An automated non-contact wall temperature measurement using thermoreflectance

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Abstract

This work presents a novel implementation of wall temperature measurement by thermoreflectance. The implementation relies on imaging light reflected from an interface between a transparent wall and liquid, and processing the resulting digital images to measure the diameter of the resulting light ring. The system is relatively simple to implement and the validation data indicate that the output is nearly linear over a wide range of temperatures. A detailed uncertainty analysis was performed, showing that uncertainties of ±1.1 K or less are possible for measurements in water using the multiple point calibration. Measurement uncertainties of less than ±0.1 K are readily attainable using fluids with high coefficients of thermal expansion.

1 Introduction

The wall temperature is one of the most important, yet most challenging, measurements in the study of convective heat transfer. Several non-intrusive techniques have been developed to obtain the wall temperature, such as embedding a constantin wire at a copper tube surface [1], micro-electromechanical sensors [2, 3], the use of infrared thermography on a very thin wall [4] and the use of temperature sensitive paint [5]. While each of these methods has been shown to produce accurate results, each has inherent limitations due to fabrication, calibration and expense that may preclude their use in some applications.

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A novel, automated, non-contact, optical wall temperature measurement technique has been developed for applications that allow for the use of a transparent sightglass or test section. It is based on the total internal reflection of light from the interface between the liquid and the transparent wall, and the known change of the refractive index of a liquid with temperature. Because of the nature of total internal reflection, the temperature of a very small volume of liquid within 100 nm or less of the wall is sampled, effectively giving the inner wall temperature due to the rapid diffusion of thermal energy over this small distance. The relatively simple apparatus may be easily set up for use on any transparent wall. The uncertainty of the temperature measurements was found to be $\pm 1.1K$ for water using an inexpensive optical setup. This can be improved by a factor of 5 for water by improving the imaging system and by a factor of 10 by using other fluids that have a greater coefficient of thermal expansion.

2 Principles of operation

2.1 Total Internal Reflection and Evanescent Waves

The non-intrusive optical technique used in this research involves the total internal reflection of light at an interface between materials with different refractive indices ($n_1$ and $n_2$). The theory of this method can be derived from Snell’s Law, Eqn. (1).

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t) \quad (1)$$

When $n_1$ is greater than $n_2$, an incident angle $\theta_i$ can be determined that will lead to the transmission angle $\theta_t$ being $90^\circ$. At this point, none of the light is transmitted and all that is not absorbed is reflected at a reflectance angle $\theta_r = \theta_i$. The angle at which this first occurs is the angle of total internal reflection.

The key to this technique is the fact that, when light is totally reflected from the interface between materials of decreasing refractive index, the light enters the lower index material for a small distance through a phenomenon called evanescent waves. When an incident light ray arrives at the interface within approximately $10^\circ$ of the critical angle, the solution to the electromagnetic wave propagation equations shows that waves form in the lower index material that travel parallel to the interface for a distance on the order of the wavelength of the incident light. These waves, called evanescent waves, decay in intensity exponentially with distance from the interface, and
thus they penetrate only 10 to 100 nm of the fluid near the wall. This behavior is described schematically in Fig. 1; for a complete description of this behavior and its applications, the reader is referred to Prieve and Frej [6], Mirabella [7], Yamada [8] and Zettner and Yoda [9].

The nature of evanescent waves in total internal reflection, then, ensures that only a very small region of the fluid near the wall is sampled by the reflected light. First-order conduction and penetration depth analyses suggest that, except for cases of extremely high heat flux or very short time scales (< 100 ns), the fluid within 100 nm of the wall can be assumed to be within 0.01 K of the wall temperature.

2.2 Temperature measurement by thermoreflectance

Fan and Longtin [10] and Chen et al. [11] describe a relatively simple method to measure the temperature at a solid/liquid interface using a so-called thermoreflectance technique. In summary, the index of refraction of a liquid is strongly dependent on its density [12], so, given a liquid’s coefficient of thermal expansion, a direct correlation between the refractive index and the temperature can be established. For water, this correlation, $dn/dT$, has been found to range from $-0.8 \times 10^{-4}$ K$^{-1}$ [13] to $-1.04 \times 10^{-4}$ K$^{-1}$ [14]. Since the coefficients of thermal expansion of most other common fluids, such as refrigerants, coolants and oils, are about 5 times greater than that of water, the $dn/dT$ for these fluids is much larger, leading to greater sensitivity in the temperature measurement [10].

