Calculation and Visualization of the Dynamic Ability of the Human Body

By Taku Komura*, Yoshihisa Shinagawa and Tosiyasu L. Kunii

There is a great demand for data on the mobility and strength capability of the human body in many areas, such as ergonomics, medical engineering, biomechanical engineering, computer graphics (CG) and virtual reality (VR). This paper proposes a new method that enables the calculation of the maximal force exertable and acceleration performable by a human body during arbitrary motion. A musculoskeletal model of the legs is used for the calculation. Using our algorithm, it is possible to evaluate whether a given posture or motion is a feasible one. A tool to visualize the calculated maximal feasibility of each posture is developed. The obtained results can be used as criteria of manipulability or strength capability of the human body, important in ergonomics and human animation. Since our model is muscle-based, it is possible to simulate and visualize biomechanical effects such as fatigue and muscle training. The solution is based on linear programming and the results can be obtained in real time. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: muscle-based model; human strength calculation; human strength visualization

Introduction

There is a great demand for information on the dynamic ability of the human body in various areas. In ergonomics such data are used to design various devices and tools for daily use. In medicine they are used for the design of rehabilitation tools. Also in industries such data are necessary to know in what kind of environment a human can work safely and comfortably without harming the body.

In computer graphics (CG) and virtual reality (VR) such data would be helpful to create realistic human animation. For example, they would help animators to make realistic postures used in key frame animation systems.

The structure of human bodies is quite different from that of robots. While robots are controlled by motors or engines which can exert a great amount of torque at the joints regardless of posture, the human body is driven by muscles attached to the bones. Since the forces exertable by the muscles depend on their length and contraction velocity, the human joints have limited ability to create joint torques dependent on posture and motion.

In this way the muscles characterize human body motion. The configuration space of a body is determined by the ligaments, tendons and passive elements of the muscles. Muscles crossing more than two joints (biarticular muscles) can simultaneously generate torque at a number of joints. For example, the hamstrings can extend the hip joint and flex the knee joint at the same time. For this reason there are dependences among the torques exertable by the joints. It is known that such muscles play important roles in motions such as jumping and gait. As a result, animation of the body becomes unnatural if the dynamics of the muscles is ignored. It is always necessary to check whether the motion of the human body model is actually realizable in dynamic environments by a real human body.

Musculoskeletal human body models have been created by many researchers in biomechanics (see Reference 8 for a review of musculoskeletal systems). Such models have been used to simulate body motion in dynamic environments, to check the influence of surgical operations or to analyse how the muscles are controlled to yield human motion.

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In CG, even though some researchers use muscle models to obtain a realistic rendering of human or animal bodies, musculoskeletal models have rarely been utilized to yield human motion data, with a few exceptions. Chen and Zeltzer created a very precise muscle model using the finite element method (FEM), but this has not been used for the control of human body models in a dynamic environment.

In this paper, using the musculoskeletal human body model, a method to calculate the maximal torque and acceleration of the lower extremities is proposed. First the user has to specify the posture of the human body, and next the direction of the torque or acceleration vector in configuration space. Then the maximal norm for this torque or acceleration is calculated. It is also possible to calculate the upper limit of force and acceleration exerted by the end effector of the body in the direction specified by the user in Cartesian space. Since the algorithm is based on linear programming, the calculation is accomplished in real time. The results of the calculation also contain the amount of force exerted by each muscle. Therefore it is possible to know which muscles are mainly used for creating motion specified by the user. The results are visualized using rods and polygons. It is also possible to take into account the effect of muscle fatigue or weight training.

The composition of the paper is as follows. Section 2 describes the background and previous work. Section 3 includes the explanation of the body model used in this paper. In Section 4 the properties of the Hill-based muscle model are explained. In Sections 5–8 the algorithms to calculate and visualize the maximum ability by the human body model are explained. In Section 9 the fatigue and recovery model of muscles proposed by Giat et al. is used to visualize the effect of fatigue of the muscles. In Section 10 an example of using our algorithm for the creation of human motion is shown.

**Related Work**

A large number of data on human mobility and strength capability have been collected by researchers in ergonomics, medicine and biomechanics. Fothergill et al. have collected one-handed maximum strength data in all directions in the fore and aft plane. Rühmann and Schmidtke have investigated the isometric maximal force by the arms under nine kinds of manipulation from people working in industries. However, since such data were collected under specific conditions, they do not necessarily represent the real mobility or capability of the human body under other conditions in daily life.

Wei and Badler have developed software that visualizes human strength using coloured 3D rods. The rods have been used to display either the joint torque or the force exertable by the end effector. The data on strength were obtained from a database system and it was not possible to calculate the torque or force for arbitrary motion.

Yoshikawa proposed the idea of manipulability of robot manipulators. The manipulability was visualized using ellipsoids, defined as manipulability ellipsoids. This method offers a good way to evaluate the posture of robot manipulators avoiding singular postures.

There has been no attempt, however, to calculate the maximum strength and mobility of human bodies by anatomy-based human models for use in either ergonomics or computer graphics. The advantages of using such a model for the calculation of maximum capability are as follows.

- Calculations for arbitrary posture are possible.
- Calculations under various conditions—such as when external forces are applied to the body, when the forces exertable by the muscles have declined because of fatigue, or when the forces exertable have increased after a long term of muscle training—are possible.

