Computational Analysis of Wall Roughness Effects for Liquid Flow in Micro-Conduits

1 Introduction

In recent reviews of experimental contributions to liquid flow in microchannels (c.f. Gad-el-Hak [1]; Sobhan and Garimella [2]; Koo and Kleinstreuer [3]), it was documented that there are two conflicting bodies of evidence (see Table 1 in [3]). For example, Peng et al. [4,5], Qu et al. [6] and Guo and Li [7] indicated that the friction factor in microchannel flow is augmented when compared to macrochannels, while Sharp [8], Judy et al. [9], Wu and Cheng [10] and Gao et al. [11] claimed that it is unaltered and hence there is no deviation from conventional theory. However, there is measured evidence, even in Sharp’s data, that wall effects may change the flow behavior; still, an enhanced form of the Navier-Stokes equation can be employed to analyze liquid flow in micro-conduits.

In general, as the flow cross section becomes smaller, the surface-to-volume ratio increases and the effects of wall phenomena, such as surface roughness, van der Waals force, and electrokinetic force, may play an important role in the fluid mechanics (c.f. Li [12]; Mala and Li [13]). Focusing on surface roughness impact, Mala and Li [13] and Qu et al. [6] proposed a “roughness viscosity” model, whereas Tichy [14] replaced the roughness surface with a porous film governed by Darcy’s law, while Li and Hwang [15] employed a Brinkman-extended Darcy model for near-wall flow simulations of rarefied gas in long microtubes (Li et al. [16]). Hamrock [17] classified the geometric characteristics of actual surfaces into three categories, i.e., error of form, waviness, and roughness. Waviness stems from relatively long waves in a surface profile, which is associated with unwanted vibrations that typically occur in machine tool systems. Roughness results from irregularities, excluding error of form and waviness, which are inherent in the cutting and polishing processes during production. Both waviness and roughness are regarded as “relative roughness” in the present model.

The varying experimental observations and data analyses are the impetus for the development of a micro-conduit flow hypothesis which correlates actual micro-scale flow behavior with significant effects of relative surface roughness as well as hydraulic diameter uncertainties, in order to provide a flexible model for proper experimental data interpretation.

2 Theory

Surface roughness can be described as random distributions of wall peaks and valleys, which, in the average, could be modeled as a homogeneous porous medium layer (PML) characterized by porosity $\alpha$ (or permeability $\kappa$) and height $h$. Figures 1(a–c) depict the evolution of the PML model from a random wall roughness region (Fig. 1(a)) via an ordered roughness layer (Fig. 1(b)) to a homogeneous porous medium layer (Fig. 1(c)). Specifically, a representative microchannel of half-height (or half-width) $H/2$ is depicted in Fig. 1(c). The nominal value of channel height is determined as the distance between the center of the layer and the center of the channel (Hamrock [17]) This thin porous layer, $h$, generates two phenomena, i.e., generally an enhanced flow resistance when $h$ and $\alpha$ are measurable as well as a reduced channel cross section when $h$ is substantial and $\alpha$ is very small. Clearly, when $h$ is finite, and $\alpha \rightarrow 1$, the surface roughness effect is negligible and the flow area increases slightly. In order to elucidate the capabilities of the proposed PML model on the friction factor, various liquid flow fields in different micro-conduits have been analyzed.

Starting with a generalized transport equation describing fluid flow in any conduit as well as porous medium layer, the systemic equations are derived, assuming steady fully developed flow in 2-D microchannels, microtubes, and micro-journal bearings.

2.1 Modeling Equation. A generalized transport equation (c.f. Kleinstreuer [18,19]) for flow in a porous medium layered micro-conduit can be written as

$$\frac{\partial}{\partial t}(\alpha \rho \Phi) + \nabla \cdot (\rho \vec{u} \cdot \nabla \vec{u}) - \nabla \cdot (\mu \frac{\vec{u}}{\kappa} \cdot \nabla \Phi) = \alpha S$$

(1)

where $\Phi$ is an arbitrary transport quantity, $\alpha$ is the porosity, $\kappa = \alpha \delta$ is an area porosity tensor, $\vec{u}$ is the velocity vector, $\rho$ is the fluid density, $\mu$ is the fluid viscosity, and $S$ is a source term.