In the method described by Fan and Longtin [10] and Chen et al. [11], a laser probe beam is directed at the interface between a solid and a liquid at a known temperature and the angle is adjusted until total internal reflection is achieved. The intensity of the reflected light is recorded using a photodiode or other optical detector. Due to the change in refractive index of a fluid with temperature, the intensity of reflected light from a liquid/solid interface will change with temperature. The resolution of this technique depends most strongly on the rate of change of the refractive index with temperature, sometimes called the temperature coefficient or the thermo-optic constant. Chen et al. found that they could achieve a measurement uncertainty of about 5 K with water and about 0.5 K with glycerol.
3 Implementation

3.1 Refractive Index Measurement

The measurement setup described by Chen et al. [11] imposes significant constraints on the optical access to the interface, and the accuracy depends on very accurate measurements of the beam angles, which are difficult to execute and may lead to high measurement uncertainty.

Rather than use a single light ray at or near the critical angle, this method uses a light ray striking a diffusing coating on the wall surface opposite the liquid. The light diffuser scatters the impinging light source in all directions. When the scattered light hits an interface at incident angles less than the critical angle, most of the light intensity is transmitted and a small part is reflected. At angles equal to and greater than the critical angle, all of the light is reflected. These behaviors are illustrated in Fig. 1. The reflected light bounces back to the outer wall surface and hits the diffusing material, causing an image of a light ring to form, as shown in Fig. 2. The diameter, $d$, of the light ring can be inferred from Fig. 1,

$$d = 2x = 4t_w \tan \theta_{cwl}$$

(2)

where $x$ is the distance from a point source of light to the first fully reflected light ray, $t_w$ is the wall thickness, and $\theta_{cwl}$ is the critical angle between the wall material and the liquid. The critical angle can be derived by letting $\theta_t$ in Eqn. (1) go to $90^\circ$ and solving for $\theta_i$. In this case,

$$\theta_{cwl} = \sin^{-1} \left( \frac{n_l}{n_w} \right)$$

(3)

where $n_l$ and $n_w$ are the liquid and wall refractive indices, respectively. Based on Eqns. (2) and (3), if the light ring diameter $d$ can be measured accurately and $t_w$ and $n_w$ are known, the liquid index of refraction $n_l$ can be found. Using a known value of $n_l$ at a given temperature and a known value of $dn/dT$, the temperature of the liquid can be determined,

$$T_{meas} = T_o + \frac{n_l - n_o}{\partial n/\partial T}$$

(4)

where $n_o$ is the refractive index at temperature $T_o$.

Automated algorithms for accurately measuring the light ring diameter are described in Shedd and Newell [15] and Pautsch [16]. In this system, the light ring is digitally imaged and a series of image processing steps are applied to the image to isolate the radial location within the ring formed by
the fully reflected light rays. This location is identified through the image processing as the radius where the light intensity increases most rapidly along a radial line moving outward from the light source. The light ring diameter can be determined to sub-pixel resolution by averaging a large number of light ring images.

3.2 Verification Apparatus

The image acquisition system used to demonstrate the temperature measurement method includes a personal computer, a universal system bus (USB) camera to obtain images, a light emitting diode (LED) as the light source, an LED mounting plate, and a camera mount/clamping mechanism. This is shown schematically in Fig. 3. The system requires that the outer wall of the transparent test section be coated with a thin, diffusive coating to diffuse the light source and to allow the reflected light to be visible to the camera. For this experiment, a portion of the tube wall was cut away and a 3 mm thick, 50.8 mm square piece of glass was epoxied over the opening. The outward facing side of the glass, which was obtained from Edmund Industrial Optics, was opalized, or treated to create a uniform light-diffusing layer. Although the exact process for creating this diffusing layer is not known, the resulting surface appears under an optical microscope to be as smooth as the untreated side of the glass.