**Musculoskeletal Model**

For the musculoskeletal human body model the data published by Delp and co-workers have been used. The data include the attachment sites of 43 muscles on each leg and physiological parameters such as the length of tendons, range of joint angles, etc. In their research, some basic properties such as the passive joint torques and maximum isometric joint torques were calculated and compared with biomechanically measured data to evaluate the validity of the model. Other necessary data, such as the mass and inertia of the body segments were obtained from Reference 21 (see Appendix 2 for data).

The lower half of the human body model is composed of 13 segments. The pelvis is at the top and each leg is composed of the femur, tibia, patella, talus, calcaneus and toes. In this research the talus, calcaneus and toes are stuck together to form a rigid foot. The joints of the legs are assumed here either as a three degree-of-freedom (3-DOF) gimbal joint (hip joint) or as a 1-DOF pin joint (knee and
The total number of DOFs is therefore 10. Four variables, $\theta_{h\text{-flex}}$, $\theta_{h\text{-add}}$, $\theta_{h\text{-rot}}$ and $\theta_{knee}$ are defined here for hip flexion, hip adduction, hip external rotation and knee flexion of the right leg as shown in Figure 1. The frontal and dorsal views of the body model with muscles are shown in Figure 2. The muscles are rendered using generalized cylinders. The radii of the generalized cylinders are calculated using the length of the muscle and the physical cross-section area (PCSA). The PCSA was calculated using the peak isometric force data included in Reference 20 and the scale factor of 25 N cm$^{-2}$ from Reference 22.

**Hill-based Muscle Model**

For each musculotendon a model based on Hill’s three-component-model (Figure 3 is used. There are numerous muscle models which are derived from Hill’s model. Since
a discussion of these models is outside the scope of this paper, interested readers may refer to the literature, e.g. Reference 23. The model used here is that from References 6 and 20. It is composed of three elements: the contractile element (CE, muscle fibres), the series elastic element (SEE, muscle tendon) and the parallel elastic element (PEE, connective tissue around fibres and fibre bundles). There is an inclination between the muscle part (CE and PEE) and the tendon part (SEE). The angle between them is called the pennation angle of the muscle and is denoted here. The force exerted by the CE (\(f^{ce}\)) is a function of its length \(l^{ce}\), contraction velocity \(v^{ce}\) and the muscle activation level \(a\) which is controlled by the central nervous system (CNS).

\[
f^{ce} = f(l^{ce}, v^{ce}, a)
\]  

(1)

The activation level is not actually controlled directly by the CNS, but through the neural control signals \(u\). The relationship between the neural signals and the muscle activation level is often modelled by the differential equation

\[
da = (1-a)u + \frac{(a_{\text{min}}-a)(1-u)}{\tau_{\text{rise}}}
\]  

(2)

where \(0 \leq u \leq 1\), \(\tau_{\text{rise}}\) and \(\tau_{\text{fall}}\) are the rise and decay time constants for muscle activation, and \(a_{\text{min}}\) is the lower bound of the muscle activation level. This relationship is omitted in this research, because only static or smooth motion is considered. Therefore it is assumed that the activation level can take an arbitrary value between zero and 1 during the motion. The muscle tendon length \(l^{mt}\) is the sum of the muscle fibre length and the tendon length:

\[
l^{mt} = l^{m} \cos \alpha + l^{t}
\]  

(3)

\[
l^{m} = l^{ce}
\]  

(4)

\[
l^{t} = l^{pe}
\]  

(5)

\[l^{mt}\] and \(s^{mt}\) can be calculated by the joint angles and angular velocities of each leg. The force exerted by the tendon part (SEE) is controlled by the central nervous system (CNS), but through the neural control signals \(u\). The relationship between the tendon strain and the force exerted by the tendon part (SEE) is considered. Therefore it is assumed that the tendon part (SEE) is omitted in this research, because only static or smooth motion is considered. The relationship between the tendon strain and the force exerted by the tendon part (SEE) is shown in Figure 4. The curve is defined here as \(f^{t} = f(l^{t})\).

\[
f^{t} = f(l^{t})
\]  

(6)

\[
f^{pe} = f(l^{pe})
\]  

(7)

The relationship between the force exerted at each element is

\[f^{e} = (f^{ce} + f^{pe}) \cos \alpha
\]  

(8)

The tendon is a passive element that exerts an elastic force \(f^{t}\) only when its length is longer than the slack length \(l^{t}\). The relationship between the tendon strain and the normalized elastic force is shown in Figure 4. The curve is defined here as \(f^{pe} = f(l^{pe})\).

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\]  

(7)
Let us now consider how to calculate the maximal and minimal amounts of force that can be exerted by the musculotendon when the length $l_{mt}$ is given. In order to solve this problem, first $v^m$ must be calculated. If the motion is a static one, the length of each element stays the same and therefore $v^m = 0$. However, if the legs are going through some motion, $v^m$ should be approximated by the finite difference

$$v^m = \frac{l^m - l^m_{prev}}{\Delta t}$$

where $l^m_{prev}$ is the length of $l^m$ at the previous time step and $\Delta t$ is the length of the time step. By substitution of equations (9)–(11) into equation (8) together with (3)–(5) and (12) the following equation is derived:

$$f_o(l^m - l^m \cos a - l^o \dot{l}^o, l^m) - [f_o(l^m/l^m^o) \cdot g^{uv}(l^m - l^m_{prev})/\Delta t \cdot a + f_o(l^m/l^m^o) \cos a] = 0$$