Setting $\Phi = 1$, $S = 0$ and $\Phi = \vec{u}$, $S = \vec{R}$, the corresponding continuity and momentum equations are as follows.
Continuity Equation.
\[ \frac{\partial}{\partial t} (\rho \vec{u}) + \nabla \cdot (\rho \vec{u} \vec{u}) = 0 \]  

Momentum Equation.
\[ \frac{\partial}{\partial t} (\rho \vec{u}) + \nabla \cdot \left( \rho (\vec{u} \vec{u}) \otimes \vec{u} \right) - \nabla \cdot \left( \mu \nabla \vec{u} + \nabla \vec{u}^T \right) = -\rho \vec{R} - \rho \nabla \vec{p} \]  

where \( \vec{p} \) is the pressure, and \( \vec{R} \) is the resistance vector to flow in the porous medium layer. The right-hand side of Eq. (3), the first term captures the augmented surface forces induced by wall effects in micro-systems, and the last term is the pressure force acting on the fluid. In the open conduit \( \alpha = 1 \) and \( \vec{R} = 0 \), whereas inside the porous medium layer, \( \alpha \) may vary, \( 0 < \alpha < 1 \), and the resistance vector, \( \vec{R} \), can be represented as
\[ \vec{R} = (R_C + R_F) \vec{u}_d \]  

where \( R_C \) is a resistance constant, \( R_F \) is the resistance speed factor, and \( \beta \) is a “resistance speed power” as summarized in Table 1.

Assuming constant fluid and material properties, Eq. (3) can be written as
\[ \alpha \frac{\partial \vec{u}}{\partial t} + \nabla \cdot ((\alpha \vec{u}) \otimes \vec{u}) = -\alpha \nabla \vec{p} + \frac{\mu}{\rho} \nabla \vec{u} \]  

\[ -\alpha \left( R_C + R_F |\vec{u}| \right) \vec{u} \]  

With fully-developed (parabolic) velocity profiles for 2-D channel flow, the viscous and resistance terms have the same sign, i.e., considering:

\[ \vec{u}(y) = c_1 y^2 + c_2 y + c_3 > 0 \]  

\[ \frac{d^2 \vec{u}}{dy^2} = 2 c_1 < 0 \]  

where the \( c_i \)’s are constants, so that:
\[ \text{sign} \left( \frac{\mu}{\rho} \alpha \nabla \cdot (\nabla \vec{u} + \nabla \vec{u}^T) \right) = \text{sign} \left( -\frac{\alpha}{\rho} R_C \vec{u} \right) \]
\[ = \text{sign} \left( -R_F \vec{u}_d \right) \]  

This implies that the porous medium generates augmented viscous flow effects in the wall layer directly proportional to channel flow rate and porous layer resistance \( R_C = R_C(\alpha, h) \).

Now, in order to readily elucidate the porous medium layer, or roughness, effects in a microchannel and the relative importance of the resistance speed factor, \( R_F \), a simplified version of Eq. (5) was numerically solved, employing MATLAB (MathWorks [20]) with a user-supplied program. Specifically, assuming steady, fully-developed 2-D flow in an isotropic porous medium, Eq. (5) reduces to the 2-D Brinkman-Forchheimer-extended-Darcy equation.
\[ 0 = -\frac{dp}{dx} + \frac{d^2 \vec{u}}{dy^2} - \left( \frac{\mu}{\kappa} + \frac{\rho C_F}{\kappa^2} u^2 \right) \]  

where, with respect to Eq. (5), the L.H.S. is zero, \( \nabla \cdot (\nabla \vec{u} + \nabla \vec{u}^T) \rightarrow \mu d^2 \vec{u}/dy^2 \), \( R_C = \mu / \kappa \), \( R_F = \rho C_F / \kappa^2 \), and \( C_F = 0.55 \) (Nield and Bejan [21]).