Water flows through the plastic tube and comes into contact with the 3 mm thick, smooth glass plate with only minor disruptions to the flow. The tube is clamped such that the glass plate is flush with the LED mounting plate and LED, as shown in Fig. 3. The camera views the glass plate from behind the LED through slots in the LED mounting plate (the camera’s view of the light ring can also be seen in Fig. 3). A variable resistor controls the brightness of the LED to enable the intensity of the light to be adjusted for maximum contrast without saturating the brightness of the image.

Figure 4 shows a schematic of the experimental setup used to verify the optical temperature measurement method. A circulating constant temperature bath (Cole-Parmer Polystat 28.4-liter model) was used to maintain a reservoir of clean, filtered water at a constant temperature. Norprene rubber hose with an outer diameter of 19 mm was used to deliver the water to the test section and carry it back to the reservoir through a 5 micron filter. In addition to maintaining clean water, the filter provided sufficient back pressure to keep the test section completely full of water at all times. The rubber hose and filter were wrapped with 13 mm thick fiberglass insulation. The test section was made of 15.1 mm inner diameter PVC tubing, modified...
as described above so that one 50.8 mm by 10 mm section of the tube wall was replaced by the opalized glass plate.

To ensure accurate temperature measurement of the water temperature at the optical measurement location, two thermistors (YSI, Inc.), were installed on either side of the camera mount. The thermistors had a flat, circular measurement surface 10 mm in diameter, which was placed parallel to the flow and in the center of the tube to obtain an averaged water temperature. The thermistor resistances were measured with an Agilent 34970A data acquisition system, and each thermistor was calibrated in the constant temperature bath with a NIST-traceable platinum RTD with an uncertainty of 0.01 K. The calibrated thermistors agreed with the RTD to within ±0.02 K. The average of the two thermistor measurements was used as the reference temperature to compare with the optical measurement.

4 Validation results

To use Eqn. (4), the index of refraction of the liquid needs to be measured. Figure 5 gives the variation of the measured refractive index with temperature determined using the apparatus described above. An automated machine vision program implemented the calculations necessary to solve Eqns. (2) and (3) for the refractive index. From the slope of this plot, $dn/dT = -0.966 \times 10^{-4} \text{ K}^{-1}$, which agrees quite well with data obtained by previous investigators for water. Note that this calibration would be required for any new fluid to be measured using this method.

The plot in Fig. 6 shows the variation in the measured light ring with temperature. The relationship is very linear over the range investigated. Equation (2) is not linear for large changes in $n_l/n_w$ due to the inverse sine function required to determine $\theta_{cwl}$, but for the small variations in $n_l$ that are seen over this temperature range, the non-linear terms of Eqns. (2) and (3) will not have a major impact on the ring diameter.

The slope of the ring diameter plot indicates that the diameter changes by 0.0098 mm for every degree of temperature change. Thus, an uncertainty of ±0.1 K can be achieved if the light ring diameter can be measured to within ±0.98 µm. As shown by Shedd and Newell [15], this can be achieved by improving the resolution of the image acquisition system and by using the average of multiple images.
5 Calibration and Uncertainty

5.1 Multiple-point calibration

One method of calibrating this system is to generate a curve similar to that in Fig. 6 by obtaining ring diameter measurements for the working fluid at known temperatures spanning the expected temperature measurement range. The total temperature measurement uncertainty may be estimated by combining the uncertainties of the temperature and diameter measurements with the RMS deviation of the calibration data points from the curve fit of the data. For the validation data above, the temperature measurement uncertainty was $\Delta T = 0.02 \, \text{K}$, the ring diameter uncertainty was $\Delta d = 0.01 \, \text{mm}$ and the RMS variation of the curve fit to the data by the method of least squares was $\Delta_{RMS} = 0.49 \, \text{K}$. Using the accepted approach for the propagation of uncertainty [17],

\[
\Delta_{tot} = \left[ \left( \frac{\partial T_{meas}}{\partial T} \right)^2 \Delta_T^2 + \left( \frac{\partial T_{meas}}{\partial T_{RMS}} \right)^2 \Delta_{RMS}^2 + \left( \frac{\partial T_{meas}}{\partial d} \right)^2 \Delta_d^2 \right]^{1/2} \quad (5)
\]

\[
\Delta_{tot} = \left[ \Delta_T^2 + \Delta_{RMS}^2 + (-101.6)^2 \Delta_d^2 \right]^{1/2} \quad (6)
\]

\[
\Delta_{tot} = 1.1 \, \text{K} \quad (7)
\]