As the muscle activation level $a$ is specified and the musculotendon length $l_{mt}$ is determined, the only unknown variable in equation (13) is $l^m$, which can be solved numerically from this equation. The maximum and minimum musculotendon forces $f^{max}$ and $f^{min}$ can be calculated by setting $a$ to either one or zero. Then the musculotendon force $f^t$ is limited by

$$f^{min}(a = 0) \leq f^t \leq f^{max}(a = 1)$$

When the motion is not a static one, $l^m_{prev}$ must be known to calculate $v^m$ and finally $f^{min}$ and $f^{max}$. Inverse dynamics is used to solve this problem. In the case of a dynamic motion the profile of the whole motion must be specified by the user, which means that the history of joint angles $\theta$, angular velocities $\dot{\theta}$ and angular accelerations $\ddot{\theta}$ since the beginning of the motion must be given. The joint torques can be calculated from $\theta, \dot{\theta}$ and $\ddot{\theta}$ using inverse dynamics. The muscle forces are predicted by decomposing the joint torques by an optimization method which takes into account $f^M$ or $l^m$ and $l^m_{prev}$ (see Appendix 1). When the muscle force $f^t$ is obtained, $l^m$ can be calculated from equations (3)–(5) and (8)–(11). Using this $l^m$ as $l^m_{prev}$ in equation (12) at the next stage, $f^{min}$ and $f^{max}$ can be calculated.

### Calculation of Maximal Moment

In this section the algorithms to calculate the maximum joint moment are presented.
by one joint and
by a group of joints or all the joints for a ratio specified
by the user
are explained.

Maximal Moment at Each Joint

The algorithm to calculate the maximal joint torque
around an axis specified by the user is proposed here. In
the beginning the user must specify the state of each leg,
I.e. the angles, angular velocities and angular accelerations
of the joints (θ, ˙θ and ¨θ). The axis around which the user
expects the joint to maximize the moment must also be
specified. The problem is solved using linear program-
ing. As the kinematic values of the body are determined,
using equation (14), the forces of the muscles are limited
by

\[ f_{\text{min}} \leq f_i \leq f_{\text{max}} \quad (i = 1, 2, \ldots, n) \]  

where \( n \) is the number of muscles on each leg.

Each muscle is a line segment which starts from the
origin point on some body segment, passing through a
number of ‘via points’, crossing a number of joints and
finally arriving at the insertion point (Figure 7) on another
segment. The torque \( \tau \) generated by the muscles crossing
the joint can be written in the form:

\[ \tau = \sum_{i} r_i \times f_i \]  

where \( r_i \) and \( f_i \) are the moment arm and the force exerted
by muscle \( i \) respectively and ‘\( \times \)’ is the outer product
operator. To calculate the maximum moment of the joint
around some axis \( a \), the following scalar \( f \) must be
maximized:

\[ f = a \cdot \tau \]  

\[ = a \cdot \left( \sum_{i} r_i \times f_i \right) \]  

\[ = a \cdot \left( \sum_{i} r_i \times f_i \cdot n_i \right) \]  

where \( n_i \) is \( f_i \)'s normal vector and ‘\( \cdot \)’ is the inner product
operator. Since the knee and the ankle have only one
DOF, \( a \) must be parallel to the joint axis. Maximizing \( f \)
with inequality constraints (15) is a linear programming
problem. The problem has been solved using the simplex
method. We have calculated the maximum moments of
the hip joint around a number of axes in various postures.
The maximum knee and ankle moments have also been
calculated. The result of the calculation is visualized using
rods or polygons. The direction of each rod indicates the
axis around which the moment is to be maximized, in a
right-handed co-ordinate system. The length of the rod
expresses the value of the maximum exertable moment
around that axis.

The maximum moment by the hip joint in a static phase
is shown using rods in Figures 8–10. They are drawn in a
coordinate system which is attached to the pelvis as
shown in Figure 8.

Figure 8 shows the maximum moments when the hip
angles are all 0° and the knee is at full extension (posture
1). The moments in the \( +x \) direction are larger than those
toward the \( -x \) direction. This means that more torque
can be generated at the hip when kicking backward than
forward. Another characteristic is that the \( +y \) and \( -y \)
components of moments are relatively small. This indica-
tes that the hip can exert small moments for internal and
external rotation.

Next, the hip joint is flexed 45° to the front with the
knee joint left extended (posture 2). The frontal view of
the maximal moment rods is shown in Figure 9(a). The
reason the rods toward the \( -x \) direction have a very
small maximal moment value is that when the hip is flexed
with the knee joint extended, the biarticular muscles that
cross both the hip and knee joint at the back of the thigh
(hamstring muscles) are fully stretched. Such muscles exert
a passive force that prevents further flexion of the hip
joint. As the knee is flexed 90° while the hip angles are
kept the same (posture 3), the maximal moment rods
change as shown in Figure 9(b). Since the hamstring
muscles are relaxed this time, the rods toward the \( -x \)
direction are longer than those in the previous posture. Therefore further flexion of the hip is possible. The hip is at 45° adduction in Figure 10 (posture 4). The maximal moment for abduction is greater in this configuration than in the first posture (Figure 8) because of the passive force of the antagonist muscles. The maximal moment values for

Figure 8. (a) Frontal and (b) lateral views of maximum moments around various axes when hip and knee are totally extended

Figure 9. Maximal moment rods when hip is flexed 45° with knee (a) fully extended and (b) flexed 90°
several axes at each posture are shown in Table 1. We rendered the maximal moment polygon by attaching quadratic patches to the tips of the rods. The maximal moment polygon from various viewpoints is shown in Figure 11.