In dimensionless form,
\[ 0 = -\frac{dp^*}{dx^*} + \frac{4 d^2 u^*}{dy^2} - \left[ \frac{4}{Da \cdot Re_{Da}} + \frac{C_F}{Da^{1/2}} u'^2 \right] \]  

where \( p^* = p / \rho U_0^2 \), \( u^* = u / U \), \( x^* = x / H \), \( y^* = y / H \), \( Da = \kappa / H^2 \), \( Re_{Da} = 4 \rho U_0 H / \mu \), \( H \) is the half-channel height, \( U_0 \) is the

Table 1 Porous medium constants, coefficients, and parameters (Reprinted from Ref. [3] with written permission from IOP, Bristol, UK)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Typical values/formats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>(Volume) porosity</td>
<td>( 0.0 &lt; \alpha &lt; 1.0 )</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Permeability</td>
<td>( 10^{-4} \text{ cm}^2 &lt; \kappa &lt; \infty )</td>
</tr>
<tr>
<td>( K )</td>
<td>Area porosity tensor</td>
<td>For isotropic porous media, ( K_j = \alpha \delta_{ij} )</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Fluid viscosity</td>
<td>For laminar flow, ( \Gamma = \mu / \rho )</td>
</tr>
<tr>
<td>( R_C )</td>
<td>Resistance constant</td>
<td>( R_C = f(Da, Re) )</td>
</tr>
<tr>
<td>( R_F )</td>
<td>Resistance speed factor</td>
<td>( R_F = f(Da,</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Resistance speed power</td>
<td>( \beta = 1.0 )</td>
</tr>
</tbody>
</table>

Table 2 Typical values of relative surface roughness (\( h / D_n \times 100\% \))

<table>
<thead>
<tr>
<th>Author</th>
<th>Relative surface roughness (%)</th>
<th>Material</th>
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</thead>
<tbody>
<tr>
<td>Pfuiler et al. [22,23]</td>
<td>~1%</td>
<td>Silicon</td>
</tr>
<tr>
<td>Peng et al. [4,5]</td>
<td>~0.6–1%</td>
<td>Silicon</td>
</tr>
<tr>
<td>Mala &amp; Li [13]</td>
<td>~3.5%</td>
<td>Stainless steel/Fused silica</td>
</tr>
<tr>
<td>Papautsky et al. [24]</td>
<td>~2%</td>
<td>Silicon</td>
</tr>
<tr>
<td>Wu &amp; Cheng [10]</td>
<td>~0.12%</td>
<td>Silicon</td>
</tr>
<tr>
<td>Guo &amp; Li [7]</td>
<td>~3–4.3%</td>
<td>Stainless steel</td>
</tr>
<tr>
<td>Xu et al. [25,26]</td>
<td>~0–1.7%</td>
<td>Aluminum, Silicon</td>
</tr>
</tbody>
</table>
Fig. 2 Flowcharts: (a) Flow field solution in a microchannel with constant homogeneous porous wall layers; and (b) Velocity profiles in the annulus between the rotor and the stator (see Fig. 3 for the definition of symbols)
average velocity, $D_h$ is the hydraulic diameter, $C_F$ is a drag coefficient, $\alpha$ is the porosity of the PML in the channel, and $\kappa$ is the permeability where $\kappa = \alpha^2 (1 - \alpha)^{-2}$.

Typical relative surface roughness values for micro-conduits are listed in Table 2. Generally, surface roughness depends on the wall material and machining process, where micromachining (~or~ micro-cutting ! results in higher roughness than etching processes; furthermore, microchannels of stainless steel or aluminum have higher values of surface roughness than those of silicon. For example, Xu et al. [25,26] measured typical roughness heights for etching processes to be 20 nm, which is negligible in most micro-channels where $D_h < 10^{-2} \text{m}$, while a channel milled into an aluminum plate by micro-end-mills produced roughness of about 0.5 $\mu\text{m}$. It shows the dependence of the relative roughness on the channel material and manufacturing method ~see Table 2!.

2.2 Microchannels. For the clear, i.e., porous medium-free region of the channel, Eq. (8) reduces to ($Da \to \infty$):

$$0 = -\frac{dp^n}{dx^n} + \frac{4}{Re_D} \frac{4 d^2 u^n}{dy^n} \left( \frac{1}{2} \right)$$

(9)

Double integration yields:

$$u^n (y^n) = \frac{Re_D}{4} \frac{dp^n}{dx^n} (y^n - \xi^2) + u_{in}$$

(10)

where $\xi$ is the non-dimensionalized coordinate for the open channel-porous medium layer interface, and $u_{in}^n$ is the flow velocity at the interface, which depends on the iterative solution to the PML-flow problem.

