“Unfairly Linear Signatures”

Adam Gibson

June 6, 2018

Outline

Commitments Key properties (Hash, Pedersen)

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ECDSA multisig With Paillier; adaptor

Commitments - 1

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This way Alice lost in a fair game.

Commitments - 2

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Can get the same effect using Elliptic Curve points, or numbers $\in \mathbb{Z}_N$, instead of hash functions. Add randomness and use hardness of (elliptic curve) discrete log.

Pedersen commitment

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But what happens to **hiding** and **binding** if something is up my sleeve?

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Pedersen commitments suffer from non-perfect binding as shown; but are **perfectly** hiding for the same reason.

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- Best we can do? One perfect, one computational
- Pedersen are perfect hiding (see previous slide)
- If you want perfect *binding*, cannot use compression (function not injective)

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$C_{\mathbf{x}} = rH + x_1 G_1 + x_2 G_2 + \ldots x_n G_n$

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Zero Knowledge Proof of Knowledge

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Game ends with Bob verifying $sG ?= R + eP$

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So: k protects Alice, e protects Bob; but extra interaction step $\rightarrow$ Alice "wins" the game without even opening the commitment!

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The description of a "Sigma protocol" in the previous *was* exactly the "Schnorr's Identity Protocol" - a method of proving knowledge of a private key corresponding to a public key P in the discrete log setting. This is all very nice but ... is it really secure?

ZKPOK - Definitions

A Zero Knowledge Proof of Knowledge must have 3 characteristics:

Completeness

If I know the secret, I can provide a valid proof

Soundness

If I don't know the secret, I can't.

Zero-Knowledgeness

My proof reveals nothing other than the **single bit** of information that I know the secret.

Soundness

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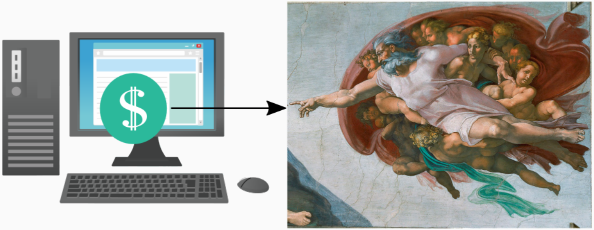
P commits; V branches the Universe and challenges in both; P responds in both.

Soundness - 2



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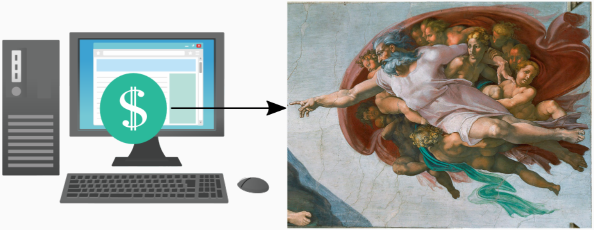

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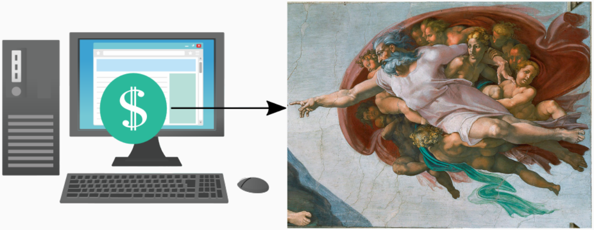

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Works due to k-reuse. The cheating verifier is called an Extractor.

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This "proves" that zero information is conveyed, if the distribution of fake transcripts is indistinguishable from the distribution of genuine ones.

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- But - random oracle and zero knowledgeness?

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Choose $s, e \leftarrow \$$; program RO to output e when input is $sG - eP$; give (R, s) to V.

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- "Elliptic Curve Discrete Logarithm Problem"
- It can be shown that: if an attacker can extract the private key from a Schnorr signature, they can also solve the ECDLP

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$P(\text{success}) \simeq \epsilon^2$; success $\Longrightarrow$ extract discrete log x.

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Digital signature security

- Previous slide(s) only discuss security of scheme against a "total break" - that is to say, the exposure of the private key from the signature.
- But there is also *security against forgery*; in particular we'd like **security against existential forgery under chosen message attack**
- In English - no matter how many signatures you get me to output for a bunch of messages you maliciously choose, you can't create your own *new* signature on a new message without my key.

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No linearity (especially over nonces due to funky use of x-coordinate).

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- Insecure! But manner of insecurity requires thinking about *interaction*

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Maintain ability to validate using *only the aggregate key* while being safe from key subtraction.
See Musig paper https://eprint.iacr.org/2018/068 for details.