Another example is shown in Figure 12, in which the leg is moving at different velocities. The leg is at the basic posture, with the hip joint angles set to 0° and the knee extended. The maximal moment polygon has different shapes, which is caused by the characteristics of muscle dynamics. In Figure 12(b) the hip joint is flexed at the rate of 1.0 rad s⁻¹. The angular accelerations of the joint angles are all zero at this moment. Since the muscles used to flex the hip joint are shortening, they can exert less force than when they are statically contracting (see Figure 6). Therefore the left side of the maximal moment polygon shrinks more than in the static phase (Figure 12(a)). On the other hand, the muscles for extension are lengthening at this moment, so they can exert a larger force than in the static phase. This is why the right side of the maximal moment polygon has more volume this time.

In Figure 12(c) the hip joint is extended backward at the rate of 1.0 rad s⁻¹, with the angular accelerations all zero. This time the left side of the maximal polygon has shrunk while the right side has expanded.

**Maximum Moment by Specifying Ratio of Joint Torques**

It is also possible to specify the ratio of the joint torques to be generated at all DOFs of the leg and to calculate the maximal amount of joint torque in this ratio:

\[
\tau_1 = a_1 \left( \sum_{i=1}^{m} r_i^1 \times f_i \right)
\]

\[
\tau_2 = a_2 \left( \sum_{i=1}^{m} r_i^2 \times f_i \right)
\]

\[
\vdots
\]

\[
\tau_m = a_m \left( \sum_{i=1}^{m} r_i^m \times f_i \right)
\]

where \(m\) is the total number of DOFs of one leg, which is five here, \(\tau_i\) is the intersegmental resultant moment at the

<table>
<thead>
<tr>
<th>Axis</th>
<th>Posture 1</th>
<th>Posture 2</th>
<th>Posture 3</th>
<th>Posture 4</th>
</tr>
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<td>224.793</td>
<td>172.332</td>
<td>82.273</td>
</tr>
<tr>
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<td>-89.432</td>
<td>94.437</td>
<td>48.187</td>
</tr>
<tr>
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<td>105.011</td>
<td>93.011</td>
<td>38.383</td>
</tr>
<tr>
<td>(0, -1, 0)</td>
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<td>36.74</td>
<td>54.3624</td>
<td>110.324</td>
</tr>
<tr>
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<td>137.633</td>
<td>116.94</td>
<td>109.84</td>
<td>203.413</td>
</tr>
<tr>
<td>(0, 0, -1)</td>
<td>103.745</td>
<td>86.1093</td>
<td>105.63</td>
<td>20.819</td>
</tr>
</tbody>
</table>

**Table 1. Maximal moment (N m) around main axes at various postures. At each posture, \((\theta_{h\text{-flex}}, \theta_{h\text{-add}}, \theta_{h\text{-rot}}, \theta_{knee})\) is (1) \((0, 0, 0, 0)\), (2) \((45, 0, 0, 0)\), (3) \((45, 0, 0, 90)\) and (4) \((0, 45, 0, 0)\)**

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In matrix form, equations (20)–(23) can be written as

$$\tau = Af$$  \hspace{1cm} (24)$$

where $\tau$ is the vector of torques at the joints, $f$ is the vector of muscle forces and $A$ is the matrix that converts muscle forces to joint torques. We define here the normal vector of $\tau$ as $N$ and its norm as $N$:

$$NN = \tau$$  \hspace{1cm} (25)$$

$$\|N\| = 1$$  \hspace{1cm} (26)$$

Then equation (24) can be rewritten as

$$NN = Af$$  \hspace{1cm} (27)$$

As a result, the calculation of maximum joint torques for a specified ratio can be considered as a linear programming problem with $n+1$ variables ($f_1, \ldots, f_n, N$) $m$ equality constraints (equation (27)), and $2n$ inequality constraints (equation (15)). The objective function that is to be maximized is $N$.

---

**Calculation of Maximal Angular Acceleration**

In this section the algorithm to calculate the maximal angular acceleration is explained. The solution is almost the same as calculating the maximal joint moment. This problem is a linear programming problem as well. The method to calculate the maximal angular acceleration

- by each DOF of a joint and
- by a group of joints or all joints for a ratio specified by the user

is explained.

---

**Maximal Angular Acceleration by Each Joint's DOF**

The relationship between the angular accelerations and the joint torques is

$$\tau = M\ddot{\theta} + V(\dot{\theta}, \dot{\theta}) + G(\theta)$$  \hspace{1cm} (28)$$
where $M$ is the mass matrix of the system, $V(Ł, Ł̇)$ is the vector of centrifugal and Coriolis terms, $G(Ł)$ is the vector of gravity terms and $Ł̇$ is the acceleration of the joint angles. By substituting (28) into (24), $Ł̇$ can be derived from the muscle forces $f$ as

$$Ł̇ = M^{-1}(Af - V - G) \tag{29}$$

The user should now choose which DOF’s acceleration is to be maximized. To maximize the acceleration of the $j$th DOF, the $j$th row of (29) should be chosen. The equation is used as the objective function for linear programming with muscle constraints (15).

The maximal acceleration of the right hip flexion/extension joint is shown in Figure 13. The posture is a static one, which means that the angular velocities are all zero. The results are visualized using rods. In posture (a) the maximal acceleration for flexion is $311.4\text{ rad s}^{-1}$, and for extension is $443.8\text{ rad s}^{-1}$. In posture (b) the maximal acceleration for flexion is $493.138\text{ rad s}^{-1}$ and for extension is $188.8\text{ rad s}^{-1}$. From these results we can conclude the following.