The velocity profile in the porous region is obtained by solving Eq. (11), using a boundary-value-problem ordinary differential equation (ODE) solver routine that is provided in MATLAB 6.0. 400 elements were evenly distributed in the porous medium layer and 2,000 elements in the open region.

$$0 = -\frac{dp^n}{dx^n} + \frac{4}{Re_D} \frac{d^2 u^n}{dy^n} - \frac{4}{Da \cdot Re_D} - \frac{C_F}{Da} u^n (1 + 2)$$

(11a)

The boundary conditions are no-slip velocity ($u=0$) at the channel wall, and matching velocity gradients at the interface between clear and porous regions, i.e.,

$$\left. \frac{du^n}{dy^n} \right|_{y^n=\xi} = \frac{Re_D}{4} \frac{dp^n}{dx^n} \xi$$

(11b)

Based on global mass conservation, the velocity gradient at the interface was iteratively obtained using the shooting method by assuming a reasonable pressure gradient for the flow field. After computing the velocity field, the average flow velocity was obtained for each iteration. Comparing it with the given average velocity, the pressure gradient, $dp^n/dx^n$, was adjusted to match the average velocity from each iteration to the given average velocity (cf. Fig. 2). It took about five minutes to obtain a solution per case on a Sun Ultra 60 workstation.

2.3 Microtubes. Equation (11) in cylindrical coordinates reads

$$0 = -\frac{dp^n}{dx^n} + \frac{2}{Re_D} \left( \frac{d^2 u^n}{dr^2} + \frac{1}{r} \frac{du^n}{dr} \right) - \frac{2}{Da \cdot Re_D} - \frac{C_F}{Da} u^n (1 + 2)$$

(12)
subject to the no-slip velocity \( u = 0 \) condition at the wall, and velocity gradient matching at the interface between clear and porous regions, i.e.,

\[
\left. \frac{d u}{d r} \right|_{r^* = \xi} = \frac{\text{Re}_D}{2} \frac{d \rho (r)}{d x} \xi
\]

The velocity profiles can be obtained with the same procedure used for the microchannel (or parallel plate) case.

2.4 Micro-Journal Bearings. Figure 3 shows the schematics for a rotating cylinder in a tube, with roughness layers on both surfaces. The PML thickness was calculated based on the nominal clearance between the rotor and the stator. Eight hundred grid points were placed in both roughness layers. Again, the flow is assumed to be steady, fully developed, and axi-symmetric and can be described by

\[
0 = \mu \left( \frac{d^2 u_\theta}{d r^2} + \frac{1}{r} \frac{du_\theta}{dr} - \frac{u_\theta}{r} \right) - \frac{\mu}{\kappa} (u_\theta - u_R) - \frac{\mu C_f}{\mu C_f} (u_\theta - u_R)^2
\]

where \( u_R \) is the rotor speed, and \( u_\theta \) is the fluid velocity in the roughness layer. In the roughness layer on the stator, \( u_R = 0 \).

3 Results and Discussion

3.1 Model Validations. For large Darcy numbers, the PML is permeable and hence the additional friction effect vanishes so that the flowing liquid experiences a half-channel of \( y^* = 0.51 \), or a fixed tube radius of \( r^* = 1.02 \). In the other extreme case when the Darcy number approaches zero, the PMLs act as solid “coatings” and hence the flow areas are slightly reduced to \( y^* = 0.49 \) or \( r^* = 0.98 \). Figures 4(a) and 4(b) depict a comparison between the modeling results and the exact flow solutions for both cases.