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- https://lists.linuxfoundation.org/pipermail/bitcoin-dev/2018-May/015951.html

Aggregation schemes - 3

Good summary of key facts at https://blockstream.com/2018/01/23/musig-key-aggregation-schnorr-signatures.html

- Break history of coins using atomicity of: spend a coin $\leftrightarrow$ reveal a secret

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- Schnorr + scriptless scripts (Poelstra); better overall features

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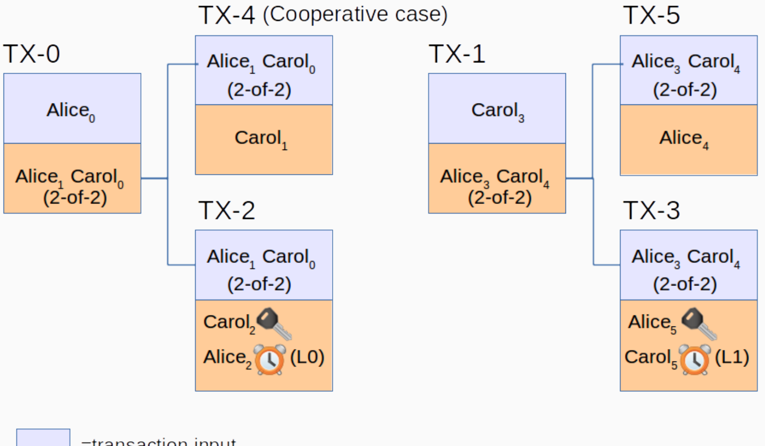

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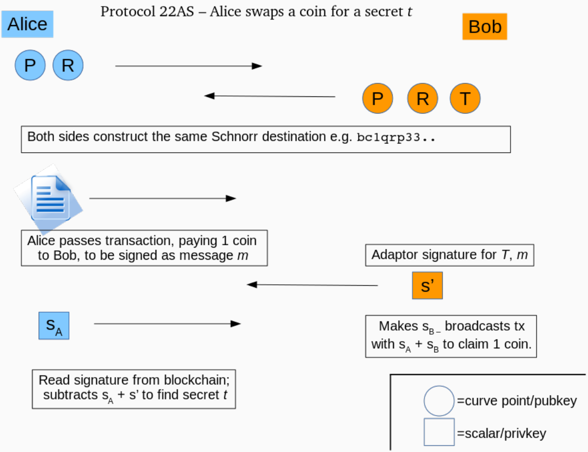
- Give $s' = k + H(m|R + T|P)x$ as incomplete **adaptor signature**

Adaptor signatures - 1

- Embed a secret in the nonce; from $s = k + H(m|R|P)x$ to $s = k + t + H(m|R+T|P)x$
- Share T as "hash" of secret
- Give $s' = k + H(m|R+T|P)x$ as incomplete **adaptor signature**
- Verifiable; you know it'll be a valid sig if you get preimage of T

A new way to swap a coin for a secret:



Adaptor sigs - 3

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4. There are *2* adaptor sigs with same T

5. When Alice claims her coins, the sig reveals t and Bob completes

Adaptor sigs - 4

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5. When Alice claims her coins, the sig reveals t and Bob completes
6. More details at https://joinmarket.me/blog/blog/flipping-the-scriptless-script-on-schnorr/
7. Huge advantage in deniability: any sig could be adaptor; Schnorr musig is 1 key

Adaptor sig in ECDSA

Recent work Malavolta et al

https://eprint.iacr.org/2018/472

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$(E(A) + E(B) = E(A + B))$

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We can recreate adaptor signatures in the above model

Adaptor sig in ECDSA - 2

Original note at https://lists.linuxfoundation.org/pipermail/lightning-dev/2018-April/001221.html

Original note at https://lists.linuxfoundation.org/pipermail/lightning-dev/2018-April/001221.html

1. Share keys, nonce points P, R, Alice sends encrypted privkey $E(x_A)$

Adaptor sig in ECDSA - 2

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1. Share keys, nonce points P, R, Alice sends encrypted privkey $E(x_A)$
2. ECDH shared nonce $R = k_A k_B G$; x-coord r

Adaptor sig in ECDSA - 2

Original note at https://lists.linuxfoundation.org/pipermail/lightning-dev/2018-April/001221.html

1. Share keys, nonce points P, R, Alice sends encrypted privkey $E(x_A)$
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3. Bob: $E(k_B^{-1} H)$, $x_B r k_B^{-1} E(x_A)$, add under enc
4. Alice: $k_A^{-1}\left(k_B^{-1}\left(H + x_A x_B r\right)\right) = s$

Adaptor sigs in ECDSA - 3

Previous slide - interactive 2 of 2 multisig for ECDSA with 1 published key – cool!

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Although it did miss tech. details - don't do that!

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How to add in adaptor (T?)

Bob tweaks his $R_B = k_B G$ to $R_B^* = k_B t G$

Needs to send PoDLE

Adaptor sigs in ECDSA - 4

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$E(adaptor) = E(s')$

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Alice decrypts and verifies s'

Alice returns $s'' = s' \times k_A^{-1}$

Bob publishes (r, s) where $s = s'' \times t^{-1}$

Alice gets $t = s'' \times s^{-1}$ from on-chain sig

Other interesting things

- Ring signatures - $s_i = k_i + \mathbb{H}(R_{i-1}| \ldots)x_i$
- AND and ORs of Sigma Protocols
- General ZKP systems - zkSNARKs, Bulletproofs, others
- Blinded Schnorr signatures

Thank you

Contact info:

waxwing (freenode IRC, reddit)

@waxwing__ (twitter)

https://github.com/AdamISZ

A blog: https://joinmarket.me/blog/blog (email in /about-me)

gpg: 4668 9728 A9F6 4B39 1FA8 71B7 B3AE 09F1 E9A3 197A