- The maximal acceleration for backward kicking is larger when the knee is totally extended.
- The maximal acceleration for forward kicking is larger when the knee is flexed.

Maximal Angular Acceleration by Joints in a Specified Ratio

If the user specifies the ratio of all the joint angles by $n$, $Ł̇$ can be rewritten as

$$Ł̇ = Ĝn \tag{30}$$

By another substitution of (29) for (30) we obtain

$$Ĝn = M^{-1}(Af - V - G) \tag{31}$$

Now we have $n+1$ variables ($f_1, f_2, \ldots, f_n, Ĝ$), $m$ equality constraints (equation (31)), and $2n$ inequality constraints (equation (15)). The problem is to calculate the maximum Ĝ, which is obviously a linear programming problem. An example of creating human animation using this algorithm is explained in Section 10.

Maximal Acceleration by End Effectors

The next problem is to calculate how much acceleration the foot can exert in various directions. First the velocity
at any point on the foot can be given as a linear function of the joint angular velocities:

\[ \dot{r} = J \dot{\theta} \]  

(32)

where \( \dot{r} \) is the velocity of some point on the foot in Cartesian space and \( J \) is a matrix, called the Jacobian, which converts the angular velocities of the joints to the linear velocity of a point on the end effector. The differentiation of equation (32) by time yields

\[ \ddot{r} = J \ddot{\theta} + \dot{\theta} \]  

(33)

By substitution of (29) for (33), we obtain

\[ \ddot{r} = JM^{-1}(Af - V)G + \dot{\theta} \]  

(34)

As the direction to maximize the acceleration of the foot is specified, \( \ddot{r} \) can be decomposed to a scalar and normal vector:

\[ \ddot{r} = \beta \left( \begin{array}{c} n_x \\ n_y \\ n_z \end{array} \right), \quad n_x^2 + n_y^2 + n_z^2 = 1 \]  

(35)

where \( \beta \) is the normal of \( \ddot{r} \), and \( (n_x, n_y, n_z) \) is the direction of the acceleration of the end effector. From equations (34) and (35) the following equation is obtained:

\[ \beta \left( \begin{array}{c} n_x \\ n_y \\ n_z \end{array} \right) = JM^{-1}(Af - V - G) + \dot{\theta} \]  

(36)

Here the problem is to maximize \( \beta \). The variables are \( \beta \) (the maximal force) and the components of \( f \) (the force exerted by the muscles). This is again a linear programming problem, with equation (36) as equality constraints and equation (15) as inequality constraints. When no solution is found, this means that the foot cannot exert any positive acceleration in the direction specified.

The results are again visualized using rods and polygons. In Figure 14 the maximum acceleration that can be generated at the ankle in various directions when flexing the hip angle 45° and the knee angle 90° is shown. The feature of the result is that the maximum frontal–backward acceleration is larger than the acceleration toward the sides. It can also be observed that in this posture, greater acceleration can be made toward the front than the back. The angular velocities of the joints are all set to zero in this example. The maximal acceleration values for several directions are shown in Table 2. The axes in the table are defined in the global co-ordinate system shown in Figure 14. The side view of the maximum acceleration rods and polygon when extending the foot is shown in Figure 15. When the knee angle is fully extended, the maximum acceleration polygon is a flat two-dimensional polygon, because the Jacobian matrix \( J \) degrades.

**Maximal Force by End Effectors**

Data on the amount of force that the body’s end effector can exert in various directions are useful in several areas. This section explains the method to calculate this force. There is a relationship between the joint torque \( \tau \) and the force exerted by the end effector, \( F_e \).
The user should specify the normal vector of the force by the end effector, \((f_x, f_y, f_z)\), and the problem is to maximize its norm \(F_r\):

\[
\tau = J^T F_r \quad (37)
\]

Using equation (24) together with equation (38), the following equation is derived:

\[
J^T F_r = Af \quad (40)
\]

Using equation (40) as equality constraints and equation (14) as inequality constraints, the norm of the end effector force, \(F_r\), is maximized.

The results are again visualized using rods and polygons. In Figure 16 the maximum force that can be exerted at the ankle in various directions is shown when flexing the hip angle 45° and the knee angle 90°. The values of the maximal force for several axes are shown in Table 3. The axes are again defined in the global co-ordinate system previously shown in Figure 14. The side view of the maximum force rods and polygon at each configuration in an extension motion of the foot are shown in Figure 17. When the knee angle is fully extended, an infinite force is exertable in the vertical direction because of the degradation of the Jacobian matrix \(J\).

### Simulation and Visualization of Effect of Fatigue

In this section the effect of fatigue on the maximal moment polygon is visualized. The model of fatigue and recovery proposed by Giat et al.\(^{13,14}\) is used to determine the maximal amount of force exertable by the muscles as time passes.

In biomechanical areas it has been revealed that muscles are composed of three types of fibres: fast glycolic (FG), fast oxidative glycolic (FOG) and slow oxidative (SO) fibres. The FG fibres can produce a large force with a low endurance, while the SO fibres have a high endurance but a small maximal force. The type of fibres recruited to exert force is determined by the activation level of the muscle. The SO fibres are recruited first, and as the activation level increases, next the FOG and finally the FG fibres are used to generate the maximal amount of force. When the FG fibres are used, the intracellular pH level inside the muscle
declines and the maximum amount of force exertable by the contractile element decreases.

