

Modeling, Model Order Reduction and Simulation of High Pass Filter Using Bond Graph Method

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Abstract: The traditional modeling and simulation techniques for dynamic systems are generally adequate for single-domain systems only, but the Bond Graph technique provides new strategies for reliable solutions of multi-domain system. They are also used for analyzing linear and non linear dynamic production system, artificial intelligence, image processing, robotics and industrial automation. In the first phase, the system is modeled using Bond Graph technique. In the second phase an approach is developed towards model order reduction. The suggested algorithm and other known modern model order reduction techniques are applied to a 11th order high pass filter. The order of the model was reduced using four methods, and maximum absolute errors were determined as a function of model order. The methods included the bond graph approach developed, balanced model reduction, the Hankel norm approximation and Schmidt norm model approximation with prescribed eigen values. The model order reduction technique developed in this paper has two principal advantages. Firstly the model order reduction process indicates which system components have the most bearing on the frequency response, secondly the final model retains structural information.

Keywords: Bond graph modeling, multi-domination system, model order reduction, frequency range of interest, balanced realization, Schmidt technique, Hankle norm and mechatronic systems simulation.

1. INTRODUCTION

Bond Graph Modeling Technique

Bond graph is an explicit graphical tool for capturing the common energy structure of systems. It increases one's insight into systems behavior. In the vector form, they give concise description of complex systems. Moreover, the notations of causality provides a tool not only for formulation of system equations, but also for intuition based discussion of system behavior, viz. controllability, observability, fault diagnosis, etc. Multi-domain problem differs from conventional design of electronic circuits,

mechanical systems, and fluid power systems, in part because of the need to integrate several types of energy behavior as part of the basic design. Multi-domain design is difficult because such systems tend to be complex and most of the simulation tools operate over only a single domain. In order to automate design of multi-domain systems, a new approach is required. The goal of the work reported in this paper is to develop an automated procedure capable of designing multidomain systems to meet given performance specifications, subject to various constraints. The most difficult aspect of the research is to develop a method that can explore the design space in a topologically open-ended manner, yet find appropriate configuration efficiently enough to be useful.

Bond graph consist of elements and bonds. There are several types of elements, each of which performs analogous roles across energy domains. The first type-- C , I , and R elements are passive one-port elements that contain no sources of power, and represent capacitors, inductors, and resistors (in the electrical domain). The second type, S_e , and S_f , are active one-port elements that are sources of power, and that represent effort sources and flow sources, respectively (for example, sources of voltage or current, respectively, in the electrical domain). The third type, TF , and GY , are two port elements, and represent transformers and gyrators, respectively, power is conserved in these elements. A fourth type, denoted as 0 and 1 on bond graph, represents junctions, which are two (or multi-) port elements. They serve to interconnect other elements into subsystem models.

Domain	Effort, $e(t)$	Flow, $f(t)$
Mechanical translation	Force component, $F(t)$	Velocity component, $V(t)$
Mechanical rotation	Torque component $\tau(t)$	Angular velocity component, $\omega(t)$
Hydraulic	Pressure, $P(t)$	Volume flow rate, $Q(t)$
Electric	Voltage, $e(t)$	Current, $i(t)$

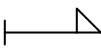
Table-1

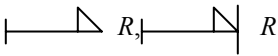
In the following table, effort and flow variables in some physical domains are listed

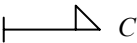
Systems	Effort (e)	Flow (f)
Mechanical	Force (F)	Velocity (v)
	Torque (τ)	Angular velocity $\omega(t)$
Electrical	Voltage (V)	Current (i)
Hydraulic	Pressure (P)	Volume flow rate (dQ/dt)
Thermal	Temperature (T)	Entropy change rate (ds/dt)
	Pressure (P)	Volume change rate (dV/dt)
Chemical	Chemical potential (μ)	Mole flow rate (dN/dt)
	Enthalpy (h)	Mass flow rate (dm/dt)
Magnetic	Magneto-motive force (e_m)	Magnetic flux (ϕ)

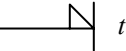
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S_e – element:

