases as the correlation increases. As the Gaussian copula correlation increases from 0 to 0.1 to 0.2 , the probability of promised return being achieved for the senior tranche decreases from 99.59% to 90.44% to 78.93%. Similarly the probability of the promised return being achieved by the mezzanine tranche decreases from 94.52% to 72.99% to 60.09%. The probability that none of the projects will succeed increases from 0.05% to 3.22% to 10.12%. It can be shown that, for a AAA-rate tranches to be viable, the correlation parameter must be quite low, about 0.03 or less.

5.5 Uncertain Payoffs

Up to now we have assumed that the payoff from a successful project is known with certainty ($10 billion in the base case.) We now relax this assumption by assuming that the cash inflows for successful

projects have a multivariate normal distribution.[5] Define the mean and standard deviation of the cash flow at time T from a successful project as μ and σ, respectively. We assume that the payoffs from different projects are independent. (This assumption is relaxed in Section 5.6.)When there are h hits the total final cash flow is normal with mean μ_h and standard deviation σ_h where

$$\begin{aligned} \mu_h &= \mu h \\ \sigma_h &= \sigma\sqrt{h} \end{aligned} \tag{3}$$

The probability that tranche j will receive its promised return when the number of hits is h is the probability that the final total cash flow is greater than K_j (j=1,2), where the K's are defines as in equation (1). This is

$$N\left(\frac{\mu_h - K_j}{\sigma_h}\right)$$

so that

$$P_j = \sum_{h=0}^{n} q(h) N\left(\frac{\mu_h - K_j}{\sigma_h}\right)$$

The probability that tranche j will lose some principal when the number of hits is h is the probability that the final total cash flow is less than L_j (j=1,2,3), where the L's are defined as in equation (2). This is

$$N\left(\frac{L_j - \mu_h}{\sigma_h}\right)$$

so that

$$Q_j = \sum_{h=0}^{n} q(h) N\left(\frac{L_j - \mu_h}{\sigma_h}\right)$$

Define $E_j(h)$ as the expected cash flow to tranche j when the number of hits is h (j=1,2,3). This can be calculated analytically:

[5] This is a convenient assumption. Other probability distribution can be used in conjunctions with copulas and Monte Carlo simulation.

$$E_1(h) = \left(\mu_h - K_1\right) N\left(\frac{K_1 - \mu_h}{\sigma_h}\right) - \frac{\sigma_h}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{K_1-\mu_h}{\sigma_h}\right)^2} + K_1$$

$$E_2(h) = \left(\mu_h - K_2\right) N\left(\frac{K_2 - \mu_h}{\sigma_h}\right) - \frac{\sigma_h}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{K_2-\mu_h}{\sigma_h}\right)^2} + K_2 - E_1(h)$$

$$E_3(h) = \mu_h - E_1(h) - E_2(h)$$

It follows that

$$R_j = \left[\frac{\sum_{h=0}^{n} E_j(h)q(h)}{nI\alpha_j}\right]^{1/T} - 1$$

Table 6 shows how the performance of the tranches in our base case is affected when the mean payoff is the same as the payoff in the base case (μ=10,000) and the standard deviation, σ, is 25% and 50% of the mean. The table shows that the impact of an uncertain payoff is relatively small and should not affect the viability of the structure in Figure 2 to any great extent.

5.6 Payoff Correlation

In practice the size of the payoff from successful projects will be affected by economic conditions. Some correlation between the payoffs from different successful projects can therefore be anticipated. Define ρ_c as the correlation between the payoff from any two successful projects. Equation (3) becomes:

$$\mu_h = \mu h$$
$$\sigma_h = \sigma\sqrt{h + \rho_c h(h-1)}$$

The rest of the analysis given in Section 5.5 is unchanged.

Table 7 shows the impact of a payoff correlations of 0.25 and 0.5 for the base case when the standard deviation of the payoff is 25% of it mean. The table indicates that payoff correlation reduces the attractiveness of the senior and mezzanine tranches, but only by a relatively small amount.

6. Conclusions

The viability of the type of structure we have proposed depends critically on the number of projects, the success probability, and the success correlation. The maximum width of the AAA-rate tranche increases as the number of projects increases and the success probability increases. It decreases quite fast as the dependence between success probabilities increases.