Giat et al.\textsuperscript{13} observed the intracellular pH level inside the electronically stimulated quadriceps muscle using $^{31}$P nuclear magnetic resonance spectroscopy and obtained the relationship between the intracellular pH and the force exerted by the muscle.

The decay of the pH level during the fatigue phase with time $t$ is calculated by

$$\text{pH}_{\text{F}}(t) = c_1 - c_2 \tanh(c_3(t - c_4))$$  \hspace{1cm} (41)

with constant parameters $c_1$, $c_2$, $c_3$ and $c_4$. The pH during the recovery phase is similarly calculated by

$$\text{pH}_{\text{R}}(t) = d_1 + d_2 \tanh(d_3(t - d_4))$$  \hspace{1cm} (42)

<table>
<thead>
<tr>
<th>Axis</th>
<th>Maximal force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.00, 0.00, 0.00)</td>
<td>70.59</td>
</tr>
<tr>
<td>(−1.00, 0.00, 0.00)</td>
<td>135.44</td>
</tr>
<tr>
<td>(0.00, 1.00, 0.00)</td>
<td>271.8</td>
</tr>
<tr>
<td>(0.00, −1.00, 0.00)</td>
<td>344.63</td>
</tr>
<tr>
<td>(0.00, 0.00, 1.00)</td>
<td>153.09</td>
</tr>
<tr>
<td>(0.00, 0.00, −1.00)</td>
<td>262.38</td>
</tr>
</tbody>
</table>

Table 3. Maximal force at ankle when hip is flexed 45° and knee 90°

---

Figure 15. Side view of maximum acceleration rods and polygon when extending foot

Figure 16. Maximum force at ankle, visualized using rods and translucent polygons
Figure 17. Side view of maximum force rods and polygon when extending foot.
with the constant parameters $d_1$, $d_2$, $d_3$ and $d_4$. The force output was fitted by the function

$$f_{\text{pH}}(\text{pH}) = d_5(1 - e^{d_6(\text{pH} - d_7)})$$

(43)

where $d_5$, $d_6$ and $d_7$ are constant values. Equation (43) is normalized by the force obtained at the beginning of the experiment:

$$f_{\text{pH}}^N(\text{pH}) = \frac{f_{\text{pH}}^{\text{max}}(\text{pH}(t_1))}{f_{\text{pH}}^{\text{max}}(\text{pH}(t_0))}$$

(44)

where $0 < f_{\text{pH}}^N < 1$. The values of the constant parameters defined here are listed in Table 4. The normalized force–pH function $f_{\text{pH}}^N(\text{pH})$ is combined with equation (1) to compose a new dynamic equation of the CE component:

$$f_{\text{CE}} = f(l_{\text{CE}}, v_{\text{CE}}, a) \cdot f_{\text{pH}}^N$$

(45)

The decay and increase of the normalized force during the fatigue and recovery phase are shown in Figure 18. The problem with this model is that the relationship between the activation level and the pH derivation is not included.

<table>
<thead>
<tr>
<th>Function</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH$^F$</td>
<td>$c_1$</td>
<td>6.70</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>0.502</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>0.0406</td>
</tr>
<tr>
<td></td>
<td>$c_4$</td>
<td>30.0</td>
</tr>
<tr>
<td>pH$^R$</td>
<td>$d_1$</td>
<td>6.55</td>
</tr>
<tr>
<td></td>
<td>$d_2$</td>
<td>0.502</td>
</tr>
<tr>
<td></td>
<td>$d_3$</td>
<td>0.0026</td>
</tr>
<tr>
<td>$f_{\text{pH}}$</td>
<td>$d_4$</td>
<td>475.806</td>
</tr>
<tr>
<td></td>
<td>$d_5$</td>
<td>1136</td>
</tr>
<tr>
<td></td>
<td>$d_6$</td>
<td>-0.0097</td>
</tr>
<tr>
<td></td>
<td>$d_7$</td>
<td>6.0934</td>
</tr>
</tbody>
</table>

Table 4. Values of constant parameters for Giat’s fatigue and recovery model

Previous experiments have shown that the FG fibres begin to be recruited when the activation level is above 50% of maximal contraction. Therefore in this research the phase of each muscle is switched between fatigue and recovery according to its activation level. The threshold is set to 0.5.

In this section the maximum moment polygon is observed while the musculotendon model isometrically exerts the maximum hip torque around the x-axis defined in Section 5.1. The isometric torque at the hip joint and the muscle force have been obtained using the maximum moment calculation algorithm explained in Section 5.1. The results are shown in Figures 19 and 20. It is possible to see the effect of fatigue by the shrinking polygon. When the maximum torque to flex the hip to the front is applied, the left side of the polygon mainly shrinks. This means that the maximum moment exertable by the hip joint for flexing the leg to the front decreases as time passes. In contrast, when the maximum torque to extend the hip backward is applied, the right side of the polygon shrinks. The effect of fatigue on the maximum acceleration and force can be observed as well.

**Application to Computer Graphics and Simulation**

This section discusses how properties such as the maximal acceleration or maximal force explained in previous sections can be used for

- creating human motion (for animation use) and
- simple simulation of the human body (for ergonomic, biomechanical and medical applications).

**Simulating Exhaustive Motion**

When creating a fully exhaustive and powerful motion, the maximal acceleration is a useful property to determine
the upcoming motion. In this subsection a kicking motion by the leg is created.