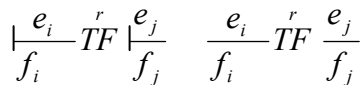
S_f – element: 

R – element: 

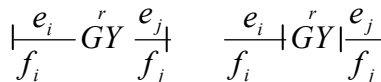
C – element: 

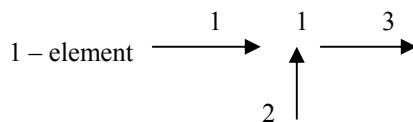
I – element: 

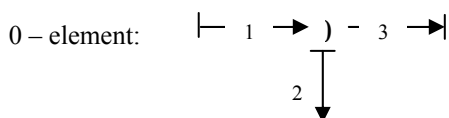
TF – element:



GY – element:



1 – element 

0 – element: 

Bond graph have three embedded strengths for design applications – the wide scope of systems that can be created because of the multi-and inter-domain nature of bond graphs, the efficiency of evaluation of design alternatives, and the natural combinational features of bond and node components for generating of design alternatives. First, multi-domain systems (electrical, mechanical, hydraulic, pneumatic, and thermal) can be modeled using a common notation, which is especially important for design of mechatronic system.

Model Order Reduction

The approximation of high-order plant and controller models by models of lower order is an integral part of control system design. Until relatively recently model reduction was often based on physical intuition. For example, chemical engineers often assume that mixing is instantaneous and that packed distillation columns may be modeled using discrete trays. Electrical engineers represent transmission lines and the eddy currents in the rotor cage of induction motors by lumped currents. Mechanical engineers remove high-frequency vibration modes form models of aircraft wings, turbine shafts and flexible structures. It may also be possible to replace high-order controllers by low-order approximations with little sacrifice in performance.

There are several procedures which seek to automate the model reduction process. Suppose a high-order, linear, time-invariant model G is given, then the Prototype H_∞ model reduction problem is to find a low-order approximation \hat{G} of G such that $\|G - \hat{G}\|_\infty$ is small. Consider the more difficult problem of selecting \hat{G} such that the problem of selecting W_1 and W_2 such that $\|W_2(G - \hat{G})W_1\|_\infty$ is small; the weighting function W_1 and W_2 are used to frequency shape the model reduction error. For example, one might select the weights so that the modeling error is small in the unity gain range of frequency. Let us discuss some model order reduction techniques used here:

State space truncation

Consider a linear, time-invariant system with the realization

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0$$

$$y(t) = Cx(t) + Du(t)$$

and divide the state vector x into components to be retained components to be discarded:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

The r -vector $x_1(t)$ contains the components to be retained, while the $(n - r)$ - vector x_2 contains the components to be discarded. Now partition the matrices A , B and C conformably with x to be obtain

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$C = [C_1 \quad C_2]$$

By omitting the states and dynamics associated with $x_2(t)$, we obtain the lower-order system

$$\begin{aligned} \dot{p}(t) &= A_{11p}(t) + B_{1u}(t) & p(0) &= p_0 \\ q(t) &= C_{1p}(t) + D_u(t). \end{aligned}$$

The r^{th} -order truncation of the realization (A, B, C, D) .

$$T_r(A, B, C, D) = (A_{11}, B_1, C_1, D)$$

In general very little can be said about the relationship between x and p , y and q or the transfer function matrices G and \hat{G} associated with (A, B, C, D) and (A_{11}, B_1, C_1, D) . In particular, the truncated system may be unstable even if the full-order system is stable, and the truncated system realization may be non minimal even if the full-order realization is minimal. One thing that clearly does hold is $\hat{G}(\infty) = G(\infty)$ which means that all reduced-order models obtained by truncation have perfect matching at infinite frequency.