Figure 5 shows comparisons between the PML model predictions and selected experimental results. Specifically, the experimental results of Mala and Li [13], which fall into the region predicted by the PML model, indicate a strong \( f^*(\text{Re}_D) \) dependence (Fig. 5(a)). The experimental data sets of Guo and Li [7] are well matched with the PML model (Fig. 5(b)). Clearly, roughness elements of the 179.8 \( \mu \)m diameter tube have a higher Darcy number when compared to the 128.8 \( \mu \)m diameter tube. The
Fig. 7 Effects of Reynolds number, surface roughness, Darcy number, and Forchheimer drag term on the change in friction factor for microtubular flows

\[ \Delta f(\%) = \left( \frac{f_{\text{measured}} - f_{\text{theoretical}}}{f_{\text{theoretical}}} \right) \times 100 \]

\[ R_{e_p} = \frac{U_D}{v} \]

\[ SR(\%) = \text{relative surface roughness} \]

\[ Da_R = \frac{v}{R} \]

Fig. 8 The effect of Darcy number, Da_R, on the velocity profiles in the gap

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Data for the 179.8 μm diameter tube seems to indicate the effect of laminar-to-turbulent flow transition, when \( \text{Re}_D > 1700 \). They claimed that “the form drag resulting from the roughness is one reason leading to the increased friction factor;” the form drag is captured by the Forchheimer term in the PML model.

### 3.2 Porous Medium Layer Effects

There are several PML effects of interest in terms of porous medium permeability, and height as well as actual flow are influencing the local velocity, friction factor, and torque. It should be noted that changes in friction factor and the influence of the Forchheimer term, \( C_f \text{Da}^{-1/2} \text{Re}^{-1/2} \), are more pronounced in microtubes than in microchannels so that only solutions to systems of sections 2.3 and 2.4 are shown. Specifically, the velocity inside the PML of a microtube is somewhat larger than in the parallel plate case (see Fig. 4 in [3]), and hence the flow resistance is greater in a microtube. Thus, the PML thickness plays a measurable role concerning the type of conduit, impact of the Forchheimer term, and ultimately the flow resistance.

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**Fig. 9** The effect of roughness layer on the torque required to maintain flow field: (a) 10% clearance case; (b) 20% clearance case; and (c) 40% clearance case.

\( f^*(\text{Re}_D) \) data for the 179.8 μm diameter tube seems to indicate the effect of laminar-to-turbulent flow transition, when \( \text{Re}_D > 1700 \). They [7] claimed that “the form drag resulting from the roughness is one reason leading to the increased friction factor;”

**Fig. 10** The effect of clearance width on the torque: \( R \) is the nominal radius of the stator.

**Fig. 11** The effect of the Darcy number difference on the required torque.
3.2.1 Microtube Results. For a given Reynolds number ($R_0 = 2000$) and PML-height $h/R = 0.04$, while neglecting for this test run the Forchheimer term in Eq. (12), the impact of the Darcy number on the axial velocity profile is twofold (Fig. 5). With increasing PML permeability, the velocity profile: (a) flattens in the tube-center region, and (b) migrates into the PML with a profile “cross-over” point at $r^* = 0.7$. Of special interest is the PML-flow effect, with and without the Forchheimer term, on the change in friction factor, i.e.,

$$f^* = \frac{f - f_{\text{theoretical}}}{f_{\text{theoretical}}} \times 100\%$$

where $f_{\text{theoretical}}$ is obtained from conventional theory, i.e.,

$$f_{\text{theoretical}} = \frac{2\tau_{\text{wall}}}{\rho u^2}$$

and $f_{\text{observed}} = f(SR, Da, Re)$ results from our computer experiments, where $SR$ represents relative surface roughness ($SR = h/D_a$). Clearly, the Forchheimer term is important for significant relative surface roughness values and Darcy numbers, i.e., $SR > 0.02$ and $10^{-2} < Da < 1$, even for low Reynolds numbers (Fig. 7). When using only the Brinkman term, $2(Da Re_p)^{-1}$, the Reynolds number has no measurable influence on $f^*$. As already implicit in Fig. 6, the Darcy number affects $f^*$ in a major fashion (see Fig. 7). For example, for $SR = 0.02$, the friction factor increases about 12.7% for low PML Darcy numbers and decreases by 3.6% for high Darcy numbers. The reason is that when $Da \to \infty$ the cross sectional area increases slightly ($r = r_0 + h/2$) while the average velocity is somewhat reduced and hence $f_{\text{observed}} < f_{\text{theoretical}}$.