In an ABS, it is difficult to choose debt instruments that have zero default correlation because default rates tend to depend on economic conditions. However, it may be easier to achieve a near-zero success correlation for the type of projects we are considering here. Consider a fund consisting of 150 clean energy projects. If the projects are chosen carefully, so that the approaches taken by the researchers are quite different, the success correlation should be close to zero.

Ideally, different types of projects should be included in the same portfolio to minimize the success correlation. For example, rather than focussing on a cure for cancer or source of clean energy, a portfolio could consist of a range of projects concerned with different biomedical challenges and different environmental challenges. Of course, this is likely to add to the administrative overhead associated with the creation of the fund.

Success correlation should be distinguished from payoff correlation. In practice, the payoffs from successful projects are likely to depend on economic conditions. There are therefore likely to be correlations between the payoffs from any two different successful projects. Our results indicate that the impact of payoff correlation on the attractiveness of the proposed structure is not as serious as the impact of success correlation. Even when payoff correlation is incorporated into the analysis it should be possible to construct a structure that is reasonably attractive to investors.

Finally we note that, as pointed out by Stein (2016), insurance companies and pension plans may be natural buyers for the tranches created from projects to develop new drugs because they have longevity risk exposure and average life expectancy can be expected to be correlated with the number of successful drugs developed. Also, as pointed out in Fagnan et al (2013), the

government may find that an efficient way of funding certain types of research is to guarantee the senior tranches of a megafund.

References

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Figure 1

A Five-Year Asset-Backed Security

Structure lasts for 5 years. Each loan pays interest at 6.5% and has a 2% chance of defaulting. The recovery rate is 20%

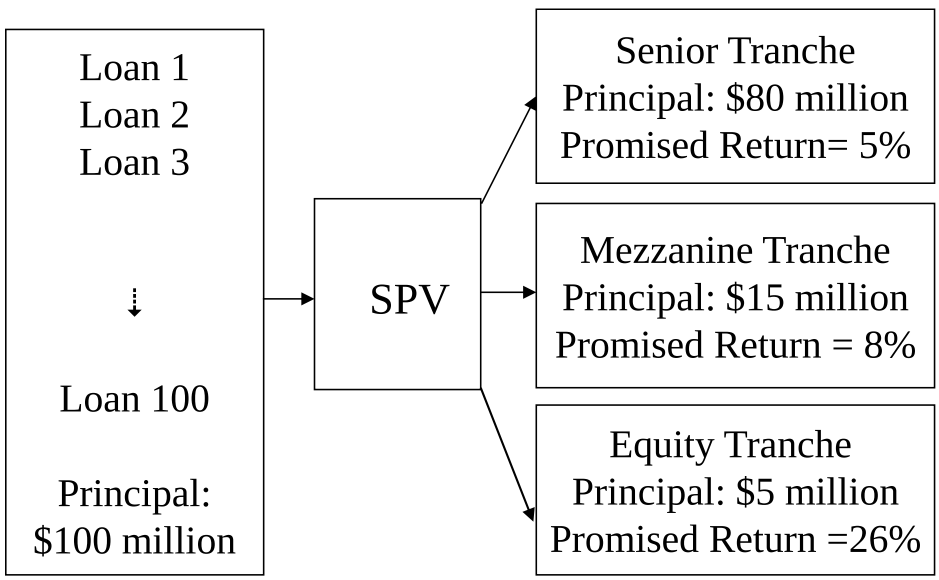

Figure 2

Megafund Example

Structure lasts for 10 years. Each project costs $200 million and has a 95% chance of providing no cash inflows and a 5% chance of providing a cash inflow of $10 billion at the end of the 10 years