This rather large uncertainty can be improved by using a camera with a higher resolution and additional samples for determining the light ring diameter. For example, if $\Delta d$ were improved to 0.005 mm, which is well within the capability of the experimental setup described above, $\Delta_{tot}$ will fall to 0.51 K.

5.2 Single-point calibration

It is also possible to calibrate this measurement system using only one diameter, $d_o$, measured at a known temperature $T_o$. At each unknown temperature, then, the difference, $(d_{meas} - d_o)$, can be used with Eqn. (2) to find the change in $\theta_{cut}$. Equation (3) gives the new critical angle and the measured temperature can be found using Eqn. (4). The total uncertainty in this case must be propagated through each of these equations and the non-linear terms make this difficult to do analytically. Instead, the Engineering Equation Solver (EES) software package can be used to perform the error propagation numerically with its built-in uncertainty propagation function [18]. Details for a specific case using water, $T_o = 293.2 \, \text{K}$ and $T_{meas} = 329.9 \, \text{K}$ are given in Table 1. The total uncertainty for this case is
±3.80 K (this is the sum of the “Measurement Uncertainty” column of the table). It is clear from this table that the uncertainty in $dn/dT$, $\Delta_{dn/dT}$, dominates the total measurement uncertainty at this condition.

The impact of $\Delta_{dn/dT}$ on $\Delta_{tot}$ becomes more pronounced as the $T_{meas}$ gets further from the calibration temperature $T_o$. The top three curves in Fig. 7 show the variation in $\Delta_{tot}$ with temperature as a function of $\Delta_{dn/dT}$, with $T_o = 293.15$ K and all of the other measurement uncertainties listed in Table 1 held constant at the listed values. The top curve, which describes the variation in uncertainty for $dn/dT = -0.0001 \pm 0.00001$, shows that the total uncertainty remains at approximately 10% of the temperature difference $T_{meas} - T_o$. If $\Delta_{dn/dT}$ can be decreased by an order of magnitude, the total uncertainty also decreases by about an order of magnitude.

Fluids with larger coefficients of thermal expansion, such as oils, coolants or refrigerants, may be measured with better accuracy using the single-point calibration, as shown by the bottom two curves in Fig. 7. For a fluid such as a refrigerant with $dn/dT \approx -0.005$, the temperature may be measured to better than 0.1 K uncertainty for a range within 20 K of $T_o$.

### 5.3 Uncertainty in the wall thickness

The remaining significant source of uncertainty identified in Table 1 that has not been discussed is that of the wall thickness, $t_w$. It has been assumed for this work that the wall thickness can be determined to within 0.010 mm at $T_o$. If the thermal expansion of the wall material is significant, this could add to the uncertainty of the temperature measurement at temperatures that are quite different from $T_o$. However, the coefficients of linear expansion of nearly all glasses are below $0.010 \text{ mm/mm-K}$ [19]. If the tube wall is constrained in such a way that the wall thickness expands at this rate with temperature, a 5 mm thick wall would expand at a rate of $5 \times 10^{-5}$ m/K, or 0.005 mm for a wall temperature that is 100 K greater than $T_o$, which is within the assumed wall thickness uncertainty. Thus, uncertainty due to thermal expansion of the wall should not affect the analysis as presented.

### 5.4 Wall refractive index variations with temperature

As is clear from Eqn. (3), the refractive index of the wall material is significant to the determination of the liquid temperature using this system. If the thermo-optic constant for the wall material ($dn_{w}/dT$) is large, the value of $n_{w}$ may change significantly over the temperature range of interest. However, $dn_{w}/dT \approx 0.02 \times 10^{-4}$ K$^{-1}$ for the soda lime glass used in this
experiment [20]. Typical values for many common glasses range from \( \approx 0.01 \) to \( 0.05 \times 10^{-4} \text{ K}^{-1} \) [20, 21]. Thus, the variation due to the temperature-induced change in wall refractive index for temperature changes of 20 to 50 K from the calibration point has already been included in the uncertainty analysis performed here (i.e., an uncertainty in \( n_w \) of \( \pm 0.0001 \) was assumed).