According to laboratory measurements by Mala and Li [13], the normalized friction constant, $C^* = f_{\text{observed}}/f_{\text{theoretical}}$, increases up to 20% for different Reynolds numbers, up to $Re = 1200$ (Above $Re \approx 1200$, additional effects, such as nonuniform inlet conditions and complex conduit geometries, may come into play [3,8]). Specifically, Mala and Li [13] reported relative roughness values varying from 1.15% for the 152 $\mu$m diameter tube to 3.5% for the 50 $\mu$m diameter tube. For a particular case study, assuming $\kappa = 10^{-3}$, the PML model predicted a 20% higher friction factor, exactly as observed by Mala and Li [13], using their definition of the nominal tube diameter. Data from Qu et al. [6], who were part of the Mala and Li research group, as well as Xu et al. [25,26] showed a measurable dependence of the friction factor on the Reynolds number. This $f^*$ (Re) effect can be captured with the Forchheimer term in Equation (12).

In contrast, Judy et al. [9] measured mostly smaller friction factors than the corresponding theoretical values without any dependence on the Reynolds number. Our PML model can reproduce this observation by means of a high Darcy number; for example, when $Da \to \infty$ their largest friction factor deviation, i.e., $f^* = -15\%$, can be simulated. Clearly, their photographs of the tube cross sections indicate “wavy” surfaces, which imply strong uncertainties with respect to the measured nominal tube diameter and hence possibly larger actual flow areas than observed.

3.2.2 Micro-Journal Bearing. Figure 8 shows the velocity profiles in micro-journal bearings. From the enlarged view of the velocity profile in the roughness layer on the rotor, the fluid is found to have higher velocity, as the PML-Darcy number increases. The effects of surface roughness thickness, Darcy number, and clearance width on the torque is shown in Fig. 9. In contrast to the previous cases, the maximum torque increase and decrease are the same, i.e., around 4% in relative magnitude. However, the surface roughness is calculated based on the clearance width which is much smaller than the hydraulic diameter as previously employed.

Figure 10 shows the effect of gap size on the torque. The change in required torque increases with gap size. The maximum Darcy number to which the required torque decreases also increases with clearance width. Furthermore, the effect of difference in roughness layer thicknesses is shown in Fig. 10. If the roughness layer on the stator is thinner than that on the rotor, the required torque change decreases; if the roughness layer on the stator is thicker than that on the rotor, the required torque change increases. The maximum Darcy number for which the required torque decreases, shifts to the left or decreases with thinner stator PMLs.

The Darcy number effect on the change in torque is shown in Fig. 11. Each line depicts the effect of the stator-side Darcy number on the required torque variation for the fixed Darcy number of the rotor PML. The maximum variation is about 1.5%, which is calculated based on the required torque for the case that the stator-side roughness layer has the same Darcy number as that for the rotor-side roughness layer. The required torque increases when the stator-Da is greater than that of the rotor-Da.

4 Conclusions

Laminar flow of liquids in micro-conduits, such as straight channels, tubes and rotating cylinders, may differ in terms of wall frictional effects, and hence flow rates, when compared to macro-channels. Specifically, some experimentalists claimed that friction factors in microchannels are higher, and perhaps require a new (molecular) modeling approach, whereas others noted that the conventional, i.e., macro-scale theory is adequate to predict all transport phenomena. Clearly, higher uncertainties are associated with micro-scale measurements, and experimental errors may lead to unrealistic interpretations of flow phenomena.

Focusing on steady laminar fully-developed flow of a liquid in different micro-conduits, relative surface roughness and actual diameter (or channel height) variations are captured in terms of a porous medium layer (PML) model. The new approach allows the evaluation of microfluidics variables as a function of PML characteristics, i.e., layer thickness and porosity, uncertainties in measuring hydraulic diameters, as well as the inlet Reynolds number. Specifically, realistic values for the PML Darcy number, $0 < Da < \infty$, relative surface roughness, $0.5 < SR < 2\%$, and actual flow area, $A = A_{n,nominal} + 4\%$, are taken into account to match observed friction factor values in micro-conduits.

The model predictions compared well with measured data sets [7–9,13] where the relative roughness were measured to be significant. Although other surface effects may have influenced the experimental results as well, surface roughness is found to affect the friction factor and hence the flow parameters in relatively rough channels, which are made of aluminum or stainless steel by way of micro-cutting processes. However, further experiments to provide accurate values for the drag coefficient, $C_f$, which depends on the configuration of the surface roughness elements, would be desirable to refine the PML model.

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References