For creating the motion, the ratio of the joint angles at each moment must be specified. This ratio is the vector $n$ in equation (30). Therefore either the trajectory of the motion or at least two postures must be determined. Here we prepared two postures of a kicking motion, one with the leg extended backward as the initial frame, and one with the leg kicked to the front (Figure 21). The ratio vector $n$ is obtained by subtracting the joint angle of the second frame, $\theta_2'$, from that of the first frame, $\theta_1'$:

$$n = \frac{\theta_2' - \theta_1'}{\|\theta_2' - \theta_1'\|}$$

Next, frame 1 is set as the initial frame. The angular velocities of the joint angles are set to zero. We define the joint angles and angular velocities as $\theta_0$ and $\dot{\theta}_0$, and the initial time as $t_0$:

$$\theta_0 = \theta_1'$$
$$\dot{\theta}_0 = 0$$

Using this initial state, the maximal acceleration by the legs, $\ddot{\theta}_0$, is calculated using the algorithm explained in Section 6:

$$\ddot{\theta}_0 = f(\theta_0, \dot{\theta}_0)$$

Then, using $(\theta_0, \dot{\theta}_0, \ddot{\theta}_0)$, the posture at the next time step is obtained:

$$\dot{\theta}_1 = \dot{\theta}_0 + \dot{\theta}_0$$
$$\theta_1 = \dot{\theta}_0 + \theta_0$$

where $\Delta t$ is the time step. This forward integration is repeated until the normal of the velocity vector $\dot{\theta}$ is small enough, i.e. when the kicking leg reaches the maximal height. The time at which the leg reaches this state is defined as $t_f$, while the joint angle and angular velocity at $t_f$ are defined as $\theta_f$ and $\dot{\theta}_f$. At each moment the maximal acceleration is calculated and used as the acceleration of the leg:

$$\ddot{\theta}_n = f(\theta_n, \dot{\theta}_n)$$

$$\dot{\theta}_{n+1} = \dot{\theta}_n + \dot{\theta}_n$$
$$\theta_{n+1} = \dot{\theta}_n + \theta_n$$

The motion obtained in this way can be considered as a quick kick done by the body with full effort. Since neither
path planning nor optimization is done, it is obvious that the algorithm explained here can be applied only to a rather simple and short motion. On the other hand, the computation for the whole motion is accomplished in real time. For example, the calculation of the first kicking motion takes 86 s using a Pentium II 450 MHz processor. The kicking motion was repeated with the musculotendon model to observe the effect of muscle fatigue. The first kicking motion and those after 50, 100 and 150 repetitions are shown in Figure 22. The joint angle and the height of the ankle at $t_f$ are shown in Table 5. $t_0$ for each motion is defined as $t_0=0$. It is possible to observe that the maximal height of the leg at $t_f$ decreases and the time for the leg to reach the maximal height from the initial state increases as the kicking motion is repeated. The obvious fact that a kicking motion by a tired leg is much slower and less powerful compared with that by a vigorous leg has been properly simulated.

**Combination with Other Methods**

The maximal properties can be utilized for other algorithms to create computer animation. First, they are useful in methods which use torque limits for motion path planning, such as in Reference 26. They could also be combined with optimization methods, such as in References 11, 27 and 28, for creating an objective function or constraints for the whole motion. Application to inverse kinematics could also be considered, since it is possible to check whether a given posture can be realized statically or dynamically.

**Discussions**

There are a number of biomechanical factors which are omitted in this research. First, as explained in Section 4, the muscle activation level is not actually a quantity which can take an arbitrary value, but is controlled by the nervous signals from the brain. In fact, the initial activity state of muscles modifies the efficiency of muscle actions during the motion. Even though this factor can be omitted for static or slow simulation, for precise simulation and calculation of maximal properties it must be taken into account.

The way the simplex method maximizes the objective function is not necessarily the way the neuromuscular system achieves that. This is because the muscles are
Figure 22. Kicking motion by musculoskeletal model (a) at first and after (b) 50, (c) 100 and (d) 150 repetitions of motion.
controlled by the CNS. The CNS is a closed-loop system, which is not a one-way controller that just send the pre-programmed nervous signals to the muscles, but accepts a lot of feedback from the peripheral sensors in order to create stable and accurate motion. There are a lot of reflexion systems in the CNS, and the force exerted by one muscle influences those exerted by other muscles. Therefore, for the precise calculation of maximal force/acceleration, some people might think modelling the CNS is inevitable. Research has been done to model such reflexion systems of the muscles. However, since quantitative data necessary to model the CNS neural network have been quite limited, there are still a lot of difficulties to complete such work. The optimal value calculated by the simplex method is at least the convex hull of the actual maximal values by the body. Therefore our results could be considered as an outermost layer of the maximal limit. Such limits are very important factors for simulation of the human body model, and our simulation results in the last section have shown that they are useful and sufficiently precise values for making realistic human animation.

Another problem is that since the geometry of the skeleton and muscle attachments differs from person to person, it is doubtful whether the characteristic pattern of our results is applicable to every human body. Since no precise comparison has been done with a real human body, giving answers to such questions requires further research. On the other hand, the maximal properties explained in this paper might be good factors to evaluate the preciseness of the muscle-based human body model by comparing the calculation results with the measured data from the target human body which one is attempting to model.