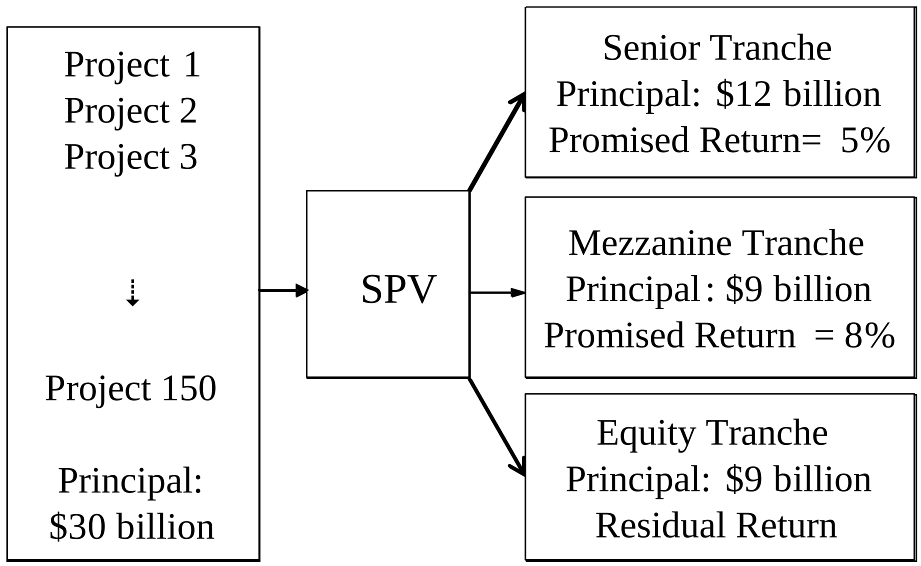

Table 1

Risks and Expected Returns for Base Case

Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
Senior	5%	99.59%	4.98%	0.41%
Mezzanine	8%	94.52%	7.62%	1.82%
Equity			15.10%	12.56%

Table 2

Risks and Expected Returns as Number of Projects is Changed

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	97.87%	4.77%	2.13%
75 Projects	Mezzanine	8%	89.44%	6.82%	10.56%
	Equity			15.63%	26.97%
	Senior	5%	99.98%	5.00%	0.00%
300 Projects	Mezzanine	8%	98.40%	7.94%	0.23%
	Equity			14.91%	6.50%

Table 3

Risks and expected returns as probability of success is changed.
The cash flow in the event of success is also changed so that the expected return remains 9.6%

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	97.76%	4.76%	2.24%
Success prob=2.5%	Mezzanine	8%	89.13%	6.79%	10.87%
	Equity			15.65%	27.34%
	Senior	5%	99.99%	5.00%	0.00%
Success Prob =10%	Mezzanine	8%	98.60%	7.95%	0.19%
	Equity			14.91%	6.00%

Table 4

Risks and expected returns as project life is changed.
The cash flow in the event of success is also changed so that the expected return remains 9.6%

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	98.18%	4.89%	0.41%
Life =5 years	Mezzanine	8%	87.44%	6.63%	5.48%
	Equity			17.07%	23.44%
	Senior	5%	99.59%	4.99%	0.05%
Life=20 years	Mezzanine	8%	98.18%	7.93%	0.41%
	Equity			13.55%	5.48%

Table 5

Impact of a correlation between the probabilities of success
ρ is the correlation in a one-factor Gaussian copula model

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	90.44%	4.32%	9.56%
ρ =0.1	Mezzanine	8%	72.99%	5.36%	17.86%
	Equity			16.52%	36.19%
	Senior	5%	78.93%	3.25%	21.07%
ρ=0.2	Mezzanine	8%	60.09%	3.46%	31.10%
	Equity			17.68%	47.54%

Table 6

Impact of Uncertainty about Payoff from a Successful Project
SD is the standard deviation of a project's cash inflow as a percent of its mean cash inflow

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	98.93%	4.97%	0.35%
SD = 25%	Mezzanine	8%	91.36%	7.56%	3.23%
	Equity			15.14%	16.42%
	Senior	5%	98.33%	4.94%	0.55%
SD = 50%	Mezzanine	8%	89.29%	7.41%	4.52%
	Equity			15.23%	18.82%

Table 7

Impact of Payoff Correlation when standard deviation of return is 25% of mean return

	Tranche	Promised Return	Probability of Promised Return	Expected Return	Probability of Any Loss of Principal
	Senior	5%	98.86%	4.97%	0.36%
Corr = 25%	Mezzanine	8%	90.52%	7.52%	3.57%
	Equity			15.16%	17.73%
	Senior	5%	98.74%	4.96%	0.38%
Corr = 50%	Mezzanine	8%	89.55%	7.46%	4.00%
	Equity			15.20%	19.07%