6 Summary

This work has presented a novel implementation of wall temperature measurement by thermoreflectance. This implementation differs from previous optical wall temperature measurements in that it relies on imaging light reflected from an interface between a transparent wall and liquid, and processing the resulting digital images to measure the diameter of the resulting light ring. The system is relatively simple to implement and the validation data indicate that the output is nearly linear over a wide range of temperatures. A detailed uncertainty analysis was performed, showing that uncertainties of \( \pm 1.1 \text{ K} \) or less are possible for measurements in water using the multiple point calibration. This is significantly lower than the uncertainty reported in previous studies. Improved uncertainty in the light ring measurement will improve the multi-point calibration uncertainty; the light ring measurement accuracy can be easily improved using a higher resolution imaging system and additional images to be averaged into each temperature measurement. Measurement uncertainties of less than \( \pm 0.1 \text{ K} \) are readily attainable using fluids with high coefficients of thermal expansion.

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References


Table 1

Table 1: Single point calibration uncertainty propagation for $T_o = 293.2$ K, $T_{meas} = 329.9$ K and total uncertainty $\Delta_{tot} = \pm 3.8$ K.

<table>
<thead>
<tr>
<th>Partial Derivative</th>
<th>Value of Derivative</th>
<th>Typical Value</th>
<th>Absolute Uncertainty</th>
<th>Measurement Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial T_{meas}}{\partial (d_{meas} - d_o)}$</td>
<td>$-93446 , \frac{K}{m}$</td>
<td>$-0.0004$ m</td>
<td>$1 \times 10^{-5}$ m</td>
<td>0.23 K</td>
</tr>
<tr>
<td>$\frac{\partial T_{meas}}{\partial (n/k)}$</td>
<td>$367261 , \frac{K}{K^\circ}$</td>
<td>$-0.0001 , K^{-1}$</td>
<td>$1 \times 10^{-5} , K^{-1}$</td>
<td>3.6 K</td>
</tr>
<tr>
<td>$\frac{\partial T_{meas}}{\partial n_{meas}}$</td>
<td>$352$ K</td>
<td>1.4800</td>
<td>$1 \times 10^{-4}$</td>
<td>0.038 K</td>
</tr>
<tr>
<td>$\frac{\partial T_{meas}}{\partial n_o}$</td>
<td>$-360$ K</td>
<td>1.3350</td>
<td>$1 \times 10^{-4}$</td>
<td>0.038 K</td>
</tr>
<tr>
<td>$\frac{\partial T_{meas}}{\partial T_o}$</td>
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<td>20.00 K</td>
<td>$2 \times 10^{-2}$ K</td>
<td>0.00 K</td>
</tr>
<tr>
<td>$\frac{\partial T_{meas}}{\partial m}$</td>
<td>$-11327 , \frac{K}{m}$</td>
<td>0.003300 m</td>
<td>$1 \times 10^{-5}$ m</td>
<td>0.34 K</td>
</tr>
</tbody>
</table>
Figure 1

Figure 1: Schematic of light rays diffusing from light source and reflecting back by total internal reflection.
Figure 2

Figure 2: Illustration of how the light ring image forms around the light source.
Figure 3

Figure 3: Schematic of the imaging setup.
Figure 4

Figure 4: Schematic of the experimental setup.
Figure 5: Measured variation in the refractive index of water with temperature.

\[ n = 1.3638 - 0.0000965999 \cdot T[K] \]
Figure 6: Measured variation in the light ring diameter with temperature.

The relationship can be described by the equation:

\[ T \text{ [K]} = 2889.2 - 101.636 \cdot D \text{ [mm]} \]
Figure 7: Variation in the total single-point calibration uncertainty, $\Delta_{tot}$, as both the uncertainty in $dn/dT$ and the fluid properties are varied ($dn/dT = -0.0001, -0.001, -0.005$).