Balanced realization

The aim of this section is to introduce balanced realizations, which are of interest because they have good absolute-error truncation properties. Suppose we are given a $G \in RH_\infty$ and our aim is to produce a reduced-order model $\hat{G} \in RH_\infty$ that approximates G . A natural criterion with which to measure the absolute error is $\|G - \hat{G}\|_\infty$. If we drive G and \hat{G} with the same input u , we get

$$y = Gu, \quad \hat{y} = \hat{G}u$$

and therefore that

$$\|G - \hat{G}\|_\infty = \sup_{u \in L_2} \frac{\|y - \hat{y}\|_2}{\|u\|_2}$$

If $G = C(sI - A)^{-1}B$, then

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

We assume that A is asymptotically stable (*i.e.*, $\text{Re} \lambda_i(A) < 0$). For $\|G - \hat{G}\|_\infty$ to be small we should delete those components of the state-vector x that are least involved in the energy transfer from the input u to the output y .

2. Model Order Reduction by Using bond Graph Method

The order of a typical full-order physical-derived model should be decreased by eliminating un-necessary state variables, provide that the removal of these state variable does not drastically increase the uncertainty associated with the model over a given frequency range of interest (FROI). In this manner a model of the appropriate order will be obtained. Various model order deduction algorithms attempt to achieve this goal by beginning with a low order approximate model and increasing the order until the model performance (eigenvalues or frequency response) no longer changes appreciable with further increases in model order. The starting point here, however, is a high order truth model. We begin with what is assumed to be a high order truth model for two reasons. First, in real application a designer could plausibly construct a model with all reasonable independent energy storage elements (each of which has an associated state variable) included and then wish to reduce the order of the model by retaining only the significant elements. Second truth model provides an absolute standard against which reduced order model can be checked.

The approach to model order reduction employed in this paper involves the following steps:

- Specifying a FROI,
- Testing the effect of removing energy storage elements on model performance over this FROI,
- Systematically eliminating those elements whose removal has little effect on model performance, and
- Continuing to remove energy storage elements until the model error becomes excessive.

3. Example of an Electrical system

An 11th Order High Pass Filter

A filter design problem was used as a test of over approach for evolving electrical circuit with bond graphs. The first phase we desired its bond graph using the theoretical background stated previously.

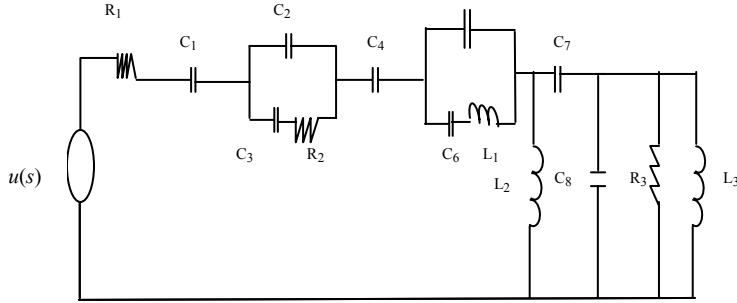
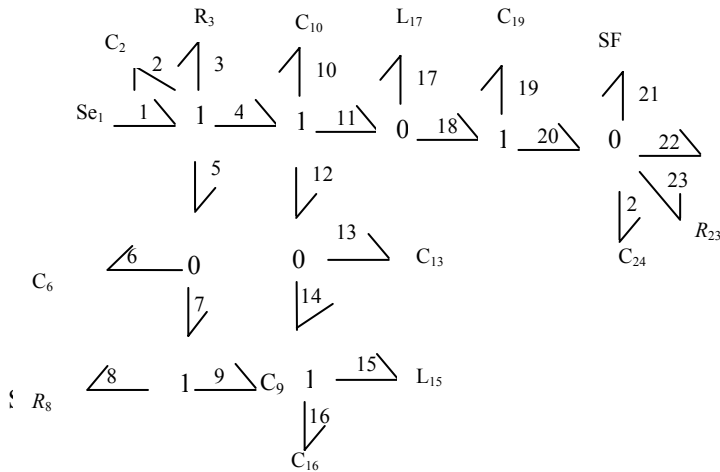


Figure 1

3. SYNTHESIS OF BOND GRAPH FOR AN ELECTRICAL SYSTEM



4. DERIVING STATE SPACE REPRESENTATION FROM BOND GRAPH

The energy variable are,

$q_{12}, q_6, q_9, q_{10}, q_{13}, q_{16}, q_{19}, q_{24}, p_{17}, p_{15}$ and p_{22}

The Co-energy variable are

$V_1(t), V_2, V_6, V_9, V_{10}, V_{13}, V_{16}, V_{19}, V_{24}, f_{15}, f_{17},$ and f_{22} .