### Conclusions and Future Work

This paper has proposed a method to calculate the maximal dynamical abilities of the human body using a musculoskeletal human body model. The calculated values have been visualized using rods and polygons. The obtained results match our experience well and are featured by the inner structure of the human body. The maximal force, torque and acceleration obtained in this research can be used as criteria for strength capability or manipulability of the leg, and as a result, they can also be utilized to develop criteria for path planning of human models in CG.

There are, however, several problems with the calculation of maximal acceleration and force of end effectors to be used in path planning.

- The maximum acceleration of the foot becomes a flat planar polygon at singular points because of the degraded Jacobian.
- The foot can exert an infinite force in the vertical direction when the leg is in a singular configuration, but such a force is not realistic with real human bodies.

To remedy these problems, it is our intention to find other criteria that give an appropriate value for manipulability and dynamic capability of the body.

Creating a model of other parts of the body and calculating the dynamic ability of the whole body are left as future work.

Another important task is to compare the results of our calculation with biomechanically obtained data to validate our method.

### Appendix 1

The method to predict the muscle forces from the joint torques and the profile of the motion is explained here.

The torque applied at joint \( i \), \( \tau_i \), is generated by the muscles crossing the joint:

\[
\tau_i = \sum_j \tau_j \times f_j
\]
where \( r_j \) and \( f_j \) are the moment arm and the force exerted by muscle \( j \) respectively and ‘\( \times \)’ is the outer product operator.

Using inverse dynamics, by specifying the joint angles \( \dot{\theta} \), angular velocities \( \ddot{\theta} \) and angular accelerations \( \dddot{\theta} \), the joint torques can be calculated. However, since the number of muscles crossing joint \( i \) is always greater than the degrees of freedom of the joint, solving \( f_j \) in equation (55) is a redundant problem. An optimization method is applied to determine the muscle force.

Crowninshield and Brand\(^2\) proposed a criterion which is based on the inversely non-linear relationship of muscle force and contraction endurance. Their criterion \( u \) was written in the form

\[
 u = \sum_{i=1}^{n} \left( \frac{f_i}{A_i} \right)^n
\]

where \( f_i = |f_i|, A_i \) is the PCSA of muscle \( i \) and \( n = 1, 2, 3 \) or 100. They compared the calculation results with electromyographic data and reported that \( n = 2 \) gave good results. In this research we set \( n = 2 \).

If at the beginning of the motion the leg is in static phase, we can determine the upper and lower limits of the force exerted by each muscle as explained in Section 4:

\[
f_{\text{min}} \leq f_i \leq f_{\text{max}}
\]

Using equation (55) as equality constraints and equation (57) as inequality constraints, \( u \) can be minimized by quadratic programming. The muscle force \( f_i \) at this moment is obtained at the same time. Using \( f_i \) and \( f_{\text{max}}, f_{\text{min}} \) is calculated. This \( f_i \) is used as \( f_{\text{prev}} \) in the next stage to calculate \( f_{\text{min}} \) by finite differentiation, then \( f_{\text{min}} \) and \( f_{\text{max}} \) can be calculated again at the next stage. By forward repetition of this calculation it is possible to calculate \( f_{\text{min}} \) and \( f_{\text{max}} \) at any stage during the motion.

### Appendix 2

The constant values of the musculoskeletal model are listed in Table 6. See Figure 23 for the definitions of the parameters.

![Figure 23. Definitions of segmental parameters.](image)

Table 6. Segment dimensions and inertial parameters from Reference 21. The centre of mass is modified to match our foot’s local co-ordinate system.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Mass (kg)</th>
<th>Centre of gravity (m)</th>
<th>Principal moments of inertia (kg m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thigh</td>
<td>7.58</td>
<td>( G_T^x = 0.000 )</td>
<td>( I_T^x = 0.126 )</td>
</tr>
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<td></td>
<td></td>
<td>( G_T^y = -0.183 )</td>
<td>( I_T^y = 0.080 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( G_T^z = 0.000 )</td>
<td>( I_T^z = 0.126 )</td>
</tr>
<tr>
<td>Shank</td>
<td>3.75</td>
<td>( G_S^x = 0.000 )</td>
<td>( I_S^x = 0.019 )</td>
</tr>
<tr>
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<td></td>
<td>( G_S^y = -0.188 )</td>
<td>( I_S^y = 0.065 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( G_S^z = 0.000 )</td>
<td>( I_S^z = 0.065 )</td>
</tr>
<tr>
<td>Foot</td>
<td>1.10</td>
<td>( G_F^x = 0.000 )</td>
<td>( I_F^x = 0.008 )</td>
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<td></td>
<td></td>
<td>( G_F^y = 0.0664 )</td>
<td>( I_F^y = 0.008 )</td>
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<td></td>
<td></td>
<td>( G_F^z = 0.0457 )</td>
<td>( I_F^z = 0.002 )</td>
</tr>
</tbody>
</table>

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Appendix 3

The average computation time for each linear programming optimization is listed in Table 7. A Pentium II 450 MHz computer was used for the calculation.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal moment (axis)</td>
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</tr>
<tr>
<td>Maximal moment (ratio)</td>
<td>364:540</td>
</tr>
<tr>
<td>Maximal acceleration (axis)</td>
<td>364:57</td>
</tr>
<tr>
<td>Maximal acceleration (ratio)</td>
<td>413:726</td>
</tr>
<tr>
<td>Maximal force by foot</td>
<td>96:7413</td>
</tr>
<tr>
<td>Maximal acceleration by foot</td>
<td>87:2813</td>
</tr>
</tbody>
</table>

Table 7. Average computation time for each calculation

References