Now

$$\dot{q}_2 = \frac{S_e}{R_3} - \frac{1}{C_2 R_3} q_2 - \frac{1}{C_{10} R_3} q_{10} - \frac{1}{C_{19} R_3} q_{19} - \frac{1}{C_{24} R_3} q_{24} - \frac{1}{C_{13} R_3} q_{13} - \frac{1}{C_6 R_3} q_6$$

2nd State

$$\dot{q}_6 = \frac{S_e}{R_3} - \frac{1}{C_2 R_3} q_2 - \frac{1}{C_{10} R_3} q_{10} - \frac{1}{C_{19} R_3} q_{19} - \frac{1}{C_{24} R_3} q_{24} - \frac{1}{C_{13} R_3} q_{13} - \frac{1}{C_6 R_3} q_6 - \frac{1}{C_9 R_8} q_9$$

3rd State

$$\dot{q}_9 = \frac{1}{C_6 R_8} q_6 - \frac{1}{C_9 R_8} q_9 \quad (3)$$

4th state

$$\dot{q}_{10} = q_2 \quad (4)$$

5th state

$$\dot{q}_2 = \frac{S_e}{R_3} - \frac{1}{C_2 R_3} q_2 - \frac{1}{C_{10} R_3} q_{10} - \frac{1}{C_{19} R_3} q_{19} - \frac{1}{C_{24} R_3} q_{24} - \frac{1}{C_{13} R_3} q_{13} - \frac{1}{C_6 R_3} q_6 - \frac{1}{L_{17}} p_{17}$$

6th state

$$\dot{q}_2 = \frac{S_e}{R_3} - \frac{1}{C_2 R_3} q_2 - \frac{1}{C_{10} R_3} q_{10} - \frac{1}{C_{19} R_3} q_{19} - \frac{1}{C_{24} R_3} q_{24} - \frac{1}{C_{13} R_3} q_{13} - \frac{1}{C_6 R_3} q_6 - \frac{1}{L_{15}} p_{15}$$

7th state

$$\dot{q}_{16} = \frac{1}{L_{15}} p_{15}$$

8th state

$$\dot{q}_2 = \frac{S_e}{R_3} - \frac{1}{C_2 R_3} q_2 - \frac{1}{C_{10} R_3} q_{10} - \frac{1}{C_{19} R_3} q_{19} - \frac{1}{C_{24} R_3} q_{24} - \frac{1}{C_{13} R_3} q_{13} - \frac{1}{C_6 R_3} q_6 - \frac{1}{L_{17}} p_{17} - SF_{21} - \frac{1}{L_{22}} p_{22} - \frac{1}{C_{24} R_{23}} q_{24} \quad (8)$$

9th state

$$p_{15} = \frac{1}{C_{13}} q_{13} = \frac{1}{C_{16}} q_6 \quad (9)$$

10th state

$$p_{17} = \frac{1}{C_{19}} q_{19} - \frac{1}{C_{24}} q_{24} \quad (10)$$

11th state

$$p_{22} = \frac{1}{C_{24}} q_{24} \quad (11)$$

Hence the state equations are derived from the bond graph.

5. MODEL-ORDER REDUCTION COMPARISON

The order of the model was reduced using four methods, and maximum absolute errors were determined as a function of model order. The methods included the bond graph approach developed here, Balanced Model Reduction, the Hankel Norm approximation and Schmidt Model approximation with prescribed eigenvalues.

However, instead of terminating the reduction process when the error became large, reduction continued until the order was the minimum possible. The absolute error between the original and reduced order models, for all four methods are shown below:

Absolute Error					Component Removed
Order	Balanced	Hankel Norm	Schmidt	Bond Graph	
11	0	0	0	0	0
10	1.6713e-013	1.3556e-009	1.7959e-010	8.2747e-018	C2
9	1.7153e-013	2.4386e-008	1.7959e-010	8.2747e-018	C6
8	1.9599e-013	2.4386e-008	1.7959e-010	1.3660e-009	C13
7	4.9048e-011	1.2828e-009	1.796e-010	3.9802e-005	C13, L17
6		2.4386e-008	2.5026e-008	1.660e-009	C6, C13
5		9.7993e-009	2.3413e-008	1.366e-009	C2, C6, C13, L15
4		9.7993e-009	2.3413e-008	2.2816e-009	C2, C6, C13, L15, L22
3		8.2048e-006	8.1565e-006		
2		4.0157e-004	4.0155e-004		

Table 3

Bond graph based model reduction is performed by selectively eliminating components from the model.

6. RESULT AND CONCLUSIONS

In this research work a new Modeling Approach using Bond Graph Method is applied. It suggested a new design methodology for automatically synthesizing design for multi-domain, lumped parameter dynamic systems, assembled from mixtures of electrical, mechanical, hydraulic, pneumatic and thermal components.

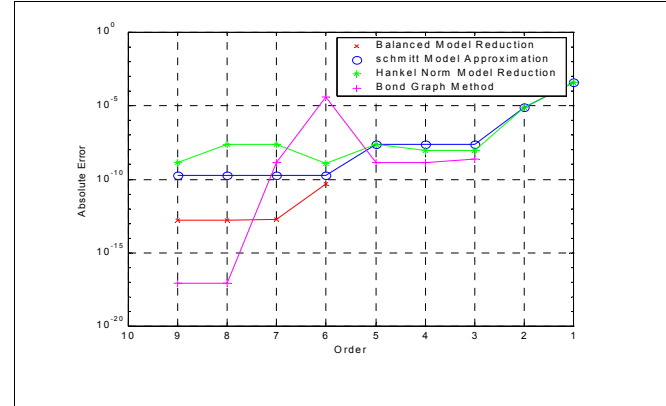


Figure 2: Absolute error as a function of model order

The second conclusion leads toward model order reduction. As the error plot indicates, the discrepancy between the original and reduced order models is smallest when using the balanced model reduction method, followed by the Hankel norm approximation, the Schmidt model approximation, and the elimination of physical elements from the model.

The ordering of errors thus obtained is expected. Balanced model reduction, the Hankel norm approximation, and the Schmidt model approximation all seek to minimize the absolute error between the original and ROM; the only constraint on this minimization is that the resulting model must be RH_∞ . In contrast to the mathematically-derived models, with the Bond Graph method, the elimination of physical elements from the model constrains the reduced order model to use state variables and coefficients from the *original* full order model. While we will not use this example to make any definitive claims regarding the efficacy of various order reduction methods, several comments are in order. The example illustrates that the errors resulting from the approach presented here, while generally larger than those encountered from purely mathematical approaches, are not so large as to render the reduced model unusable. The example also indicates that for the Bond Graph method developed here, model error increases monotonically with the amount by which the order has been reduced. In contrast, both balanced model reduction and Schmidt model approximation result in non-monotonic behavior for the model error. This suggests another advantage of the new approach, once the model errors have become too large, a user of the algorithm knows there is no point in reducing the order further. The error in Figure 2. indicate only the maximum error over the entire FROI. For the example here, the Bond Graph method actually produced lower errors than the other methods across the majority of the FROI. The relatively large errors indicate in table 3

result from model errors that are largely confined to a narrow frequency band within the FROI.

The Model Order Reduction Technique developed in this Thesis has two principal advantages:

1. The Model Order Reduction process indicates which system components have the most bearing on the frequency response.
2. The final model retains structural information.

The first of these features provides a designer with insight in the system behaviour. This indicates to a designer that such elements have little bearing on the response within a FROI.

In contrast to other model reduction approaches, which only provide unique input output information this approach allows a designer to analyze all the remaining state variables and combination thereof.

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